Formulaire Dynamique des structures

Oscillateur simple, 1 D.D.L

Notations

$\omega = \sqrt{\frac{K}{M}} \quad \xi = \frac{C}{2\omega M} \qquad \omega_D = \omega\sqrt{1-\xi^2}$

Ocillateur simple, libre , non amortie (C=0)

$$u(t) = (A_1.\cos\omega\, t + A_2 \sin\omega t) = A\cos(\omega\, t + \varphi) = \frac{V_0}{\omega}\sin\omega t + u_0\cos\omega t =$$

$$\sqrt{u_0^2 + \left(\frac{V_0}{\omega}\right)^2}\cos\left(\omega\, t - arctg\left(\frac{V_0}{\omega.u_0}\right)\right)$$

Oscillateur simple, libre , amortie

- Amortissement critique $\xi = 1$
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    - $u(t) = [u_0 + (\dot{u}_0 + \xi\omega u_0).t]e^{-\xi\omega t}$
- Amortissement subcritique $\xi < 1$
    - $u(t) = e^{-\xi\omega t}(A_1.\cos\omega_D t + A_2\sin\omega_D t)$
      $= e^{-\xi\omega t}.A\cos(\omega_D t - \phi)$

      avec $A = \sqrt{u_0^2 + \left(\frac{V_0+\xi\omega u_0}{\omega_D}\right)^2} \qquad \phi = arctan\left(\frac{V_0+\xi\omega u_0}{\omega_D}/u_0\right)$
- Amortissement supercritique $\xi > 1$
    - $u(t) = e^{-\xi\omega t}(A_1.\cosh\omega_D t + A_2\sinh\omega_D t)$
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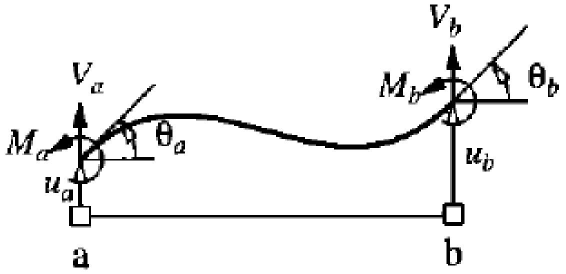
Oscillateur simple, forcé , amortie, sous sollicitation harmonique

$$u(t) = (A\cos\omega_D t + B\sin\omega_D t)e^{-\xi\omega t} + U\cos(\varpi t - \phi)$$

$$u(t) = (A\cos\omega_D t + B\sin\omega_D t)e^{-\xi\omega t} + U\cos\varpi t\cos\phi - U\sin\varpi t\sin\phi$$

$$U = \frac{P_0}{K}\left[\frac{1}{(1-\beta^2)^2 + (2\xi\beta)^2}\right]^{\frac{1}{2}} \qquad R_d = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}}$$

$$R_f = \frac{F_{dyn}}{F_{st}} = \frac{x_{max}\sqrt{k^2 + \omega^2 c^2}}{k.x_{st}} = \frac{x_{max}}{x_{st}}\frac{\sqrt{k^2 + \omega^2\frac{k^2}{\omega_n^2}}}{k} = \frac{\sqrt{1+\omega^2 c^2}}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} = \frac{\sqrt{1+(2\xi\beta)^2}}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}}$$



(M = *moments* ; V = réactions)

$(\theta_a, \theta_b = rotations \; ::: \; u_a, u_b = déplacements)$

$$M_a = \frac{4EI}{L}\theta_a + \frac{2EI}{L}\theta_b + \frac{6EI}{L^2}u_a - \frac{6EI}{L^2}u_b$$

$$M_b = \frac{2EI}{L}\theta_a + \frac{4EI}{L}\theta_b + \frac{6EI}{L^2}u_a - \frac{6EI}{L^2}u_b$$

$$V_a = \frac{6EI}{L^2}\theta_a + \frac{6EI}{L^2}\theta_b + \frac{12EI}{L^3}u_a - \frac{12EI}{L^3}u_b$$

$$V_b = -\frac{6EI}{L^2}\theta_a - \frac{6EI}{L^2}\theta_b - \frac{12EI}{L^3}u_a + \frac{12EI}{L^3}u_b$$