Ben Orlin's "Just-so Stories of Mathematical Scaling"

"Fables and math have a lot in common. Both come from dusty, moth-eaten books. Both are inflicted upon children. And both seek to explain the world through radical acts of simplification. […] By exaggerating a few features and neglecting all the rest, they help explain why or world is the way it is." Ben Orlin

What about you? Can you explain the following statements?

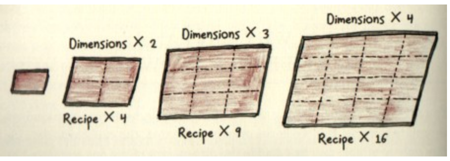
1. Big pans make better brownies. **Right.**	**2. Chares, the sculptor of the Colossus of Rhodes, took his own life before his masterpiece was finished.** **Right.**
3. Giants exist. **Wrong.**	**4. No ant fears heights.** **Right.**

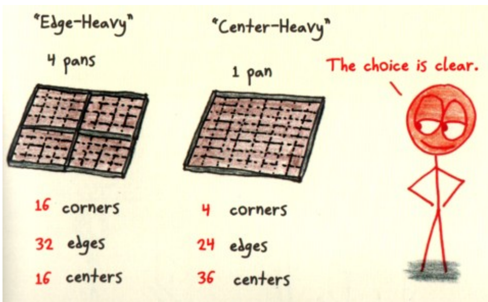
1. Why do big pans make better brownies?

You and I are baking. But when the oven is preheated, the cupboards defy us. The only available pan has double the dimensions of the one suggested by the cookbook. We adjust the recipe and when we have just finished a forgotten cupboard reveals the pans we'd been looking for all along. We blame each other, then laugh, because who can stay mad when chocolate glory is so close at hand?

We now face a choice: **Shall we bake the brownies in the one big pan, or in the small ones?**

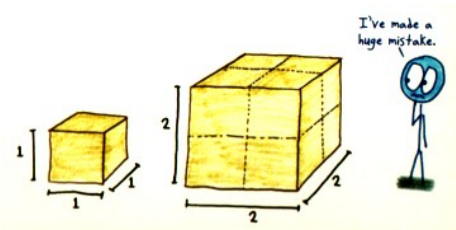
This is a fable, so we shall ignore the details. Forget over temperature, cooking times, heat flow, and minimizing the dishes to clean. Focus instead on one matter: size itself.

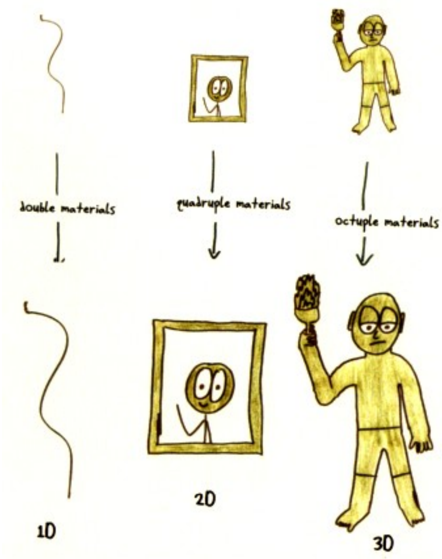


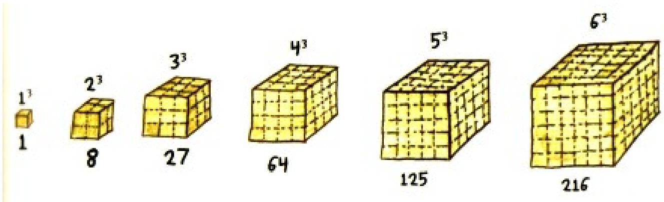


2. Why did the ambitious sculptor take his own life?

About 2300 years ago, the Greek island of Rhodes repelled an attack from Alexander the Great. In a mood of self-congratulation, the people commissioned local sculptor Chares to build a magnificent commemorative statue. Legend tells that Chares originally planned a 50-foot bronze sculpture. "What if we supersize it?" said the Rhodians. "You know, double the height? How much would that cost?"
"Double the price, of course," said Chares.
"We'll take it!" said the Rhodians.
So, why did Chares take his own life before the masterpiece was finished?



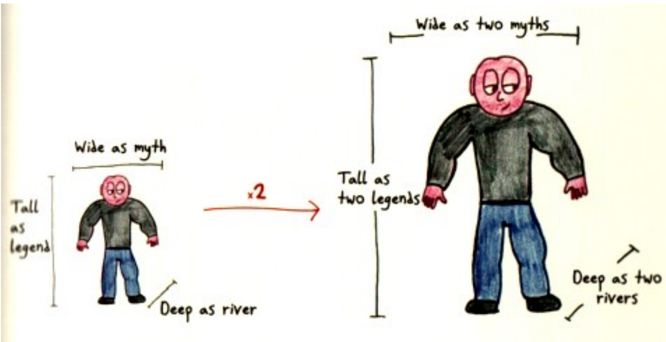


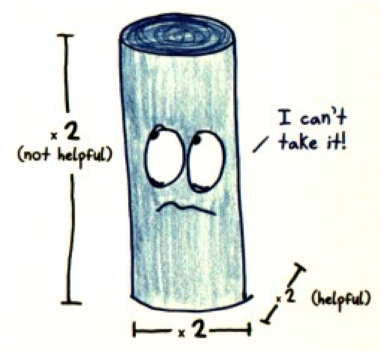


Multiply the length by r, and you multiply the volume by r^3.

3. Why aren't there giants?

King Kong, the three-story ape. Paul Bunyan, the lumberjack whose footsteps carved out lakes. Shaquille O'Neal, the mythical seven-foot one, 325-pound basketball player who could do everything except make three throws. You know these stories, and you know just as well that they are fantasies, legends, wide-eyed fictions. **There's no such thing as giants. Why?**



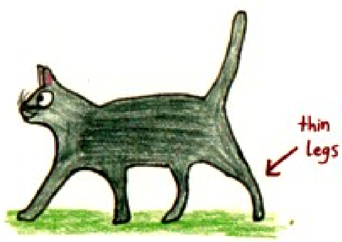


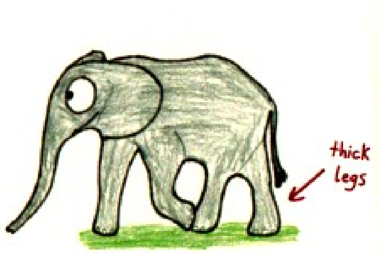
Suppose we take exemplary human specimen Dwayne Johnson and double his dimensions. Now with double the height, double the width, and double the depth, Dwayne's total body mass has grown by a factor of eight. But take a glance at his legs. To keep our man standing, his bones will need eight times the strength. Can they muster it?

They've undergone two helpful doublings (in width and depth) but one useless doubling: length. Just as you don't reinforce a column by making it taller, you can't bolster a leg by making it longer. Extra length confers no extra strength, just extra strain, as the bone's base must now support the greater mass above. Dwayne's leg bones won't keep pace with the demands placed on them: A factor of four can't match a factor of eight. Eventually, Dwayne Johnson will reach a critical breaking point. His limbs will buckle and splinter beneath the overwhelming weight of his torso.

We're dealing with a process called *isometric scaling*: growing a shape while preserving its proportions. It's a lousy method for making big animals. Instead, we need *allometric scaling*: growing a shape while altering its proportions accordingly.

When an animal's height grows by 50%, its legs can keep pace with the strain only by growing 83% thicker. That's why cats can survive on slender limbs, whereas elephants need pillars to stand on.

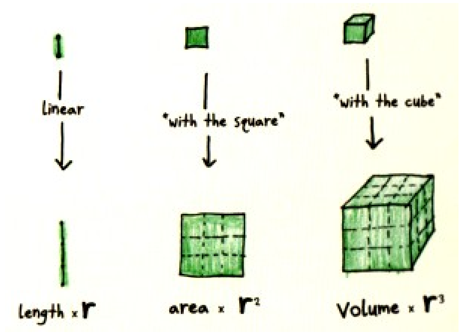


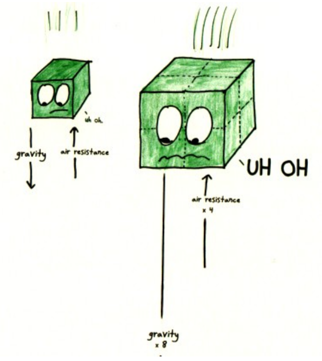


4. Why does no ant fear heights?

Ants are horrifying. They lift objects 50 times their body mass, work together in flawless coordination, and thrive on every corner of the planet. This global army of weightlifting pincer-jawed telepaths outnumbers humans by a factor of millions. Visions of their alien faces would keep me up at night if not for one redeeming fact: Ants are very, very small.

So, why does no ant fear heights? And what could they actually fear?





When a shape's length grows, its surface area grows faster, and its volume grows faster still. This means that big shapes (like humans) are "interior-heavy." We have lots of inner volume per unit of surface area. Small shapes (like ants) are the opposite: "surface-heavy." Which means they never need to fear heights.

When you fall from a great height, two forces play tug-of-war: downward gravity and upward air resistance. Gravity works by pulling on your mass, so its strength depend on your interior. Air resistance works by pushing against your skin, so its strength depends on your surface.

In short, your mass speeds up your fall, while your surface area slows it down. That's why bricks plummet and paper flutters, why penguins can't fly and eagles can.

You and I are like penguins: lots of mass, not much surface area. When falling, we accelerate to a top speed of nearly 120 miles per hour, which is a rather disagreeable way to hit the ground.

By contrast, ants are like paper eagles: lots of surface area, not much mass. Their terminal velocity is just 4 miles per hour.

So is being surface-heavy all fun, games, and parachute-free skydiving? Of course not. Ants have woes, too, and by "woes" I mean a "debilitating and wholly justified fear of water."

The trick is surface tension. Water molecules love sticking together, and they're willing to defy small amounts of gravity to do it. Thus, when any object emerges from a bath, it carries with it a thin layer of water, about half a millimeter thick, held in place by surface tension. For us, that's a trifle: a pound or so, less than 1% of our body weight. But for an ant, the clinging water outweighs its body by an order of magnitude; a single wetting can be fatal.