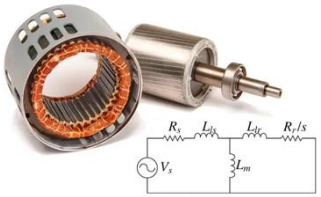
Polytech Orléans – M1 AESM



Electrical Engineering





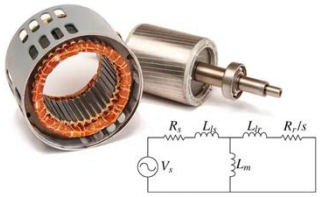


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Year 2026/2027

Original material: Emmanuel BEURUAY
English version & complementary material: Thomas TILLOCHER



Introduction

Schedule

	M1 AESM
Lectures	9 x 1 h15 + 2 evaluations
Tutorials	8 x 1,25 h
Lab work	1 x 2,50 h + 6 x 3,75 h

Outline

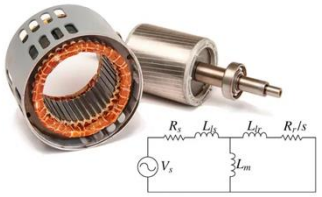
Introduction

I) Reminders (electricity)

II) Power in sinusoidal regime (single-phase and 3-phase)

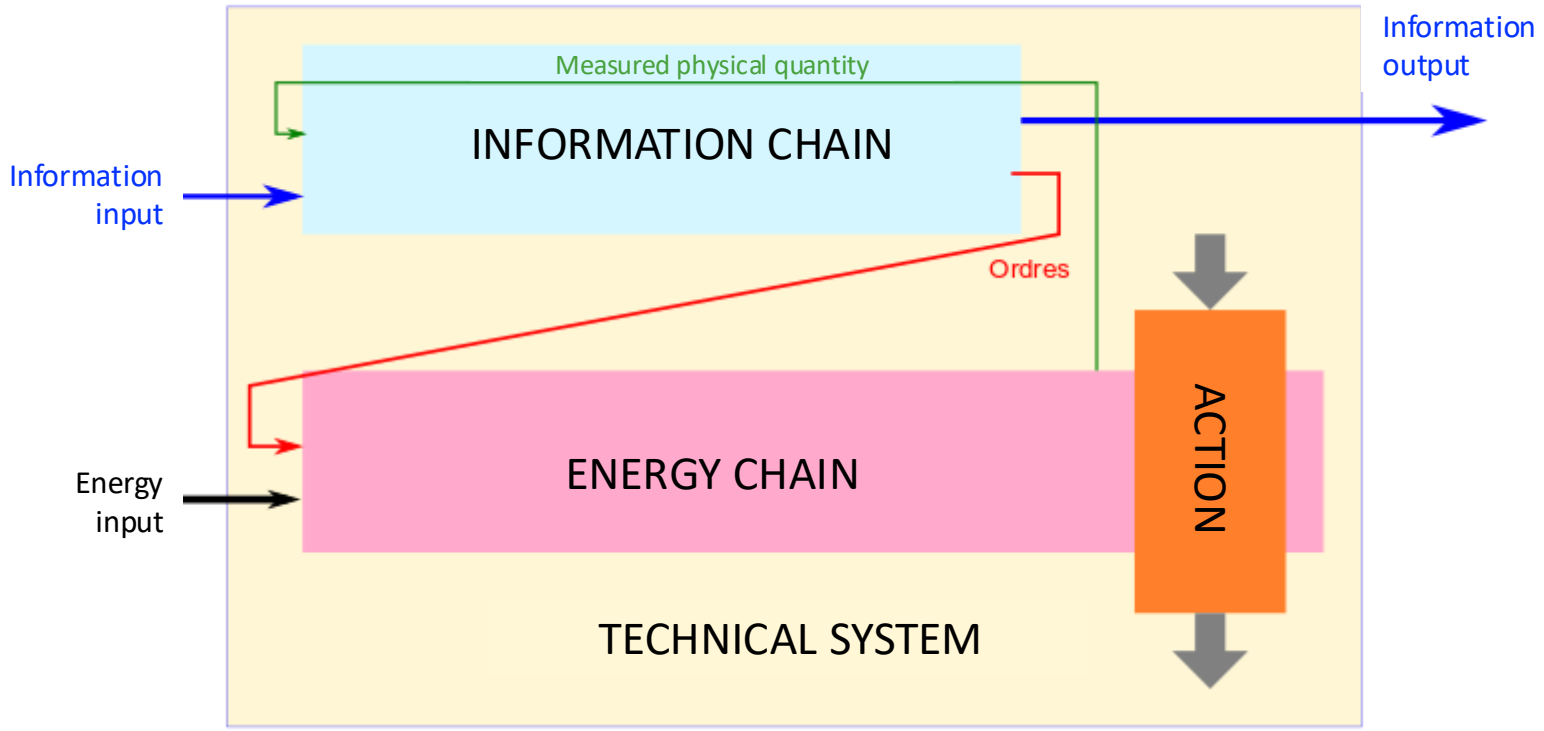
III) Transformers

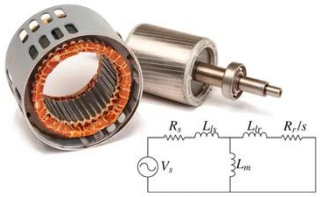
IV) Electric motors



Functional chain systems

- Technical systems, mechatronic systems, industrial systems





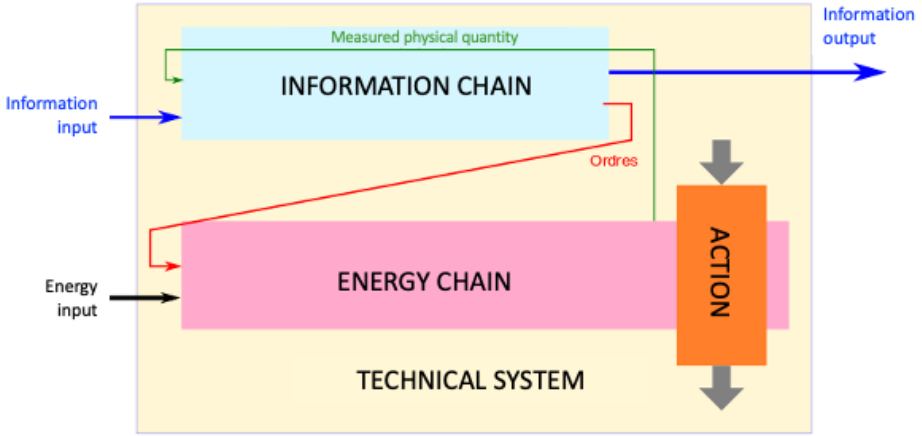
Introduction

Electricity related fieds

Electronics:

⇒ Current lower than 1 A

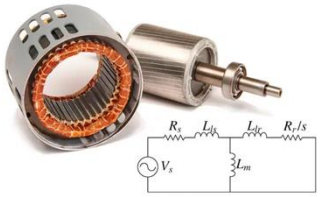
⇒ Information chain: data flow management



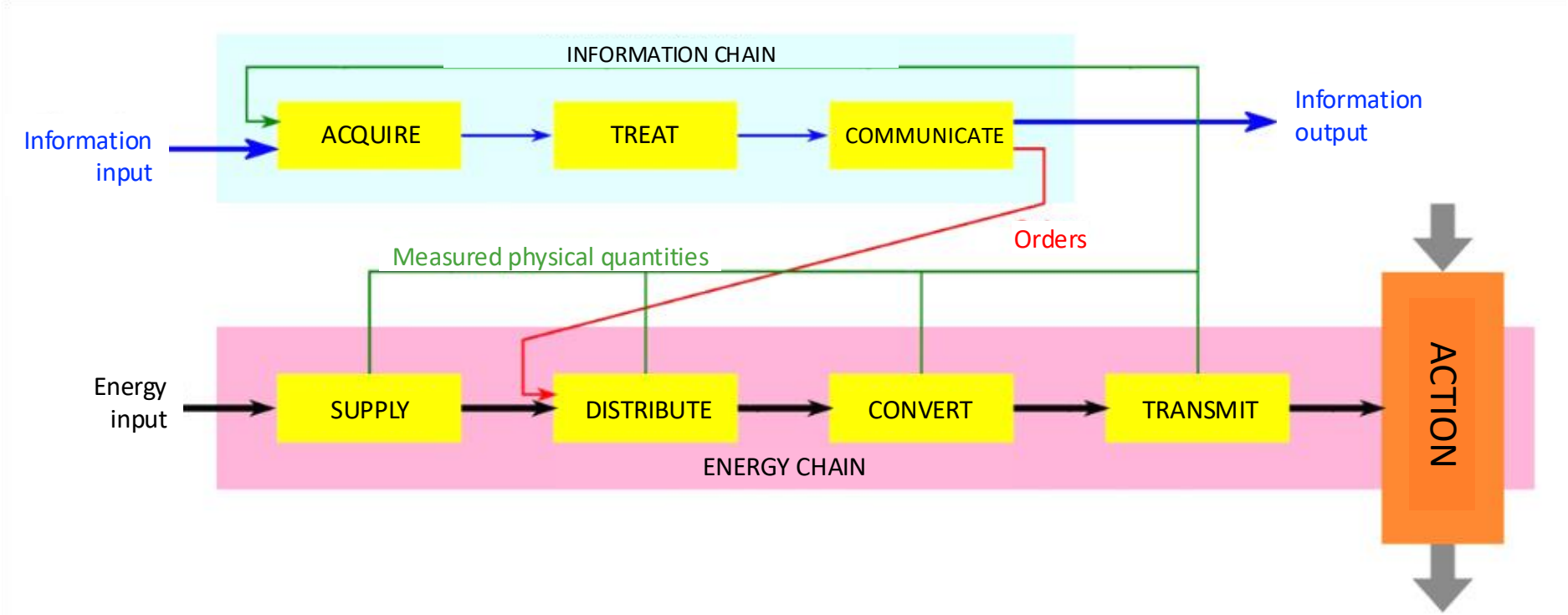
Electrical engineering:

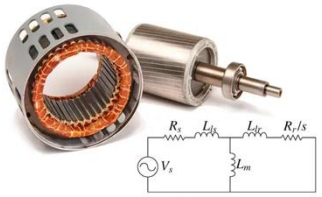
⇒ Current higher than 1 A

⇒ Energy chain: energy flow management

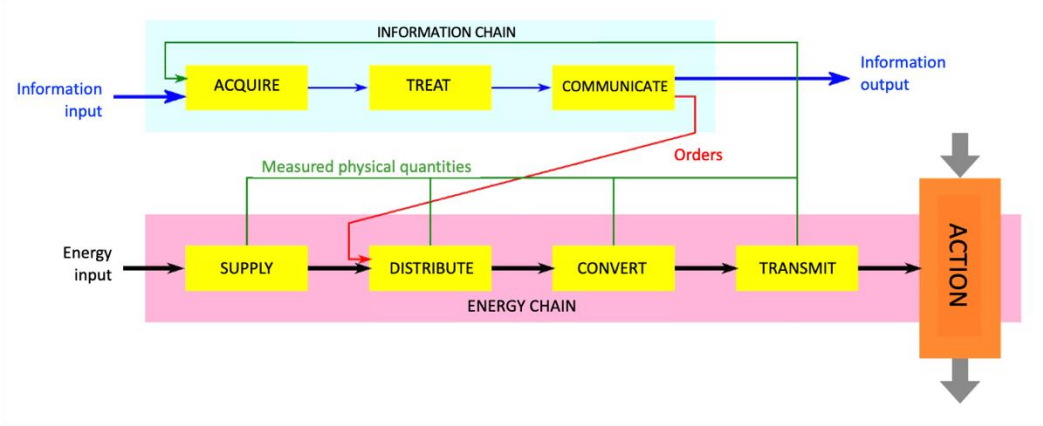


Energy chain

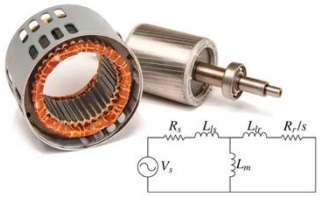




Energy chain



Supply	Distribute	Convert	Transmit
• Production • Transport • Distribution • Installation design • Storage • Embedded power supply	• Equipement • Motor start • Power electronics	• Electric motors • Lighting • Electrothermics	• Motion transformation • Light transmission • Heat transfert



From primary energy to its use

- Energy utility:







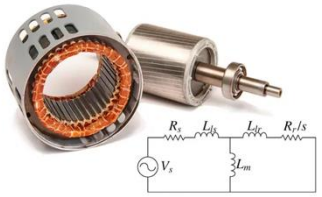


HEAT

WORK

LIGHT

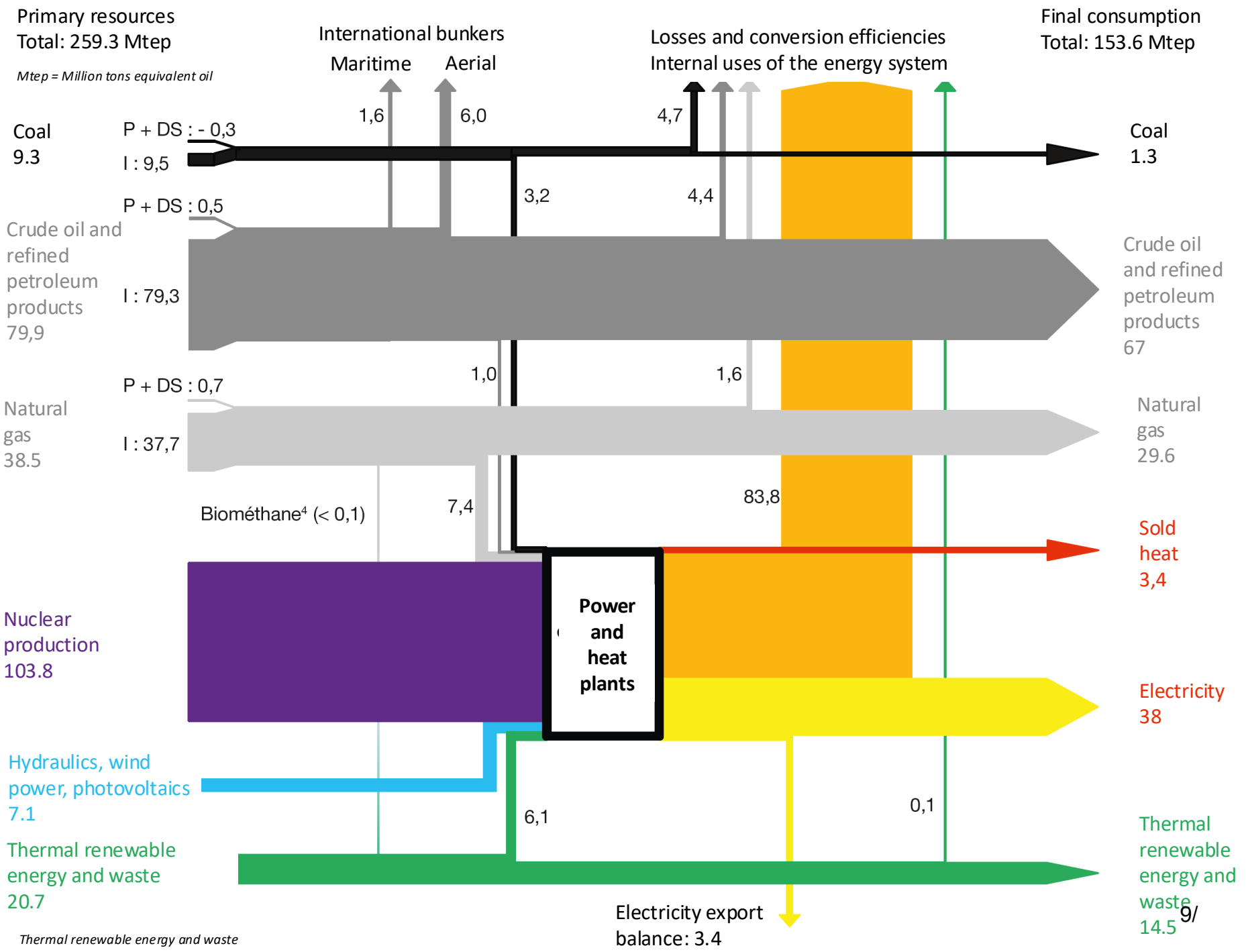
COMMUNICATION

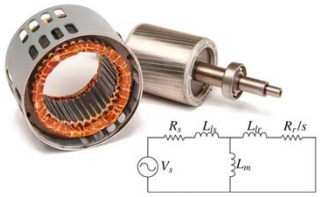


Introduction

From primary energy to its use

Primary energies	Storage	Carriers	Use
Non-renewable fossil fuels • Oil • Natural gas • Coal • Uranium	Natural or controlled storage	Oil products Gas Electricity Steam Compressed air Hydrogen Microwaves	HEAT WORK LIGHT COMMUNICATION
Continous renewable energies • Biomass • Urban waste • Geothermal energy • Hydraulic energy (dam, tidal power, water turbine)	Natural or controlled storage		
Non continous renewable energies • Solar energy (thermal, photovoltaics) • Wind energy	Mandatory storage in another form of energy		

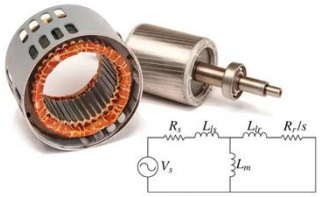

Primary resources
Total: 259.3 Mtep
Mtep = Million tons equivalent oil
International bunkers
Maritime
Aerial
Losses and conversion efficiencies
Internal uses of the energy system
Final consumption
Total: 153.6 Mtep
Coal
9.3
P + DS : - 0,3
I : 9,5
1,6
6,0
4,7
Coal
1.3
3,2
4,4
P + DS : 0,5
Crude oil and refined petroleum products
79,9
I : 79,3
Crude oil and refined petroleum products
67
1,0
1,6
P + DS : 0,7
Natural gas
38.5
I : 37,7
Natural gas
29.6
Biométhane[4] (< 0,1)
7,4
83,8
Sold heat
3,4
Nuclear production
103.8
Power and heat plants
Electricity
38
Hydraulics, wind power, photovoltaics
7.1
6,1
0,1
Thermal renewable energy and waste
20.7
Thermal renewable energy and waste
14.5
Electricity export balance: 3.4
Thermal renewable energy and waste



Introduction

Energy: 4 activities, many actors

French law of February 10th, 2000		
Production	EDF, Engie…	Competitive business
Transport	French power grid	Regulated business
Distribution	ENEDIS (ex ERDF), public companies	Regulated business
Marketing	EDF, Engie, Poweo, Total, …	Competitive business
Inspection authority	*French Energy Regulatory Commission*	COMMISSION DE RÉGULATION DE L'ENERGIE

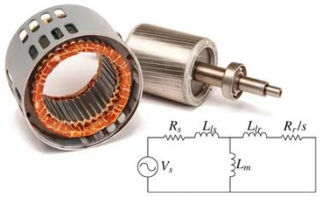


Electrical energy

- Consumed as soon as it is produced

- Distributed instantly to the point of use

- No mass storage is currently possible



Introduction

Electricity production in France



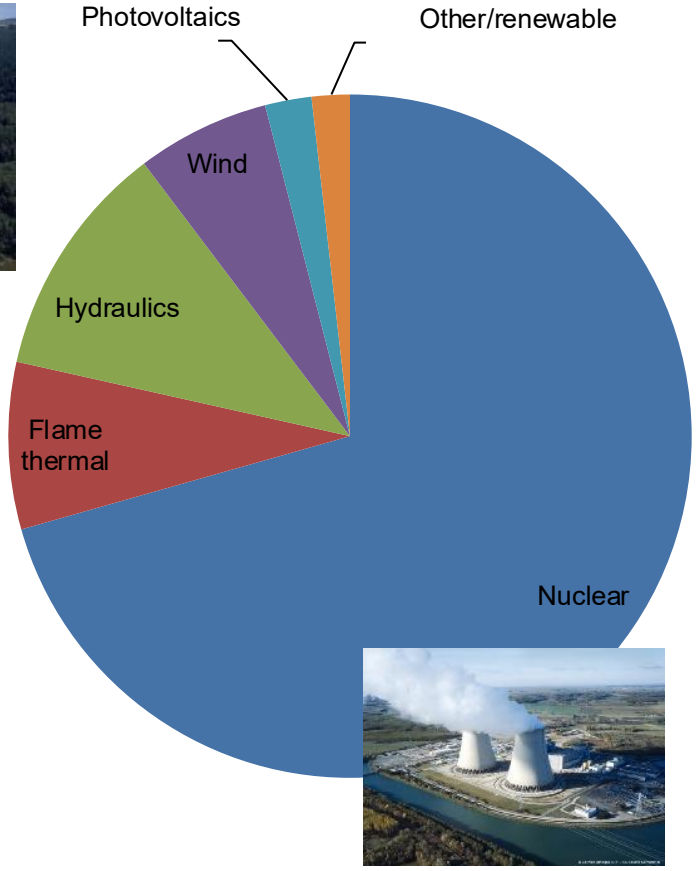
In 2019 : 538 TW.h

(46.3 Mtep)

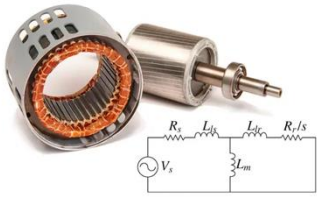




- Nuclear: 70,6 %
- Flame thermal: 7,9 %
- Hydraulics: 11,2 %
- Wind: 6,3%
- Photovoltaics: 2,2 %
- Other/renewable: 1,8 %



132 MTep were **required** to produce these **46.3 MTep**.



Introduction

Electricity production in France

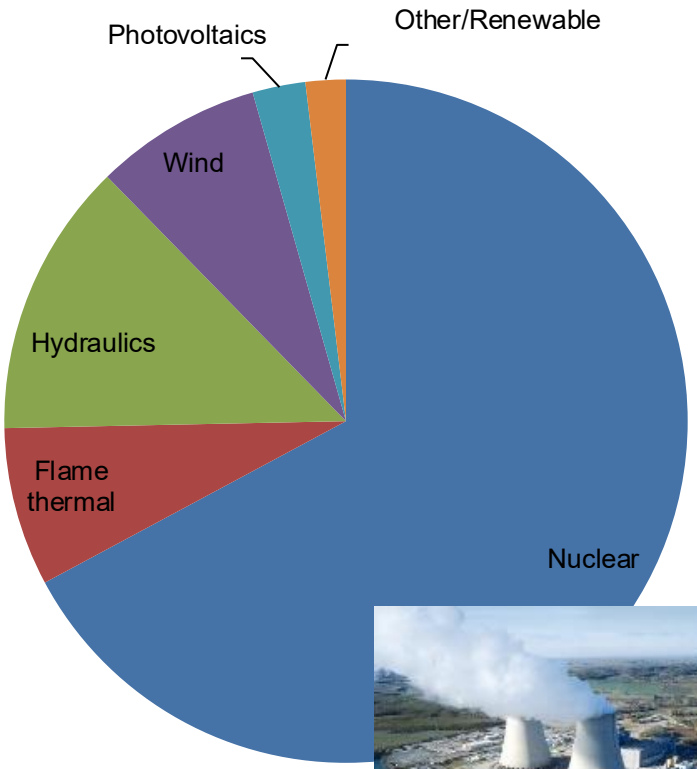


In 2020 : 500 TW.h

(43 Mtep)



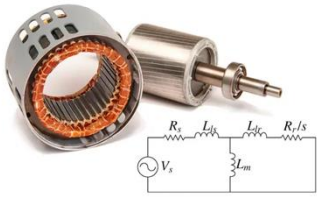




- Nuclear : 67,1 %
- Flame thermal: 7,5 %
- Hydraulics : 13 %
- Wind : 7,9%
- Photovoltaics : 2,5 %
- Other/Renewable: 1,9 %



123 MTep were **required** to produce these **43 MTep**.



Electricity production in France



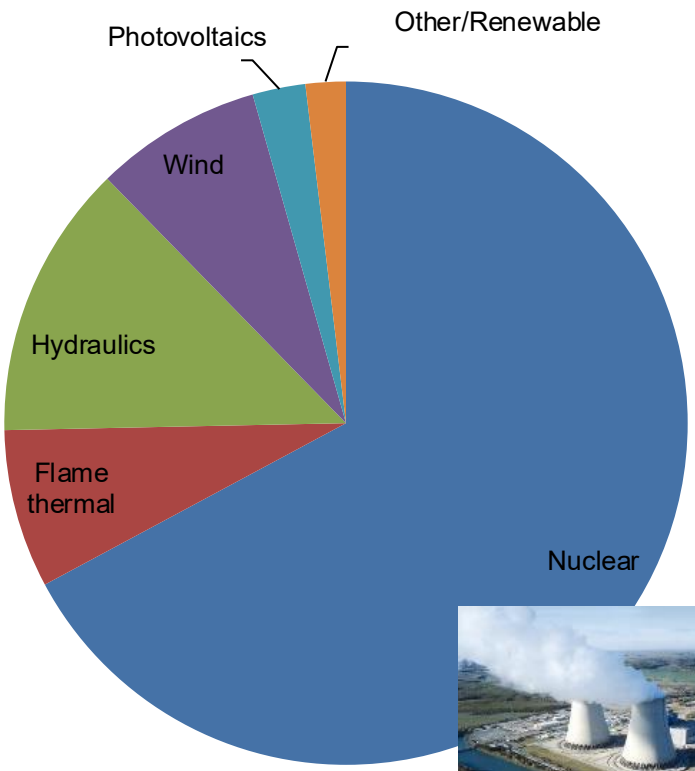
In 2021 : 523 TW.h

(45 Mtep)

- Nuclear : 69 %
- Flame thermal: 7,4 %
- Hydraulics : 12 %
- Wind : 7%
- Photovoltaics : 2,7 %
- Other/Renewable : 1,9 %

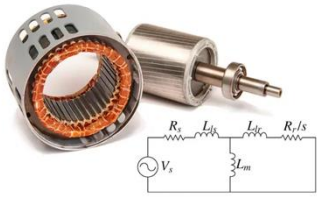








129 MTep were **required** to produce these **45 MTep**.



Electricity production in France



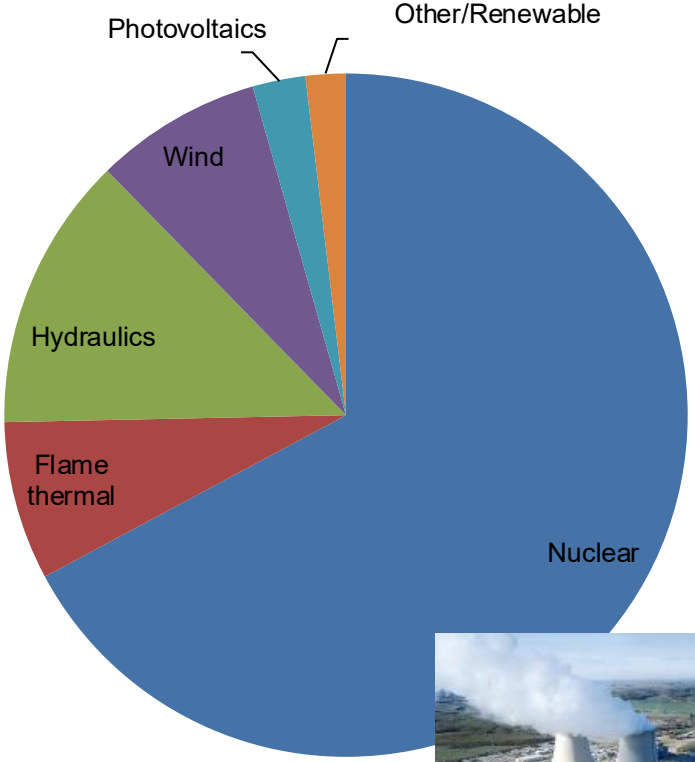
In 2022 : 445 TW.h

(38.3 Mtep)



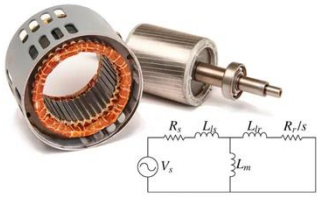


- Nuclear: 62,7 %
- Flame thermal: 11,1 %
- Hydraulics: 11,1 %
- Wind: 8,6%
- Photovoltaics: 4,2 %
- Other/Renewable: 2,4 %

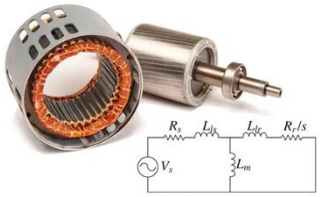




110 MTep were **required** to produce these **38.3 MTep**.

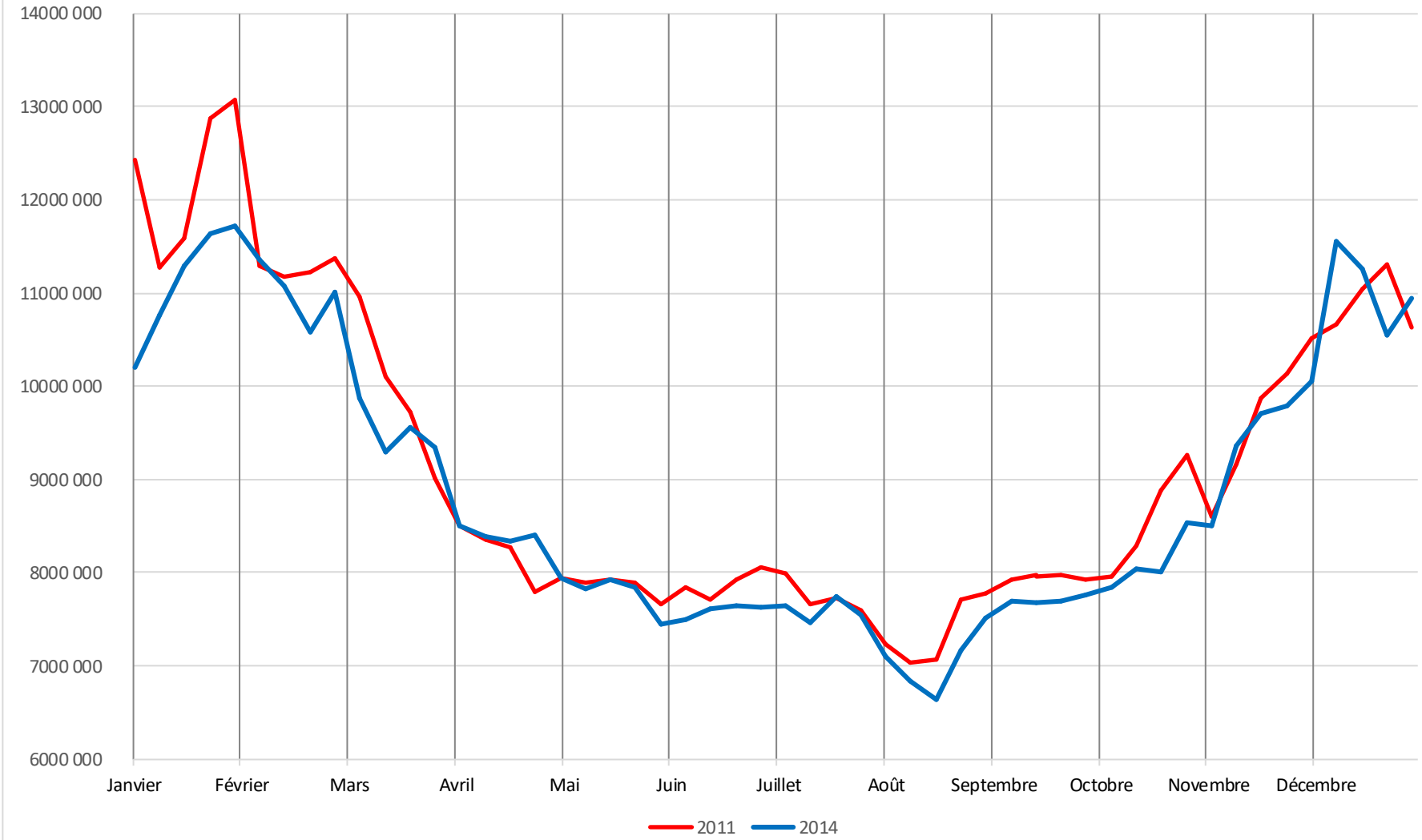


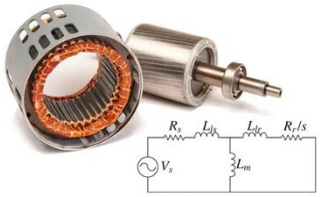
Electricity consumption in France in 2011 (in MW.h)



Introduction

Electricity consumption in France in 2011 and 2014 (in MW.h)





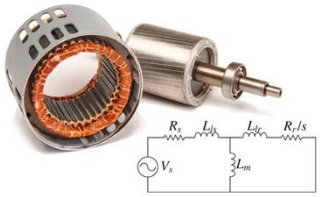
Real time production in France

https://www.rte-france.com/eco2mix/la-production-delectricite-par-filiere#

Electrical energy production in France

- “Grand Maison” dam





Electrical energy production in France

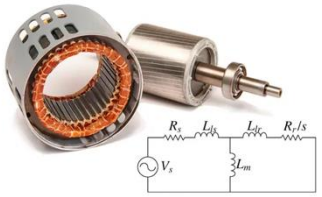
- “Grand Maison” dam



Pumped-storage power station

- Gravity dam, 550 m long et 140 m high (160 m on foundation),
- Reservoir 137 millions m^3 water
- Built at 1700 m altitude from 1978 to 1988
- Flow: **216 m^3/s** (76 and 140 m^3/s)
- Drop height: **922 et 955 m**
- Verney plant : **12 turbo-alternators** of **153 MW** – 15500V, Pelton turbines
- Total installed power: **1820 MW** equivalent to **two nuclear units**.
- Extraction : 6 elevating transformers 15500V / 405000V

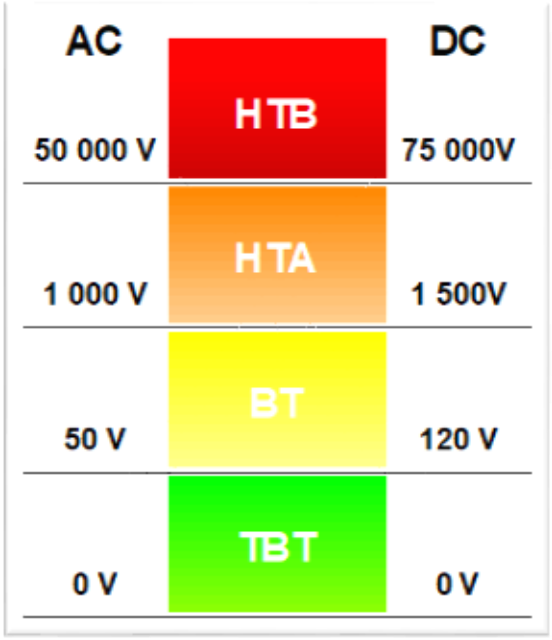




Introduction

Electrical voltage ranges in France

- HTA and HTB: High voltage (“Haute Tension”)
- BT: Low voltage (“Basse Tension”)
- TBT: Very low voltage (“Très Basse Tension”)



HTB



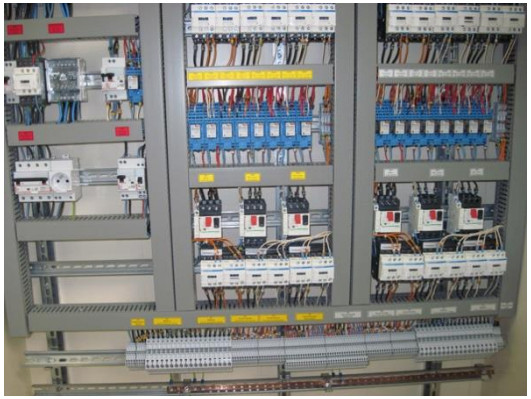
TBT



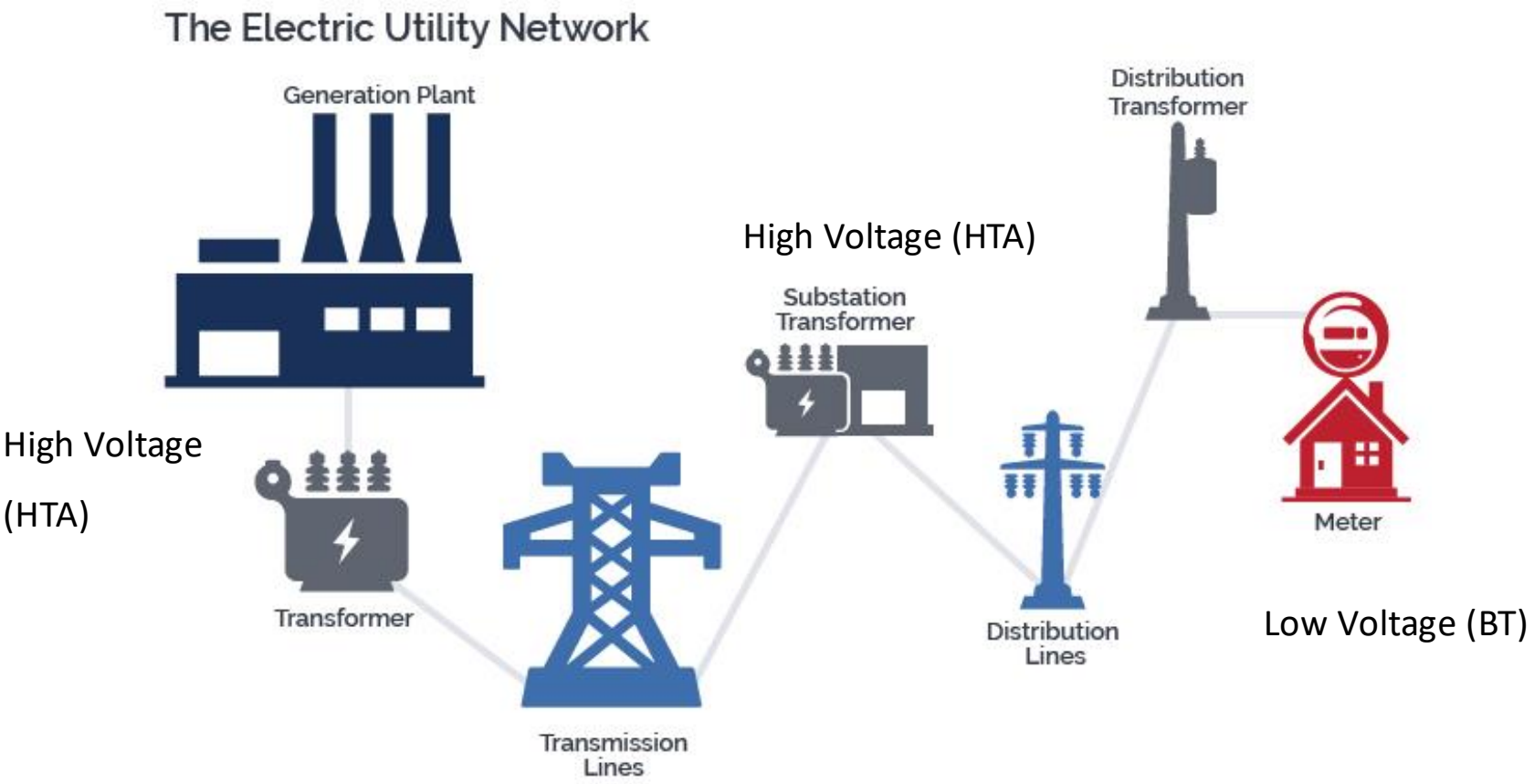
HTA

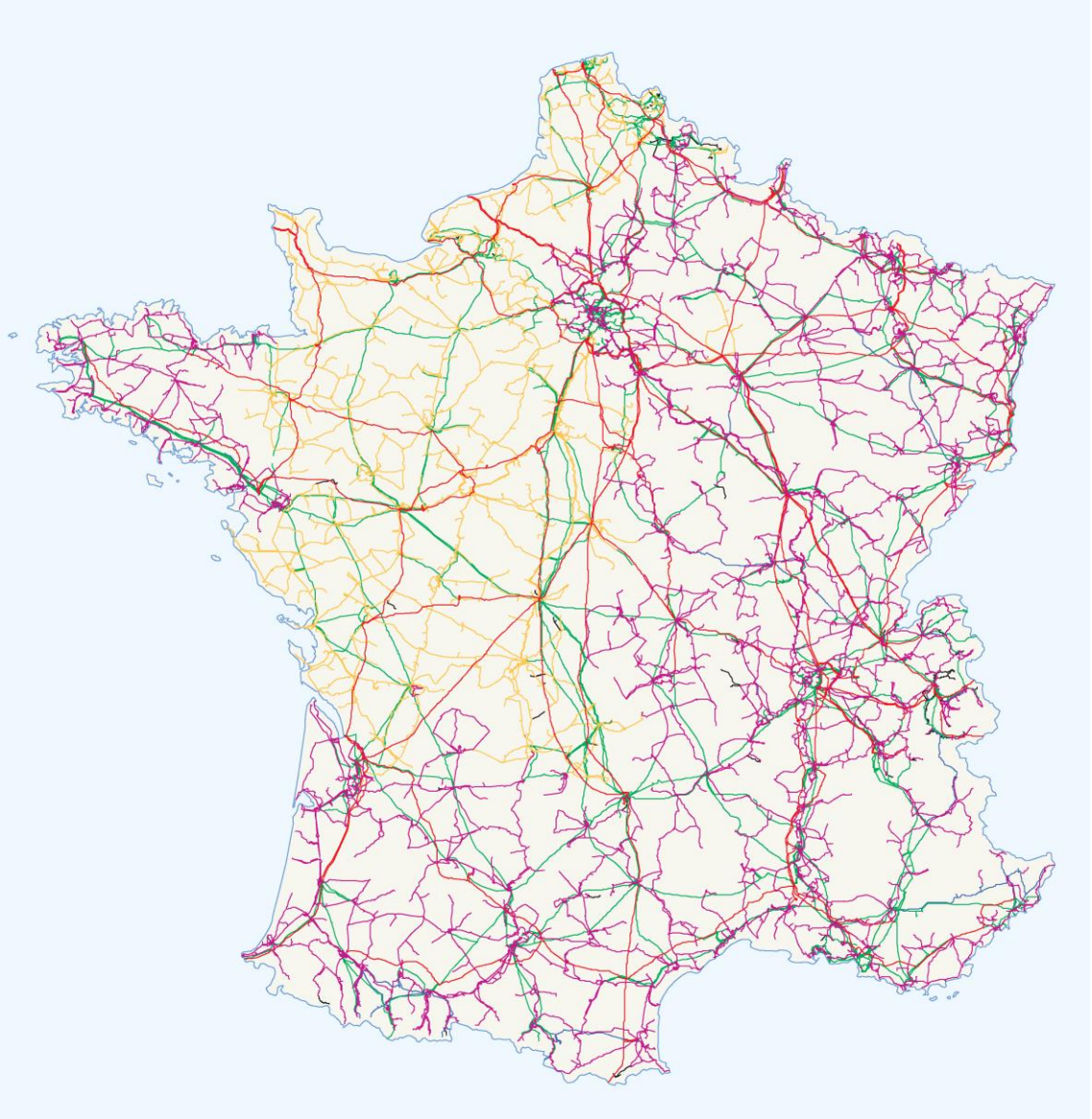


BT



Power grids : basic structure

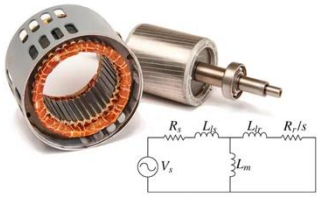




Distribution RTE grid



1.6 million km long



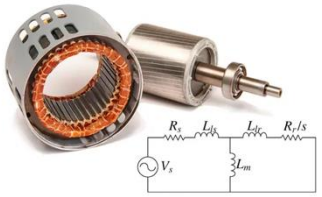
Introduction



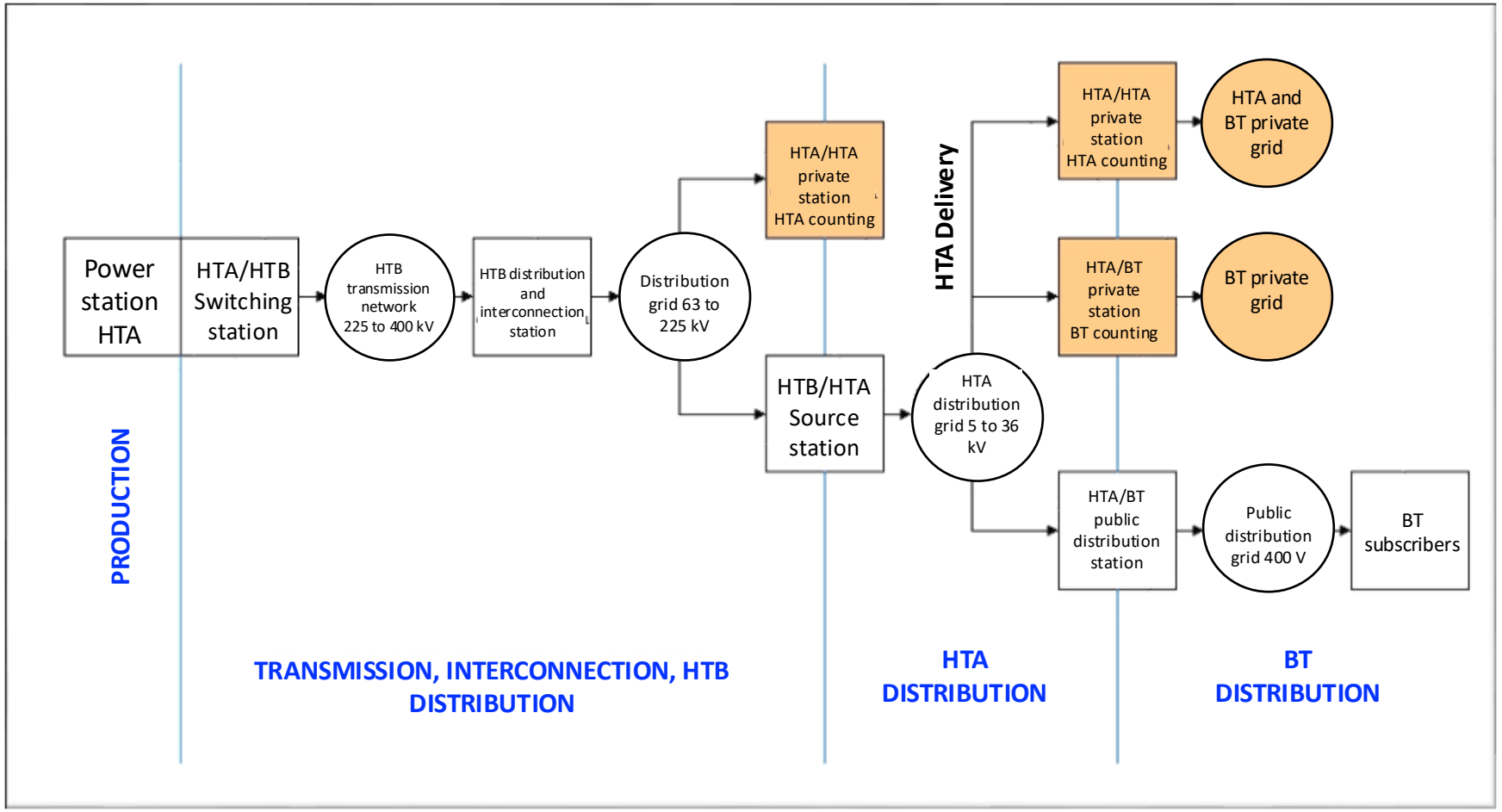
RTE grid 400 kV

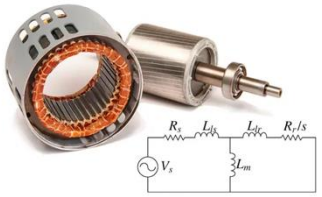






Network interconnection



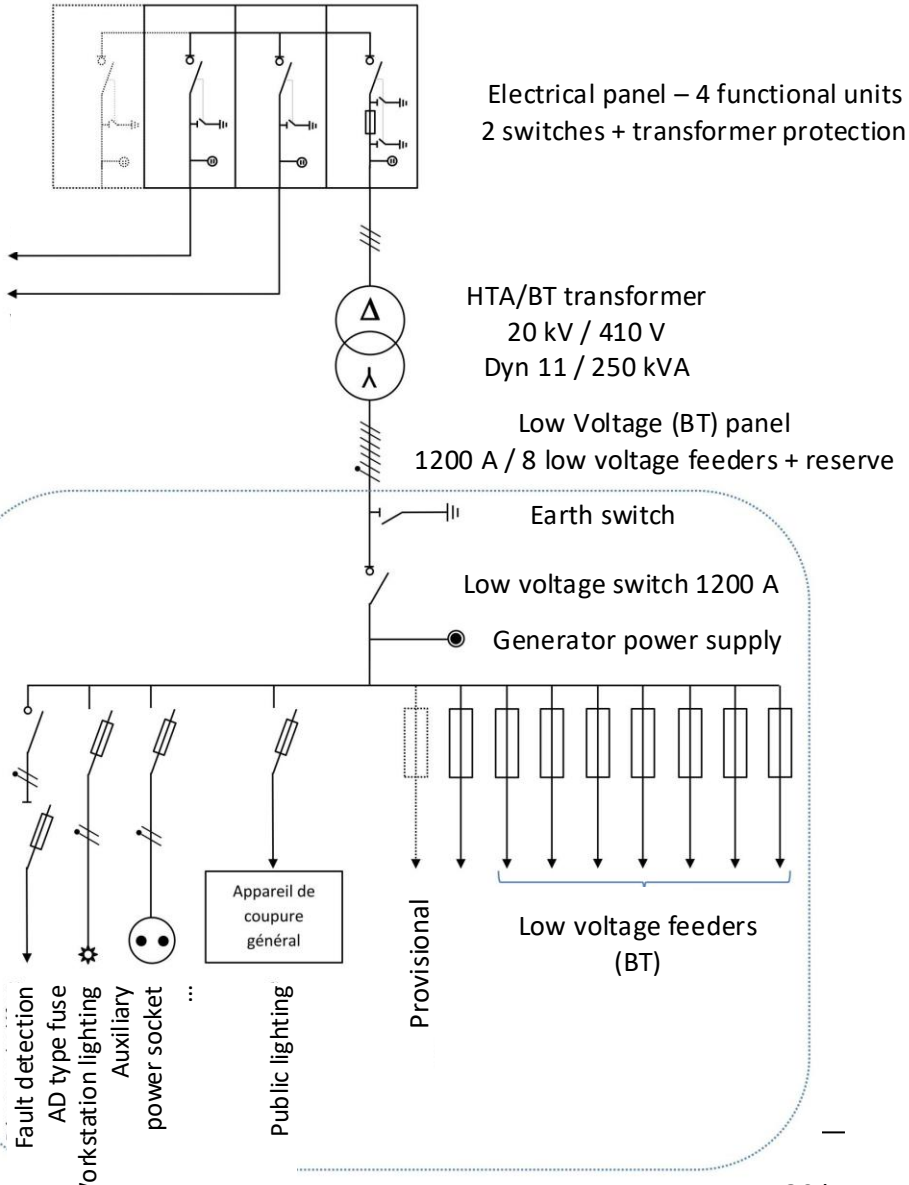


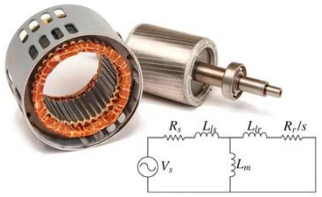
Introduction

- Urban transformer station HTA/BT







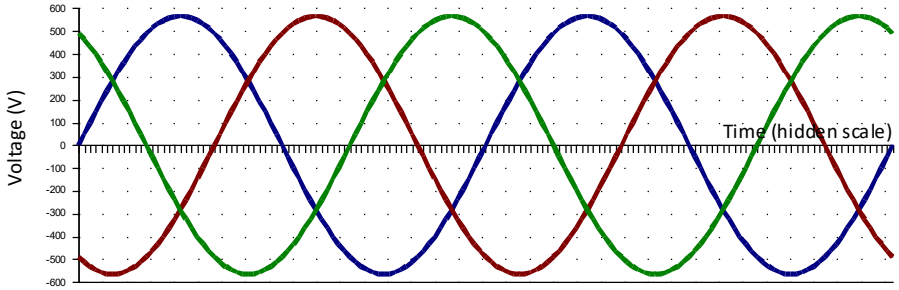


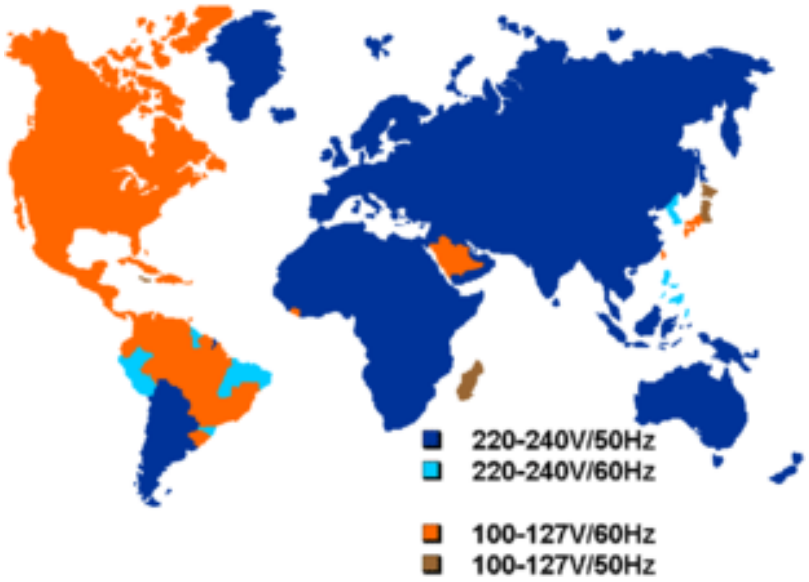
Introduction

Distribution

- In France, three-phase distribution 400 V/230 V

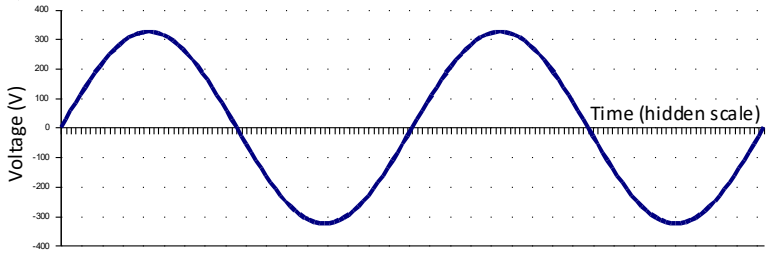
at 50 Hz (sine-wave)



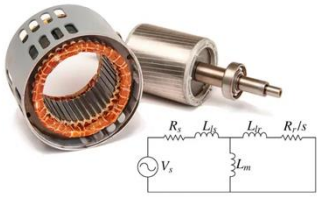


- In the world, both voltage and frequency can be slightly different

- Delivery to the customer : single-phase distribution



=> In France: 230 V, 50Hz

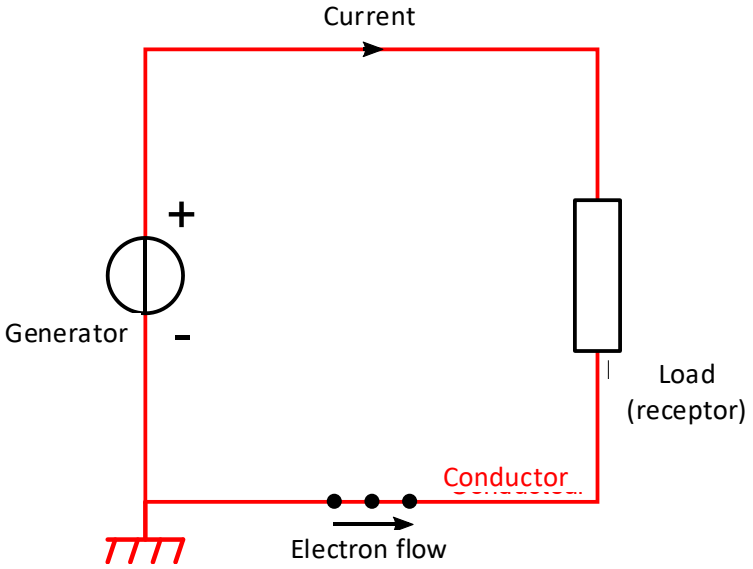


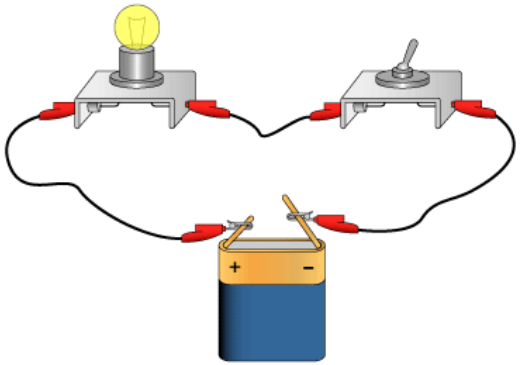
The electrical circuit

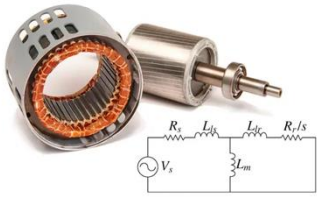
- Electrical circuit = electrical generator + load(s) (electrical receptor)

- Each element of the circuit is connected by metal conductors (usually copper wires)

=> When the circuit is closed, the generator makes the electrical charges flow







Electric quantities

Electrical current

- Electrical current i(t) = electrical charge q(t) flow

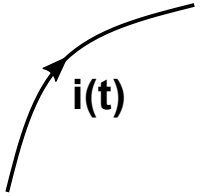
$$i(t) = \frac{dq(t)}{dt}$$

- i(t) characterized by its direction and its intensity

=> i(t) is measured in Amperes (A)

- i(t) has the same value at each point of a single circuit

=> No charge accumulation at one point, the flow is the same everywhere



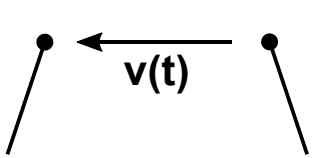
Voltage

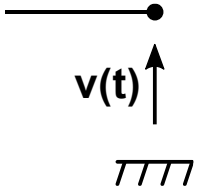
-The electrical generator creates a potential difference that makes electrical charges flow through the circuit

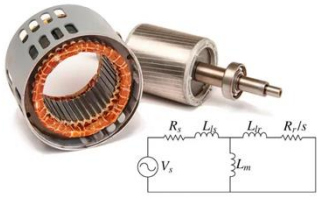
- Voltage v(t) = potential difference

=> v(t) is measured in Volts (V)

- All voltages in a circuit are defined in relation to a reference zero potential usually called the ground







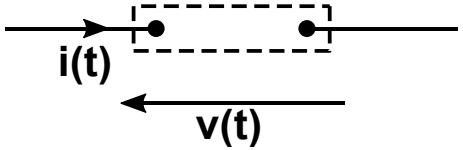
Convention

- Electric dipole = element of the circuit having two terminals through which a current flows when a voltage is applied

Receptor convention

- Current and voltage in opposite directions for each receptor

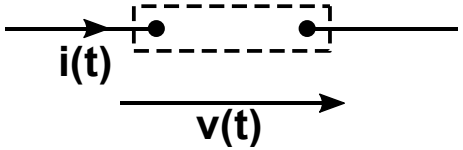
- The dipole receives power

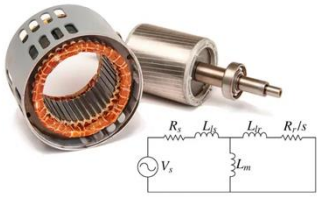


Generator convention

- Current and voltage in same directions for each generator

- The dipole supplies or generates power





Electrical power

- A dipole exchanges energy with other dipoles to which it is connected. It can transform one form of energy into electrical energy, or convert electrical energy into another form of energy

- The exchanged or transformed energy written dw(t) (in joule (J)), is related to the operating time written dt in second (s), by :

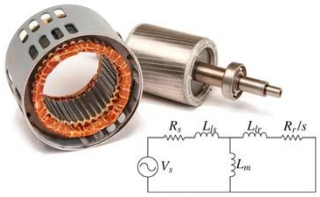
$$p(t) = \frac{dw(t)}{dt}$$ with p(t) the power in Watt (W)

- A receptor under a voltage v(t) and carrying a current i(t) generates or receives an instantaneous power p(t).
=> This power reflects the energy exchanges between the generator and the receptor at each instant t.

$$p(t) = v(t) \times i(t)$$

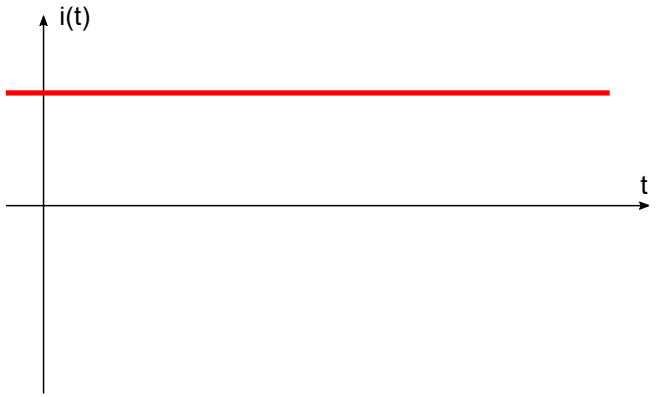
- Current and voltage are algebraic quantities => power is also an algebraic quantity.

- With a given convention, generator or receptor, the nature of the dipole, generator or receptor, is the same as that of the convention if the sign of the power is positive.



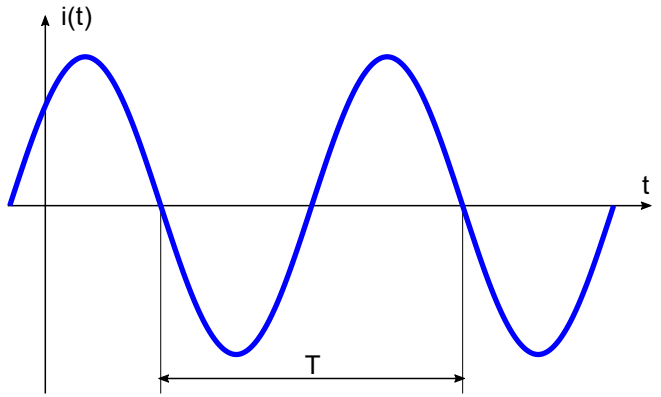
AC/DC operating regimes

DC current

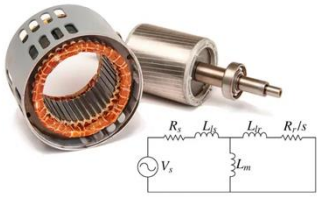


- Current intensity remains constant with time

AC current



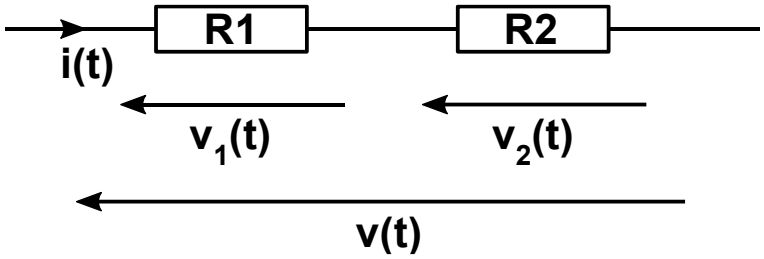
- Intensity is variable and changes direction periodically.
- Period T is measured in seconds.
- Frequency f, number of periods described by the current in one second, measured in Hertz (Hz).
- Frequency of 50 Hertz in France and Europe.



Dipole connection

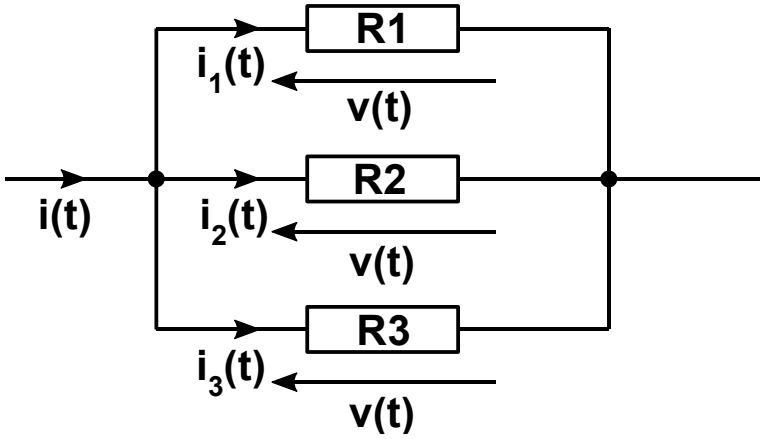
Receptors in series

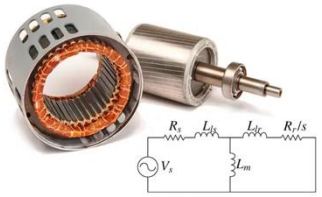
- Dipoles are connected in series when the same current flows through them



Receptors in parallel

- Dipoles are connected in parallel when they are subjected to the same voltage.

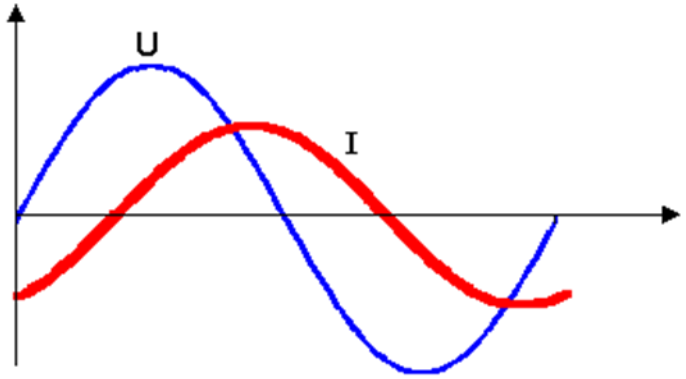




Linear dipoles

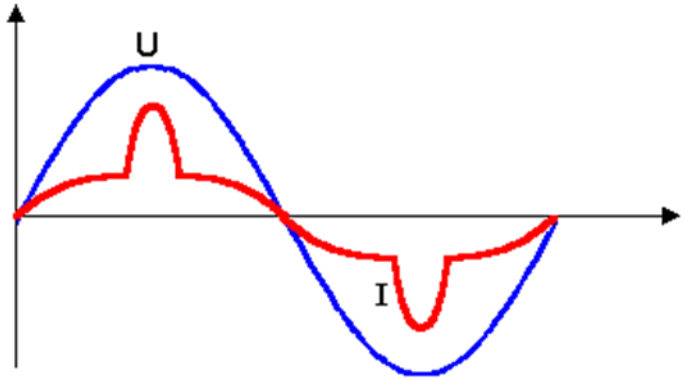
- A dipole is linear if the relation between the voltage at its terminals and the current flowing through it is linear.

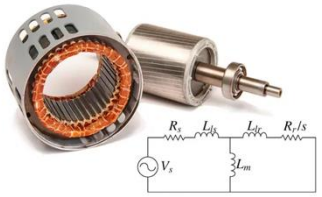
$$v(t) = f(i(t))$$



- Function f is linear if:

$$f(\alpha_1.i_1(t) + \alpha_2.i_2(t)) = \alpha_1.f(i_1(t)) + \alpha_2.f(i_2(t))$$



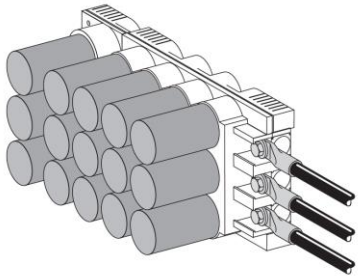


Passive dipoles

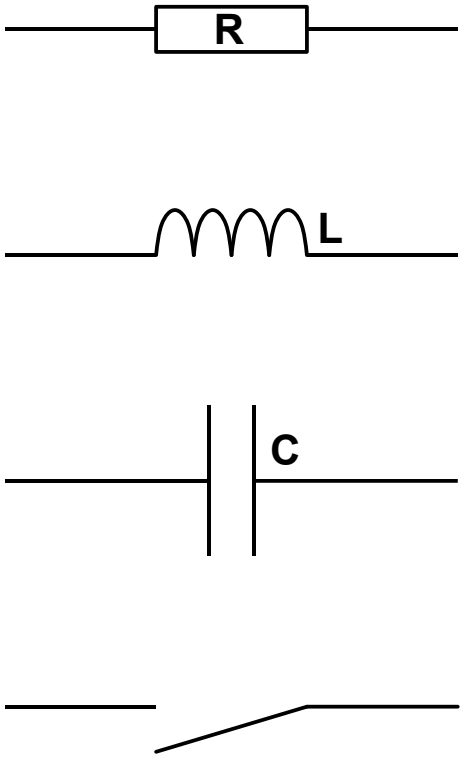
- A passive dipole is an electric component that dissipates or temporarily stores electrical energy.

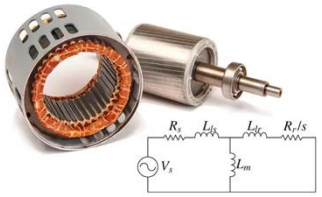












The resistor

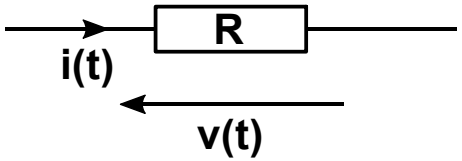
- A resistor is an electric dipole that opposes the current. R, the resistance, is expressed in Ohm (symbol: Ω)

- Ohm's law:

$$v(t) = R.i(t)$$

R: resistance in Ω (Ohm)
v(t): voltage across the resistor in V (Volt)
i(t): current through resistor in A (Ampere)

- Resistance symbol in receptor convention:



- Remarks:

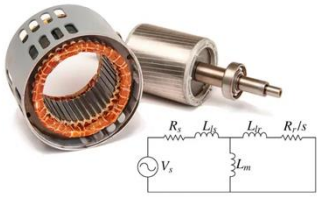
- G, the inverse of resistance, is called conductance and is expressed in Siemens (S).

$$i(t) = G.v(t)$$

- Electric resistance in Ω :

$$R = \rho \times \frac{l}{s}$$

ρ: electrical resistivity in Ω.m
l: conductor length in m
s: conductor cross-section in m²





The capacitor

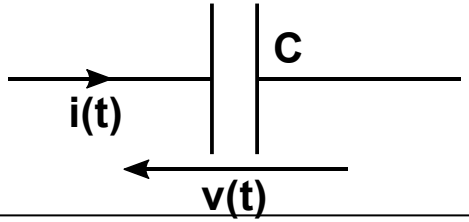
- A capacitor is an electric dipole that opposes changes in voltage and can store energy in electrostatic form. C, the capacitance of a capacitor, is independent of time and is expressed in Farads (F).

- Relation between current and voltage:

$$i(t) = C \cdot \frac{dv(t)}{dt}$$

C: capacitance in F (Farad)
v(t): voltage across capacitor in V (Volt)
i(t) : current in A (Ampere)

- Relation between current and voltage:



- Remarks:

- We can also write:

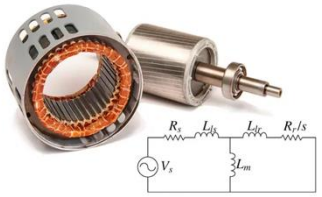
$$V_C(t) = \frac{1}{C} \cdot \int i(t) \cdot dt$$

- No voltage discontinuity across a capacitor.

- Capacitance of a planar capacitor in Farad

$$C = \varepsilon_0 . \varepsilon_r \frac{S}{e}$$

ε_0: vacuum permittivity (8.85×10^{-12} F.m^{-1})
ε_r: relative dielectric permittivity
S : surface area of armatures in m²
e : dielectric thickness in m



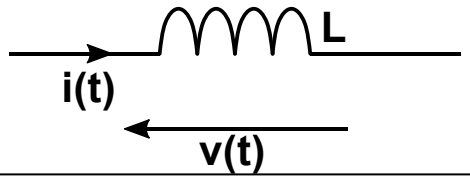
The coil

- A coil is an electric dipole that opposes the variation of electric current and can store energy in electromagnetic form. L, the coil inductance, is independent of time and is expressed in Henry (H).

- Relation between voltage and current:

$$v(t) = L \cdot \frac{di(t)}{dt}$$

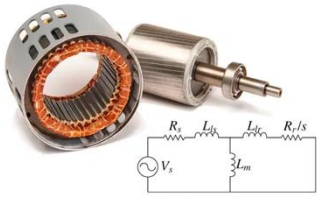
L: inductance in H (Henry)
v(t): voltage across coil in V (Volt)
i(t): current in A (Ampere)



- Remark:

- No discontinuity in the current flowing through an inductor.



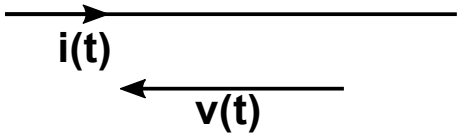


Short and open circuits

The short circuit or the wire

- The short-circuit, the wire, the closed contact (switch or any other electric device) are dipoles with zero resistance. The voltage at their terminals is zero, whatever the current flowing through them.

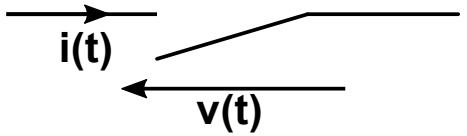
$$v = 0 \,, \forall\, i$$



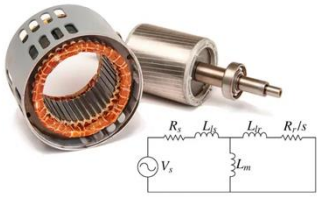
The switch or the open circuit

- The open circuit, the cut wire or the open contact (switch or other electric device) are dipoles with infinitely high resistance. The current flowing through them is zero, whatever the voltage at their terminals.

$$i = 0 \,, \forall\, v$$







Passive dipole power

- Time-dependent power: $$p(t) = v(t) \times i(t)$$

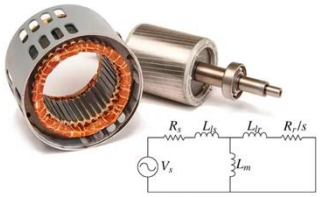
- For a resistor: $$p_R(t) = v(t) \cdot i(t) = R \cdot i^2(t)$$

- For a capacitor: $$p_C(t) = v(t) \cdot C \cdot \frac{dv(t)}{dt} = \frac{1}{2} \cdot C \cdot \frac{dv^2(t)}{dt}$$

- For a coil: $$p_L(t) = L \cdot \frac{di(t)}{dt} \cdot i(t) = \frac{1}{2} \cdot L \cdot \frac{di^2(t)}{dt}$$

- The power in a resistor is always positive. This dipole cannot restore it. It is said to be dissipative (Joule effect). => active component

- The power in a capacitor or a coil can be positive or negative. These two dipoles can store and release energy. They are said to be reactive.



Passive dipole energy

- Energy of a dipole at time t:

$$w(t) = w(0) + \int_0^t p(x)dx$$

- For a resistor:

$$w_R(t) = R \cdot \int_{-\infty}^t i^2(x)\,dx$$

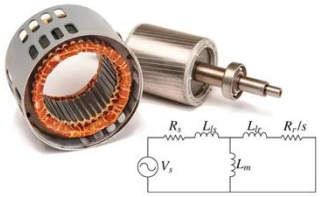
- For a capacitor:

$$w_C(t) = \frac{1}{2} \cdot C \cdot v^2(t)$$

- For a coil:

$$w_L(t) = \frac{1}{2} \cdot L \cdot i^2(t)$$

- Energy is always positive. This property is characteristic of passive elements

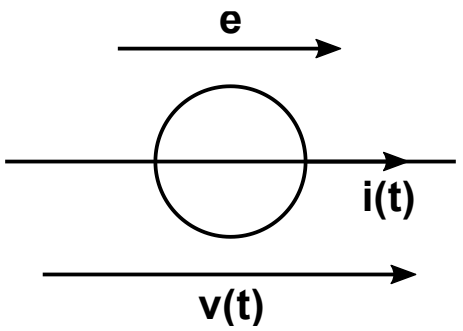


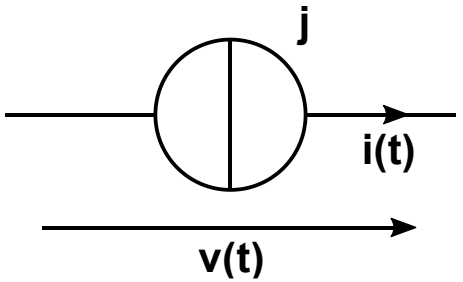
Active dipoles

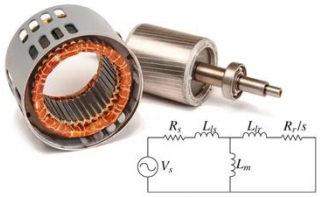
- Dipole able to supply energy to a load connected to it. They are electrical or electronic components where electromotive phenomena can permanently convert electrical energy.





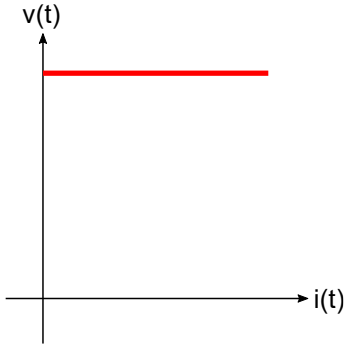




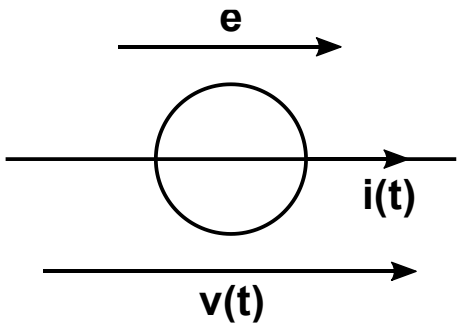


Voltage generator

- A voltage generator is a dipole able to impose a constant voltage regardless of the current it supplies



$$v(t) = e, \forall\, i$$



Thevenin model

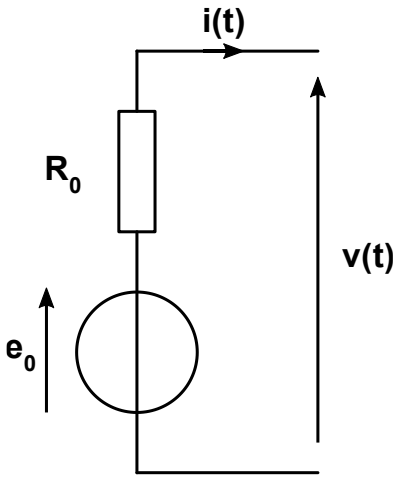
- Model used for the ideal voltage source

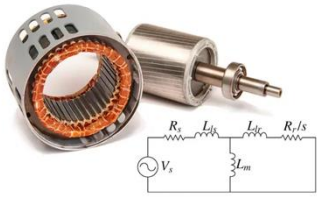
- Ideal voltage generator associated with a series resistor

No-load voltage: e_0
Internal resistor: R_0
Short-circuit current : I_{CC} (v = 0)

$$v(t) = e_0 - R_0 . i(t)$$

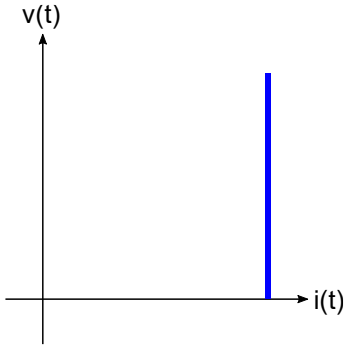
$$I_{CC} = \frac{e_0}{R_0}$$



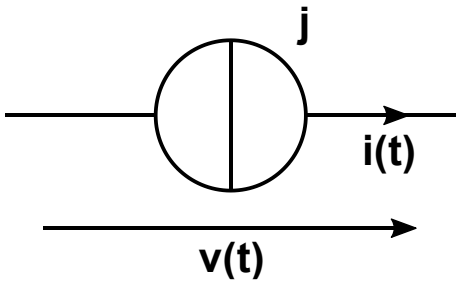


Current generator

- A current generator that can impose a constant current whatever the voltage at its terminals



$$i(t) = j, \forall\, v$$



Norton model

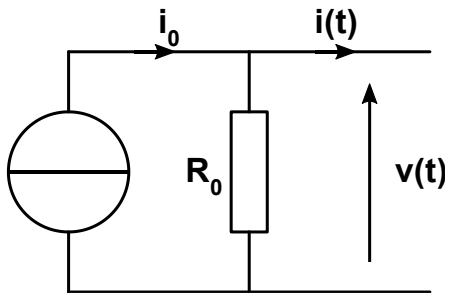
- Model used for the real current source

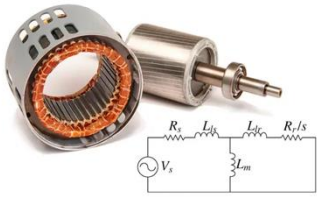
- Ideal current generator associated with a parallel resistor

Short circuit current: i_0
Internal resistor: R_0
No-load voltage: e_0 (i = 0)

$$i(t) = i_0 - \frac{v(t)}{R_0}$$

$$e_0 = R_0 . i_0$$





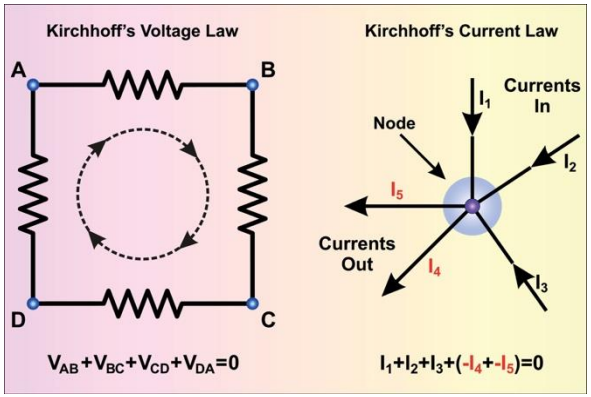
Kirchhoff's laws

Definitions

- A node is a connection between more than two dipoles

- A branch is a portion of the circuit between two consecutive nodes.

- A mesh is a set of branches forming a closed path



Kirchhoff's current law

- The algebraic sum of currents arriving at a node is zero

$$\sum i_{entrant} = \sum i_{sortant}$$

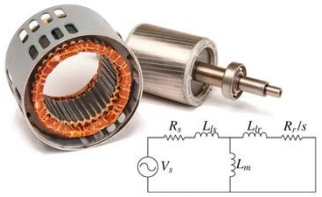
Kirchhoff's voltage law

- The sum of the voltages in a mesh is zero

$$\sum v_i = 0$$

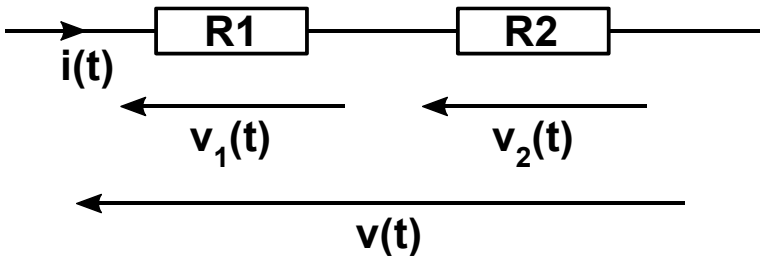
Application rule

An arbitrary direction is chosen. The mesh is described in the chosen direction. Voltages in the same direction must be counted as positive, and voltages in the opposite direction must be counted as negative



Divider bridges

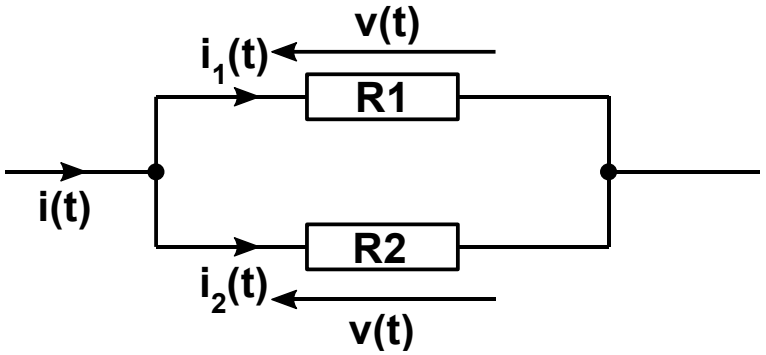
Voltage divider



$$v_1 = v \cdot \frac{R_1}{R_1 + R_2}$$

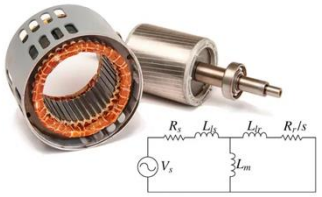
$$v_2 = v \cdot \frac{R_2}{R_1 + R_2}$$

Current divider



$$i_1 = i \cdot \frac{R_2}{R_1 + R_2}$$

$$i_2 = i \cdot \frac{R_1}{R_1 + R_2}$$



Millman's theorem

For voltage generators

- n parallel voltage generators with resistance R_k and electromotive force e_k can be replaced by a single voltage generator with internal resistance R and electromotive force e

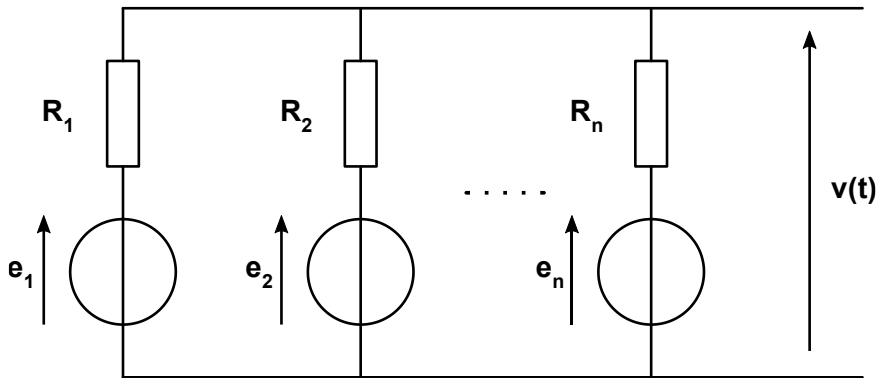
$$R = \frac{1}{\sum_{k=1}^{n} \frac{1}{R_k}}$$

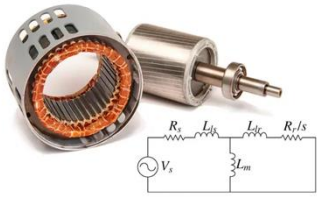
$$e = R \cdot \sum_{k=1}^{n} \frac{e_k}{R_k}$$

For a potential at one point

- Consider a current node of potential v in a network. This node is the junction point of n resistors R_K, subjected to potentials v_K on the other side of the node. The expression of the potential v is :

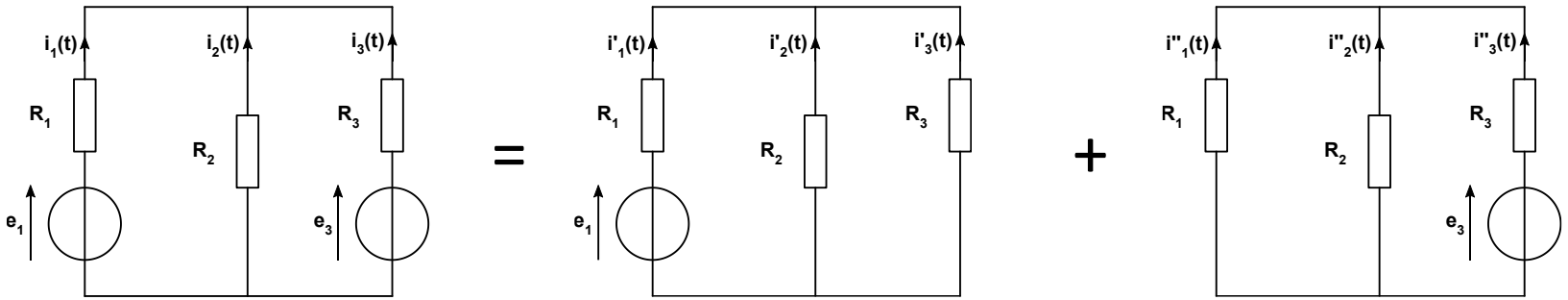
$$v = \frac{\sum_{k=1}^{n} \frac{e_k}{R_k}}{\sum_{k=1}^{n} \frac{1}{R_k}}$$



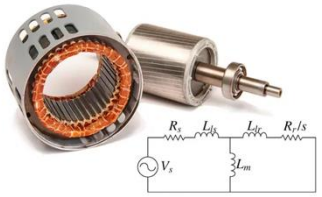


Superposition theorem

- The current or voltage response of a network containing several independent sources acting simultaneously is equal to the sum of the current or voltage responses due to each source acting alone.



$$i_1 = i'_1 + i''_1 \qquad i_2 = i'_2 + i''_2 \qquad i_3 = i'_3 + i''_3$$



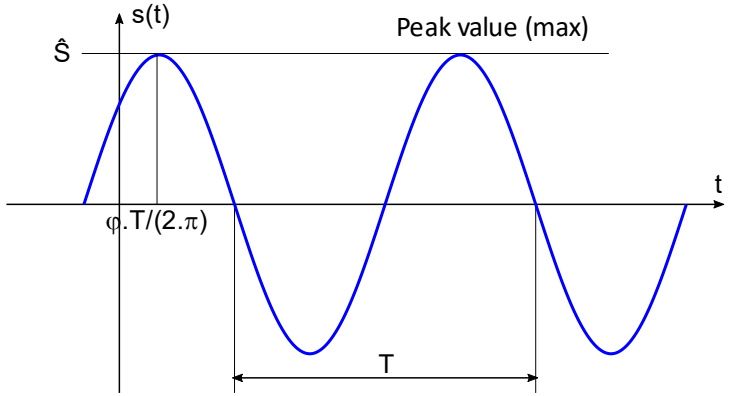
II – Power in sinusoidal regime

Sine-wave: definitions

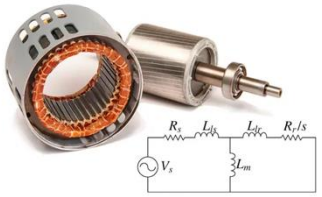
- A sinusoidal quantity s(t) (voltage, current, flux or magnetic field) is written:

$$s(t) = \hat{S}.\cos(\omega t +/-\varphi)$$

$$s(t) = S\sqrt{2}.\cos(\omega t +/-\varphi)$$



- s(t): quantity which evolves with time in sinusoidal form
- S: RMS value of the quantity,
- $\hat{S}$: maximum value reached by the quantity s(t) => $\hat{S}=S.\sqrt{2}$,
- ω : electrical pulsation in rad/s of the magnitude, $\omega= 2.\pi.f = 2.\pi/T$,
- f: signal frequency and T: signal period,
- φ : phase at origin (at t = 0).

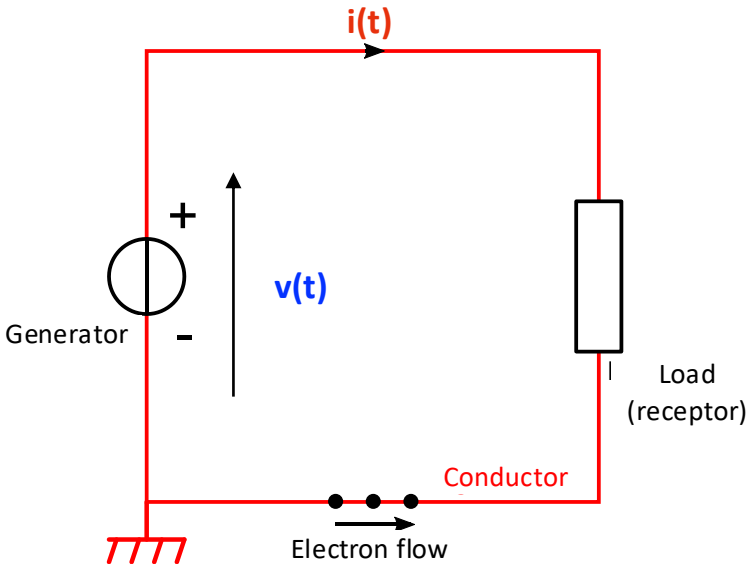


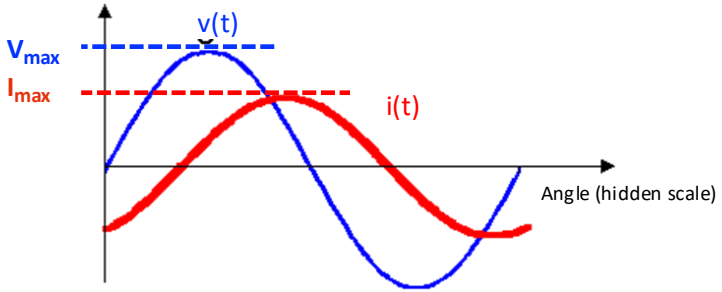
II – Power in sinusoidal regime

Single-phase

- Most of the world's electrical energy is generated, transmitted and distributed in the form of sinusoidal voltages.

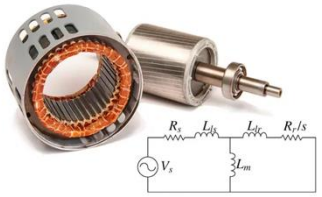
- Any periodic signal can be studied by decomposing it down into sinusoidal signals using a Fourier transform.





$$v(t) = \hat{V}.sin(\omega t)$$

$$i(t) = \hat{I}.sin(\omega t - \varphi)$$

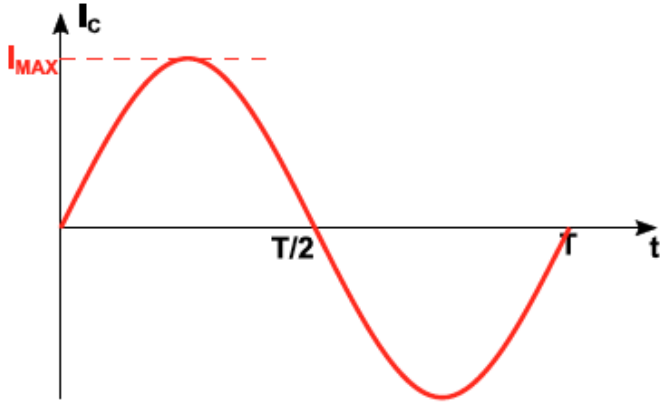


II – Power in sinusoidal regime

Single-phase – Average and RMS values

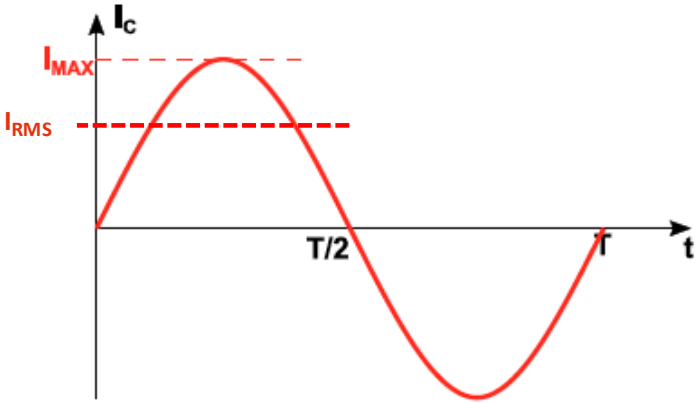
- The AVERAGE VALUE of any current (same for the voltage) is the value that a direct current carrying the same amount of electricity would have.

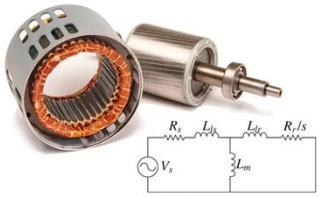
$$I_{MOY} = \frac{1}{T}\int i(t)dt \qquad I_{MOY} = 0$$



- The RMS (Root Main Square) VALUE of any current is the value that a direct current carrying the same amount of energy would have.

$$I_{RMS} = \sqrt{\frac{1}{T}\int i^2(t)dt} \qquad I_{RMS} = \frac{I_{MAX}}{\sqrt{2}}$$





II – Power in sinusoidal regime

Single-phase – Average and RMS values

- Exercise: demonstrate the expression of I_{moy} and I_{RMS}

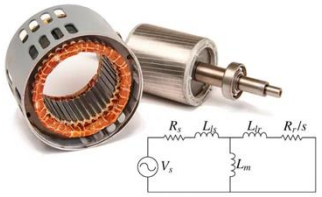
Average value

- $I_{MOY} = \frac{1}{T}\int_0^T i(t)\,dt$
- $I_{MOY} = \frac{1}{T}\int_0^T I_{MAX}.\sin(\omega t)\,dt$
- $I_{MOY} = \frac{1}{T}\left[I_{MAX}.\frac{-\cos(\omega t)}{\omega}\right]_0^T$
- $I_{MOY} = \frac{I_{MAX}}{\omega T}.[-\cos(\omega T) + \cos(0)]$
- $I_{MOY} = \frac{I_{MAX}}{\omega T}.(-1+1)$
- $\boldsymbol{I_{MOY} = 0}$

$cos2a = 1 - 2sin^2a \;\; et \;\; sin^2a = \frac{1-cos2a}{2}$

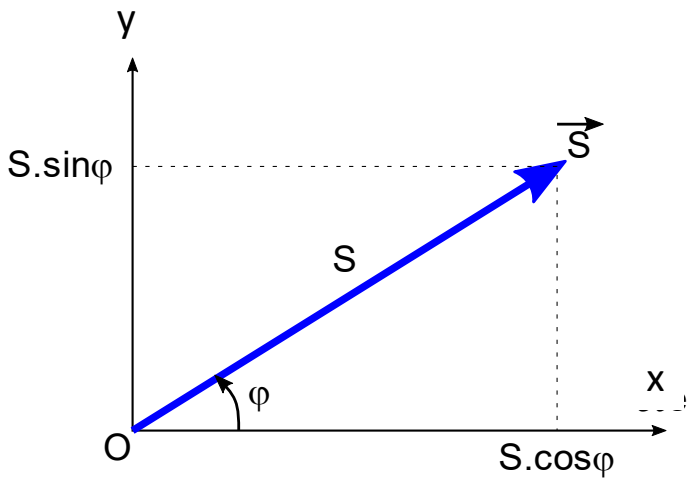
RMS value

- $I_{EFF} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$
- ${I_{EFF}}^2 = \frac{1}{T}\int_0^T i^2(t)\,dt$
- ${I_{EFF}}^2 = \frac{1}{T}\int_0^T (I_{MAX}.\sin(\omega t))^2 dt$
- ${I_{EFF}}^2 = \frac{1}{T}\int_0^T (I_{MAX})^2.(\sin(\omega t))^2 dt$
- ${I_{EFF}}^2 = \frac{1}{T}\int_0^T (I_{MAX})^2 \frac{(1-\cos 2\omega t)}{2}\,dt$
- ${I_{EFF}}^2 = \frac{{I_{MAX}}^2}{2.T}\int_0^T (1-\cos 2\omega t)\,dt$
- ${I_{EFF}}^2 = \frac{{I_{MAX}}^2}{2.T}\left[t - \frac{\sin(2\omega t)}{2\omega}\right]_0^T$
- ${I_{EFF}}^2 = \frac{{I_{MAX}}^2}{2}\left[\frac{T-0}{T} - \frac{\sin(2\omega T)-\sin(0)}{2\omega T}\right]$
- ${I_{EFF}}^2 = \frac{{I_{MAX}}^2}{2}\left[1 - \frac{0-0}{4\pi}\right]$
- ${I_{EFF}}^2 = \frac{{I_{MAX}}^2}{2}$
- $\boldsymbol{I_{EFF} = \frac{I_{MAX}}{\sqrt{2}}}$

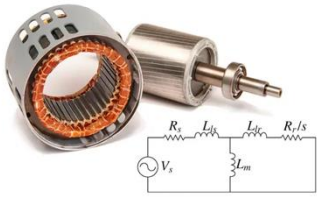


Single-phase – Fresnel vector representation

- Associated with s(t) is a vector $\vec{S}$ known as the Fresnel vector, of norm S (RMS value) rotating around the origin point O at an angular frequency w.



- Since all signals have the same angular frequency w, vectors in the same Fresnel diagram rotate at the same speed. Therefore, they are represented at t = 0.



II – Power in sinusoidal regime

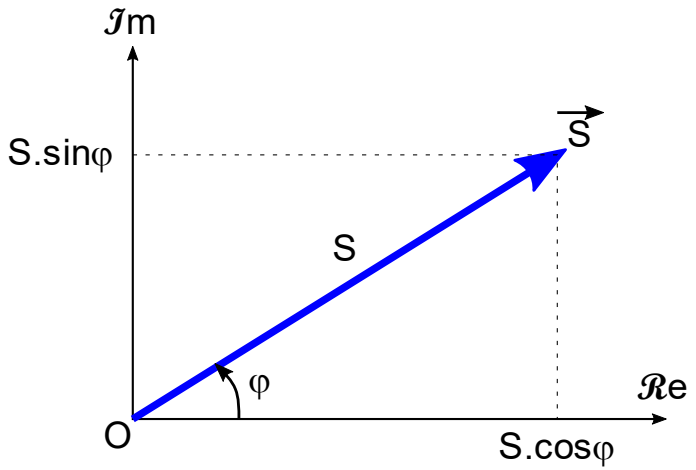
Single-phase – Complex representation

- Complex representation:

- Reminder: Euler's formula

$$e^{ix} = cos\, x + i.sin\, x$$

$$S.\cos\varphi + j.S.\sin\varphi = S.e^{j\varphi} = \underline{S}$$



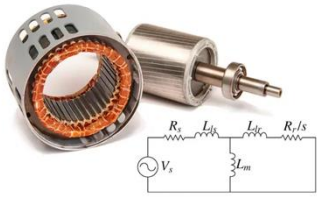
- Complex quantity

$$\begin{cases} \underline{s}(t) = S\sqrt{2}.e^{j(\omega t+\varphi)} \\ s(t) = \Re e\left(\underline{s}(t)\right) \end{cases}$$

- Complex amplitude

$$\underline{s} = \underline{S}\sqrt{2}.e^{j\omega t}$$

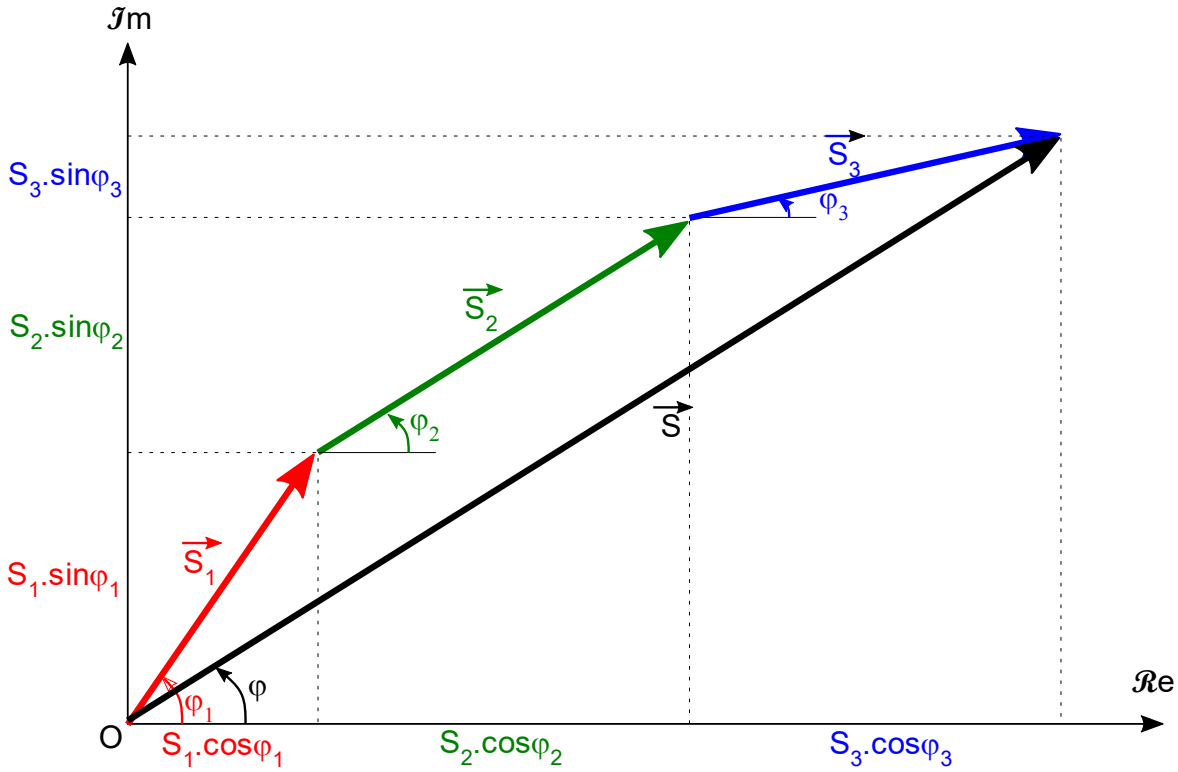
where $\underline{S} = S.e^{j\varphi} = [S;\varphi]$



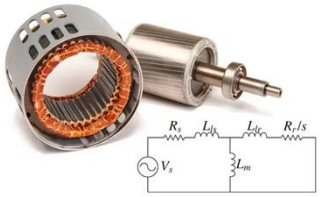
II – Power in sinusoidal regime

Single-phase – Complex representation

- Sum:



$$S = \sqrt{(\sum S_i.\cos\varphi_i)^2 + (\sum S_i.\sin\varphi_i)^2} \qquad \tan\varphi = \frac{\sum S_i.\sin\varphi_i}{\sum S_i.\cos\varphi_i}$$



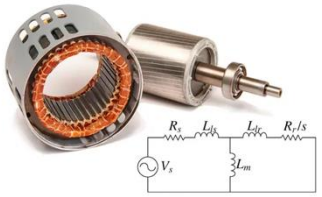
II – Power in sinusoidal regime

Single-phase – Complex representation

- Derivate:

$$\underline{s}(t) = \hat{S}.e^{j(\omega t+\varphi)} = \hat{S}.\cos(\omega t+\varphi) + j.\hat{S}.\sin(\omega t+\varphi)$$

$$\frac{d\underline{s}(t)}{dt} = \frac{d\left(\hat{S}.\cos(\omega t+\varphi) + j.\hat{S}.\sin(\omega t+\varphi)\right)}{dt}$$



Single-phase – Complex representation

- Derivate: $\underline{s}(t) = \hat{S}.e^{j(\omega t+\varphi)} = \hat{S}.\cos(\omega t+\varphi) + j.\hat{S}.\sin(\omega t+\varphi)$

$$\frac{d\underline{s}(t)}{dt} = \frac{d\left(\hat{S}.\cos(\omega t+\varphi) + j.\hat{S}.\sin(\omega t+\varphi)\right)}{dt}$$

$$\frac{d\underline{s}(t)}{dt} = \hat{S}.[-\omega.\sin(\omega t+\varphi) + j.\omega.\cos(\omega t+\varphi)]$$

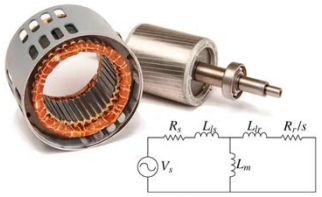
$$\frac{d\underline{s}(t)}{dt} = j.\omega.\hat{S}.\left[-\frac{\sin(\omega t+\varphi)}{j} + \cos(\omega t+\varphi)\right]$$

$$\frac{d\underline{s}(t)}{dt} = j.\omega.\hat{S}.[\cos(\omega t+\varphi) + j\sin(\omega t+\varphi)]$$

$$\boldsymbol{\frac{d\underline{s}(t)}{dt} = j.\omega.\underline{s}(t)}$$

Derivative with respect to time: **rotation of +π/2 rad** in the complex plane

$$\boldsymbol{\frac{d\underline{s}(t)}{dt} = j.\omega.\underline{s}(t) = \left[S.\omega; \varphi + \frac{\pi}{2}\right]}$$

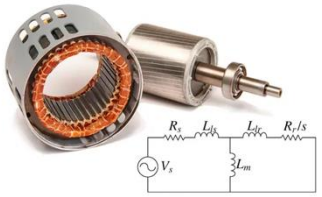


Single-phase – Complex representation

- Integrate:

$$\underline{s}(t) = \hat{S}.e^{j(\omega t+\varphi)} = \hat{S}.\cos(\omega t+\varphi) + j.\hat{S}.\sin(\omega t+\varphi)$$

$$\int \underline{s}(t).dt = \int \left(\hat{S}.\cos(\omega t+\varphi) + j.\hat{S}.\sin(\omega t+\varphi)\right).dt$$



II – Power in sinusoidal regime

Single-phase – Complex representation

- Integrate:

$$\int \underline{s}(t).dt = \int \left(\hat{S}.\cos(\omega t+\varphi) + j.\hat{S}.\sin(\omega t+\varphi)\right).dt$$

$$\int \underline{s}(t).dt = \hat{S}.\left[\int (\cos(\omega t+\varphi)).dt + j.\int (\sin(\omega t+\varphi)).dt\right] = \hat{S}.\left[\frac{\sin(\omega t+\varphi)}{\omega} + j.\frac{-\cos(\omega t+\varphi)}{\omega}\right]$$

$$\int \underline{s}(t).dt = \frac{\hat{S}}{j\omega}.[j.\sin(\omega t+\varphi) + j.j.(-\cos(\omega t+\varphi))]$$

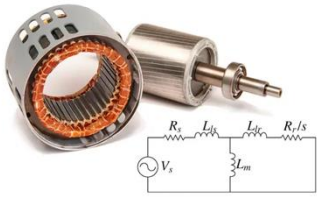
$$\int \underline{s}(t).dt = \frac{\hat{S}}{j\omega}.[j.\sin(\omega t+\varphi) + \cos(\omega t+\varphi)]$$

$$\int \underline{s}(t).dt = \frac{1}{j\omega}.\hat{S}.[\cos(\omega t+\varphi) + j.\sin(\omega t+\varphi)]$$

$$\int \underline{s}(t).dt = \frac{\underline{s}(t)}{j.\omega}$$

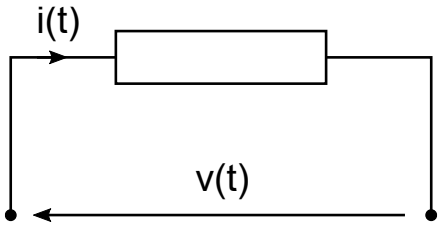
Time integration: **rotation of -π/2 rad** in the complex plane

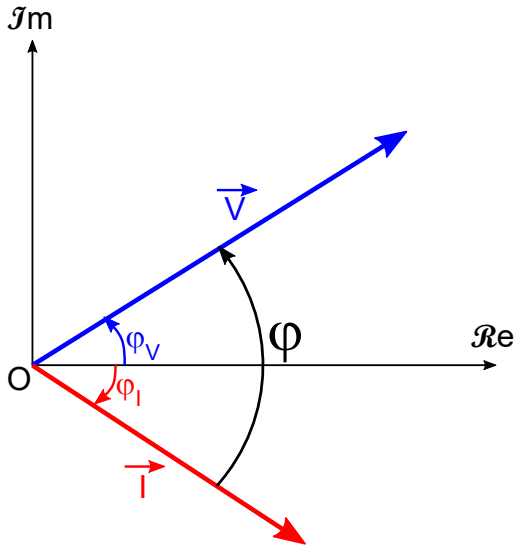
$$\int \underline{s}(t).dt = \frac{\underline{s}(t)}{j.\omega} = \left[\frac{S}{\omega}; \varphi - \frac{\pi}{2}\right]$$



Single-phase – Complex impedances

- Let us consider a passive linear dipole, in receptor convention, subjected to a sinusoidal voltage v(t) and flowed by a sinusoidal current i(t):

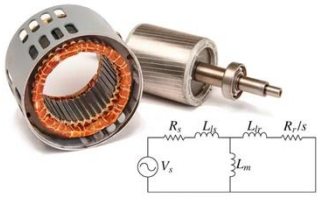




- We write:

$$\begin{cases} v(t) = V\sqrt{2}.\cos(\omega t + \varphi_V) \\ i(t) = I\sqrt{2}.\cos(\omega t + \varphi_I) \end{cases}$$

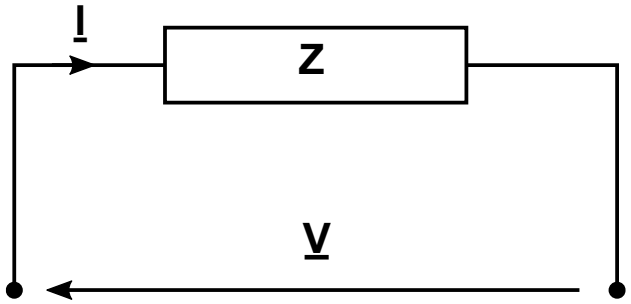
$$\begin{cases} \underline{V} = V.e^{j\varphi_V} = [V;\varphi_V] \\ \underline{I} = I.e^{j\varphi_I} = [I;\varphi_I] \end{cases}$$



Single-phase – Complex impedances

- The complex impedance is defined as:

$$\underline{Z} = \frac{\underline{V}}{\underline{I}} = R + jX = \underline{Z} = Z.e^{j\varphi}$$

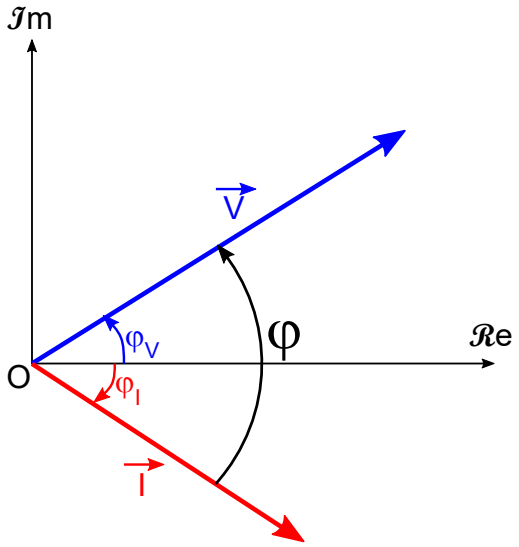


- where:

- Z : dipole impedance $Z = \|\underline{Z}\| = \frac{V}{I}$

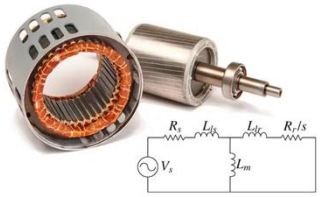
- φ : phase shift angle of voltage with respect to current:

$$\varphi = Arg\underline{Z} = \varphi_V - \varphi_I$$



- R: resistance in Ohms of the dipole, real part of the complex impedance: $R = \Re e(\underline{Z}) = Z.cos\varphi$

- X : reactance in Ohms of the dipole, imaginary part of the complex impedance $X = \Im m(\underline{Z}) = Z.sin\varphi$



II – Power in sinusoidal regime

Single-phase – Complex impedances

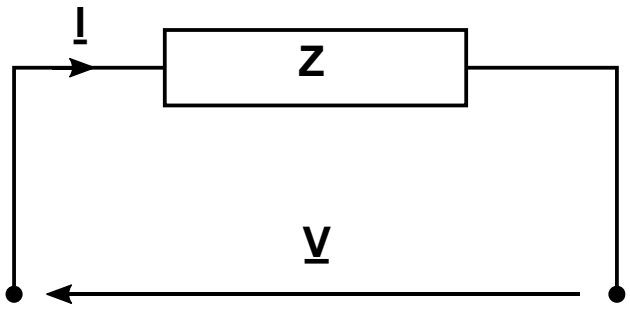
$$\underline{Z} = \frac{\underline{V}}{\underline{I}} = R + jX = \underline{Z} = Z.e^{j\varphi}$$

- where:

- Z : dipole impedance $Z = \|\underline{Z}\| = \frac{V}{I}$

- φ : phase shift angle of voltage with respect to current:

$$\varphi = Arg\underline{Z} = \varphi_V - \varphi_I$$



- If $\varphi > 0$, the voltage leads the current, impedance or load is said to be inductive.

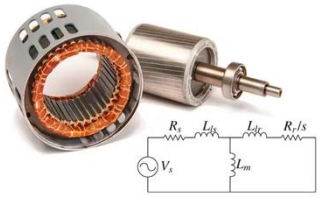
(Motors, Transformers, Electromagnets)

- If $\varphi = 0$, the voltage in in phase with the current, impedance or load is said to be resistive.

(Resistors, Furnaces, Regulated baths)

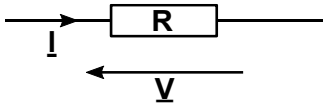
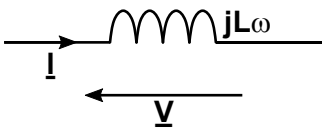
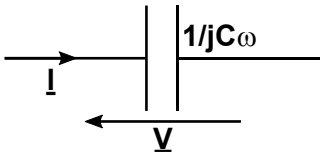
- If $\varphi < 0$, the voltage lags the current, impedance or load is said to be capacitive.

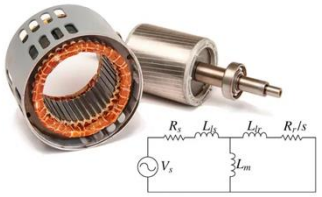
(Capacitors)



II – Power in sinusoidal regime

Single-phase – linear dipoles

	Resistor	Ideal coil	Ideal capacitor
Definition	An electrical resistor is an electrical dipole that opposes the flow of current	A “coil” is an electrical dipole that opposes the variation of electric current and can store energy in electromagnetic form.	A “capacitor” is an electrical dipole that resists variation in the voltage to which it is subjected, and can store energy in electrostatic form.
Characteristics	R : resistance in Ω (Ohm)	L : Coil inductance in H (Henry)	C : Capacitance in F (Farad)
AC regime	$V_R(t) = R.i(t)$	$V_L(t) = L.\frac{di(t)}{dt}$	$V_C(t) = \frac{1}{C}.\int i(t).dt$
Symbolic representation in receptor convention			
Sinusoidal regime	$\underline{V_R} = R.\underline{I}$	$\underline{V_L} = jL\omega.\underline{I}$	$\underline{V_C} = \frac{1}{jC\omega}.\underline{I}$
Impedance	$\underline{Z} = \mathrm{R} = [R; 0]$	$\underline{Z} = \mathrm{jL}\omega = \left[L\omega; \frac{\pi}{2}\right]$	$\underline{Z} = \frac{1}{jC\omega} = \left[\frac{1}{jC\omega}; -\frac{\pi}{2}\right]$



Single-phase – Fresnel (vector) representation

$$\underline{V_R} = [R.I; 0]$$

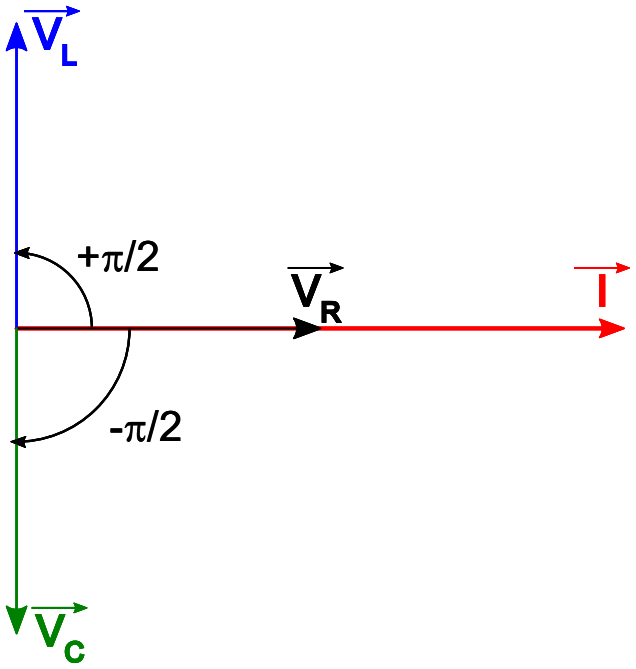
V_R and I are in phase

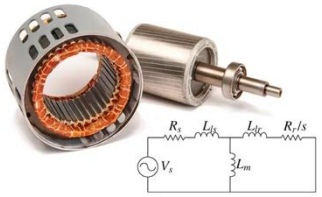
$$\underline{V_L} = [L\omega.I; \frac{\pi}{2}]$$

V_L leads I in quadrature

$$\underline{V_C} = [\frac{I}{C\omega}; -\frac{\pi}{2}]$$

V_C lags I in quadrature





II – Power in sinusoidal regime

Single-phase – Expressions of powers

- Instantaneous power (in W): $\boldsymbol{p(t) = v(t) \times i(t)}$

=> exchange of energy, heat or work, between the network and the dipole at each instant

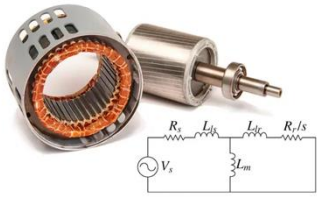
- Active power or average power (in W) – General expression:

$$\boldsymbol{P = < p(t) > = \frac{1}{T} \int_0^T p(t).dt = \frac{1}{T} \int_0^T v(t).i(t).dt}$$

=> Balance of energy exchanged between the network and the dipole over a period T

=> If P > 0, the dipole consumes energy, it operates a receptor

=> If P < 0, the dipole supplies energy, it operates a generator



II – Power in sinusoidal regime

Single-phase – Expressions of powers

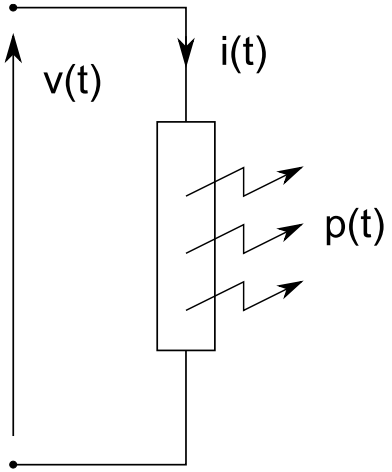
- *Linear receptor in sinusoidal regime*

- Through which a current i(t) flows

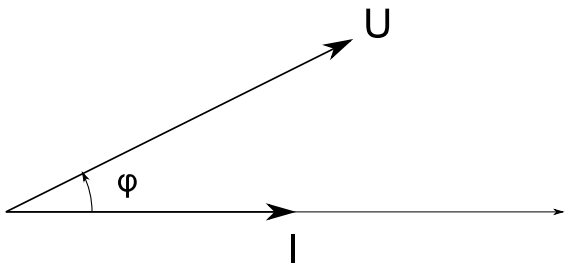
$$i(t) = I\sqrt{2}\times sin(\omega t)$$

- Subjected to a voltage v(t)

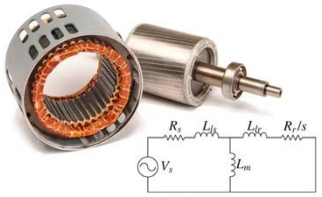
$$v(t) = V\sqrt{2}\times \sin(\omega t + \varphi)$$



φ positive, voltage ahead of current, inductive receiver
φ zero, voltage in phase with current, resistive receiver
φ negative, voltage lagging current, capacitive receiver



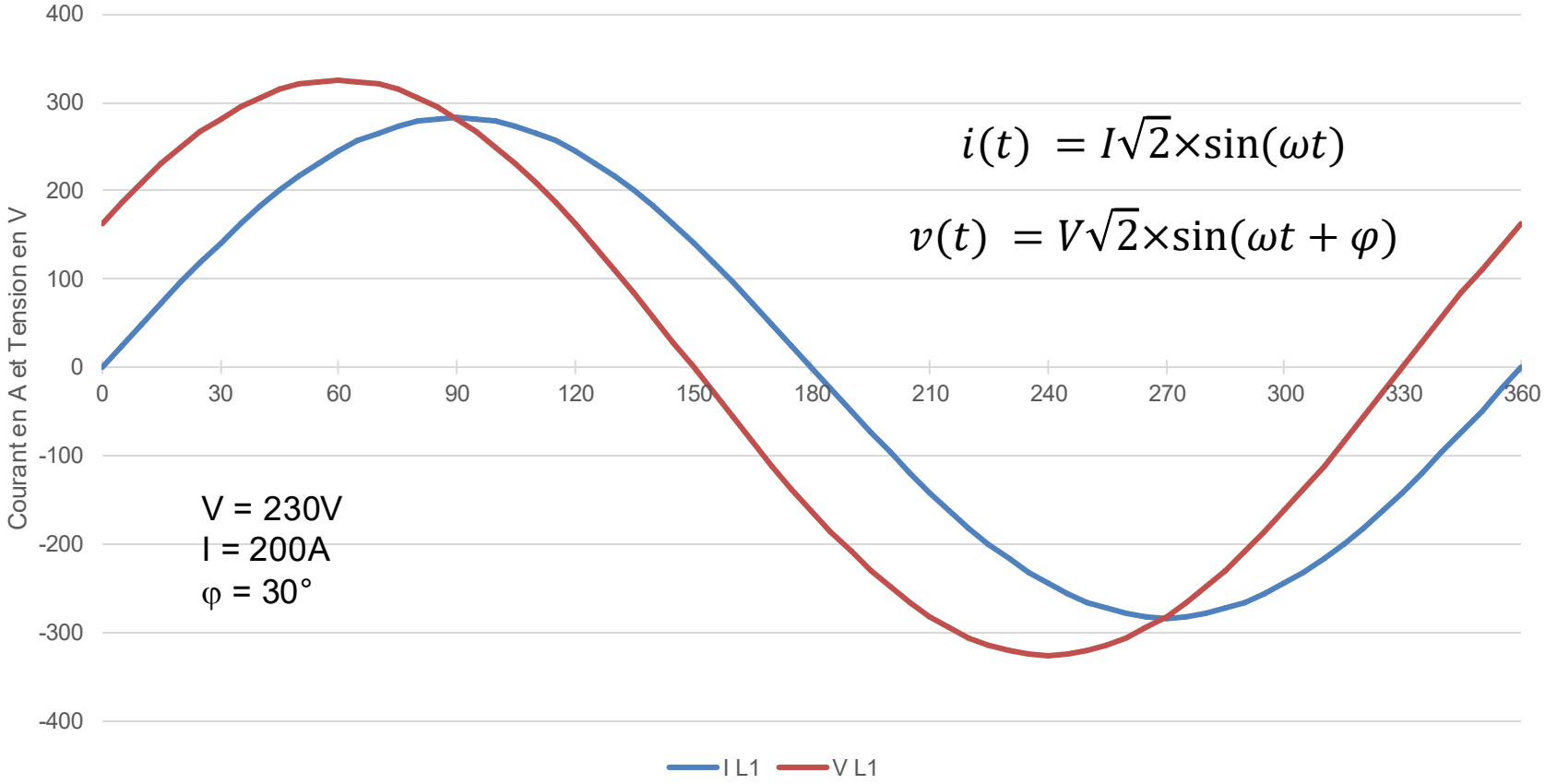
- Instantaneous power $p(t) = v(t)\times i(t)$

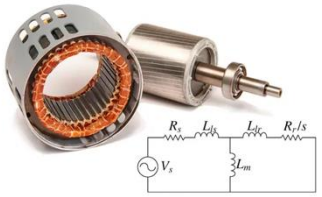


II – Power in sinusoidal regime

Single-phase – Expressions of powers

- Linear receptor in sinusoidal regime: current and voltages





II – Power in sinusoidal regime

Single-phase – Expressions of powers

- *Linear receptor in sinusoidal regime*

- Instantaneous power:

$$p(t) = v(t) \times i(t)$$

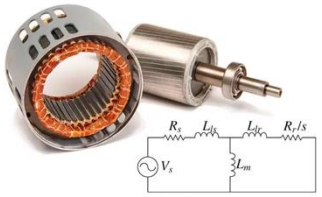
$$p(t) = V\sqrt{2} \times \sin(\omega t + \varphi) \times I\sqrt{2} \times \sin(\omega t)$$

$$p(t) = 2 \times V \times I \times \sin(\omega t + \varphi) \times \sin(\omega t)$$

$$\sin a \times \sin b = \frac{1}{2}(\cos(a - b) + \cos(a + b))$$

$$p(t) = 2 \times V \times I \times \frac{1}{2}(\cos(\omega t + \varphi - \omega t) + \cos(\omega t + \varphi + \omega t))$$

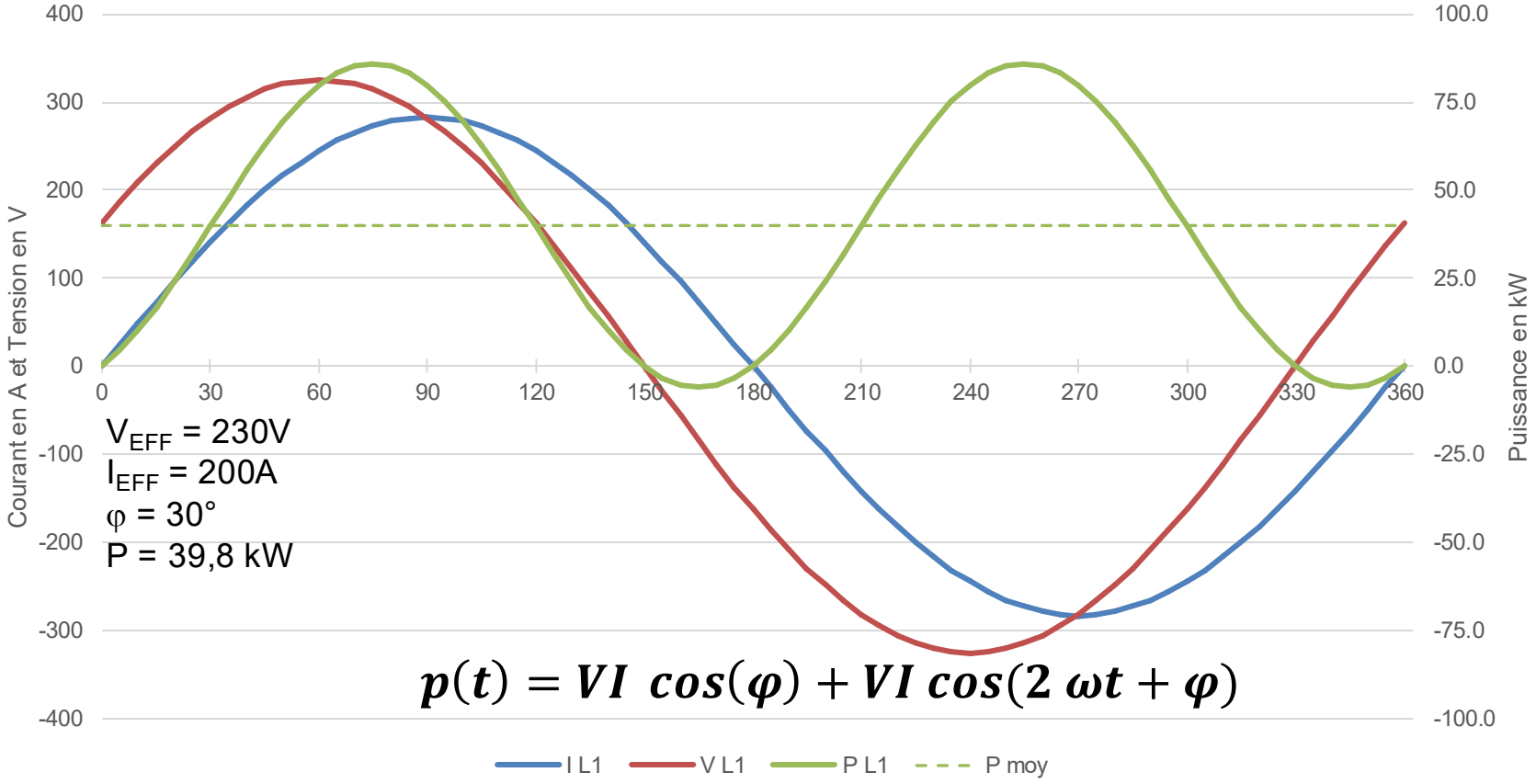
$$\boldsymbol{p(t) = V\,I\,\cos(\varphi) + V\,I\,\cos(2\,\omega t + \varphi)}$$

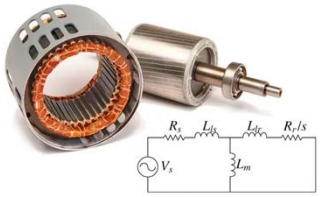


II – Power in sinusoidal regime

Single-phase – Expressions of powers

- Linear receptor in sinusoidal regime: current, voltage and power





II – Power in sinusoidal regime

Single-phase – Expressions of powers

- Instantaneous power (in W): $\boldsymbol{p(t) = v(t) \times i(t)}$

=> exchange of energy, heat or work, between the network and the dipole at each instant

- Active power or average power (in W): $\boldsymbol{P = < p(t) > = V\ I\ cos(\varphi)}$

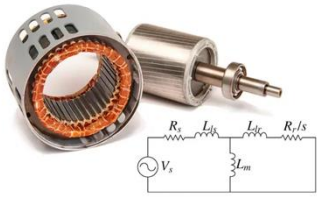
=> Balance of energy exchanged between the network and the dipole over a period T

- Apparent power (in VA): $\boldsymbol{S = V.I}$

=> Design value (voltage for insulation, current for conductor cross-section)

- Power factor (general expression):

$$\boldsymbol{FP = \frac{P}{S}} \qquad FP = cos(\varphi)$$ => Sinusoidal regime only

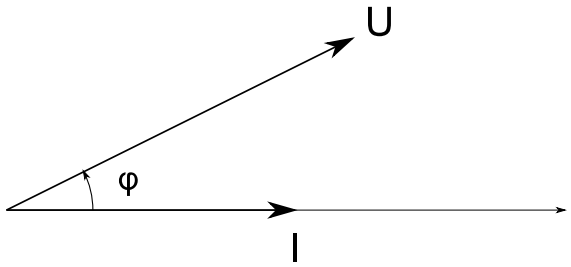


II – Power in sinusoidal regime

Single-phase – Expressions of powers

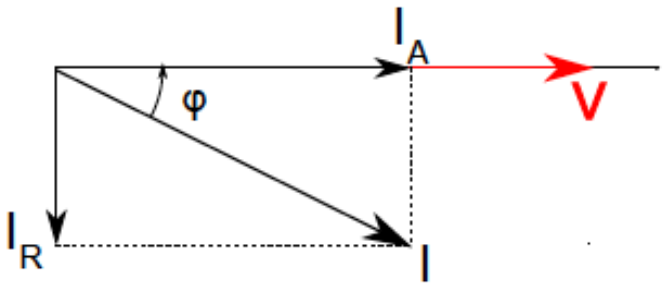
- Reactive power:

- Projection of I on the axes: $\vec{I} = \overrightarrow{I_A} + \overrightarrow{I_R}$



=> Active current: $I_A = I.cos(\varphi)$

=> Reactive current: $I_R = I.sin(\varphi)$

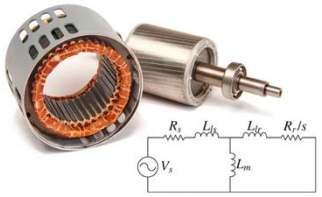


- Reminder of the active power (W):

$$P = V\,.I.cos(\varphi) = V.I_A$$

- By analogy, the reactive power (given in VAR) can be written:

$$\boldsymbol{Q = V\,.I.sin(\varphi) = V.I_R}$$



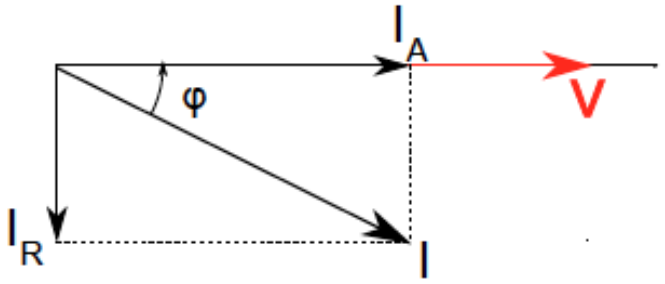
Single-phase – Expressions of powers

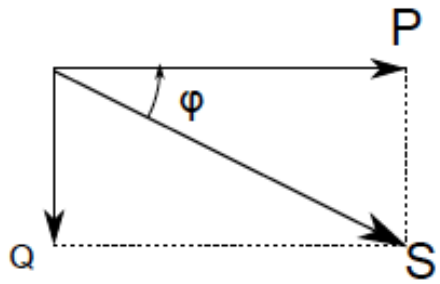
- Relation between powers S, P and Q:

$$P = V \,.\, I.\, cos(\varphi)$$

$$Q = V \,.\, I.\, sin(\varphi) \; = P.\, tan(\varphi)$$

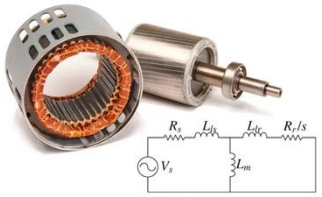
$$S^2 = P^2 + Q^2$$





- Note that: $S \neq P + Q$

- But: $\underline{S} = \underline{V}\,\underline{I}^{*} = P + jQ$



II – Power in sinusoidal regime

Single-phase – Expressions of powers

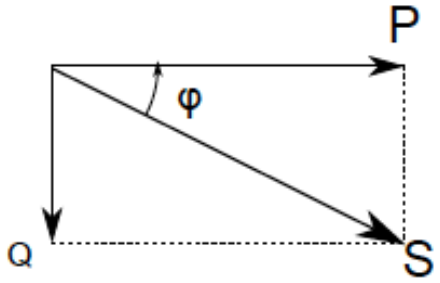
- The resistive load (e.g. a furnace)

=> $\varphi = 0$, voltage in phase with current.

=> $\cos(\varphi) = 1$, $\sin(\varphi) = 0$

=> P = V . I and Q = 0

A resistive dipole consumes no reactive power



- The inductive load (e.g. a motor)

=> The voltage leads the current, $\varphi > 0$

=> $\sin(\varphi) > 0$, **Q > 0**

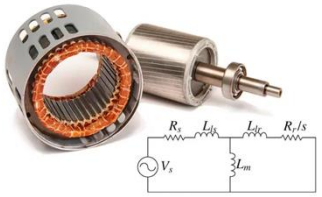
An inductive dipole consumes reactive power

- The capacitive load (e.g. a capacitor)

=> The voltage lags the current, $\varphi < 0$

=> $\sin(\varphi) < 0$, **Q < 0**

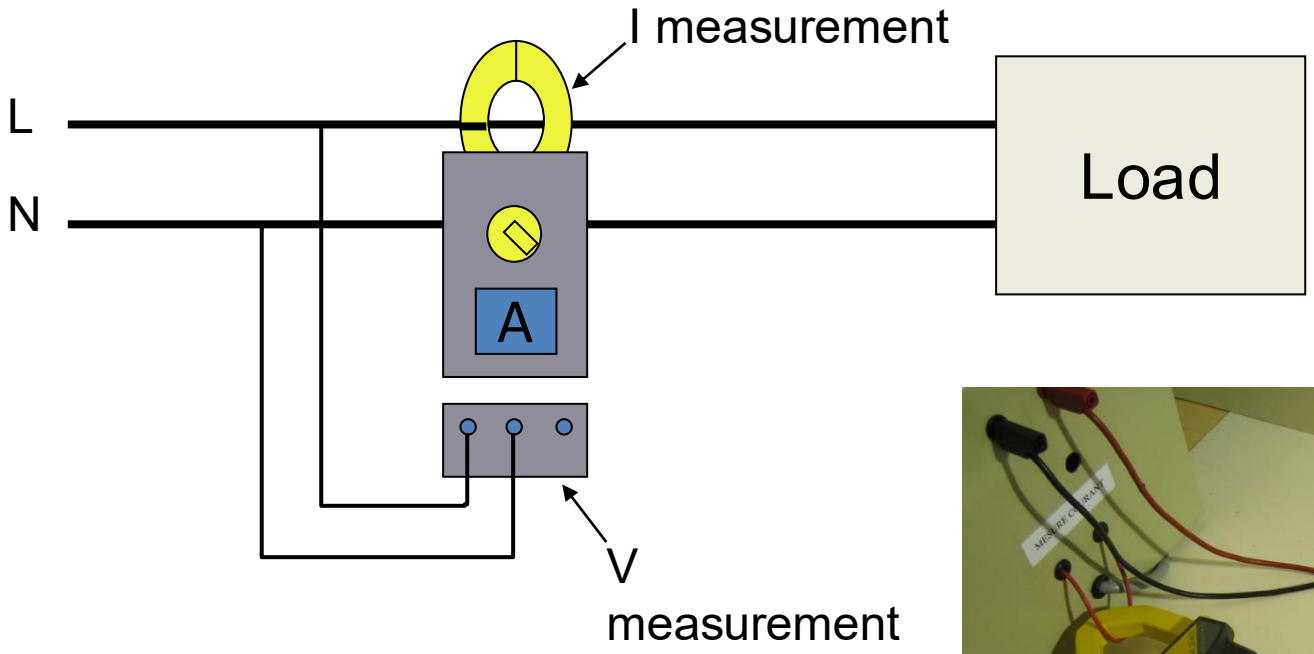
A capacitive dipole supplies reactive power

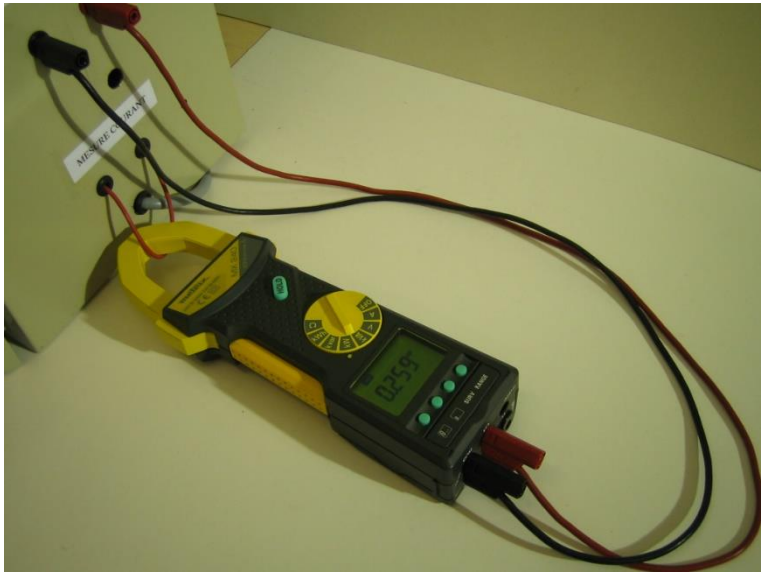


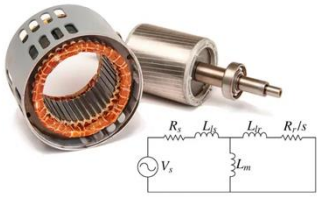
II – Power in sinusoidal regime

Single-phase

- Measurement of powers







II – Power in sinusoidal regime

Single-phase

- Measurement of powers

