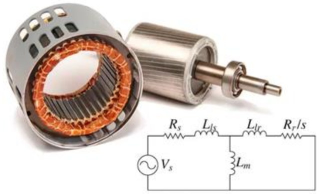
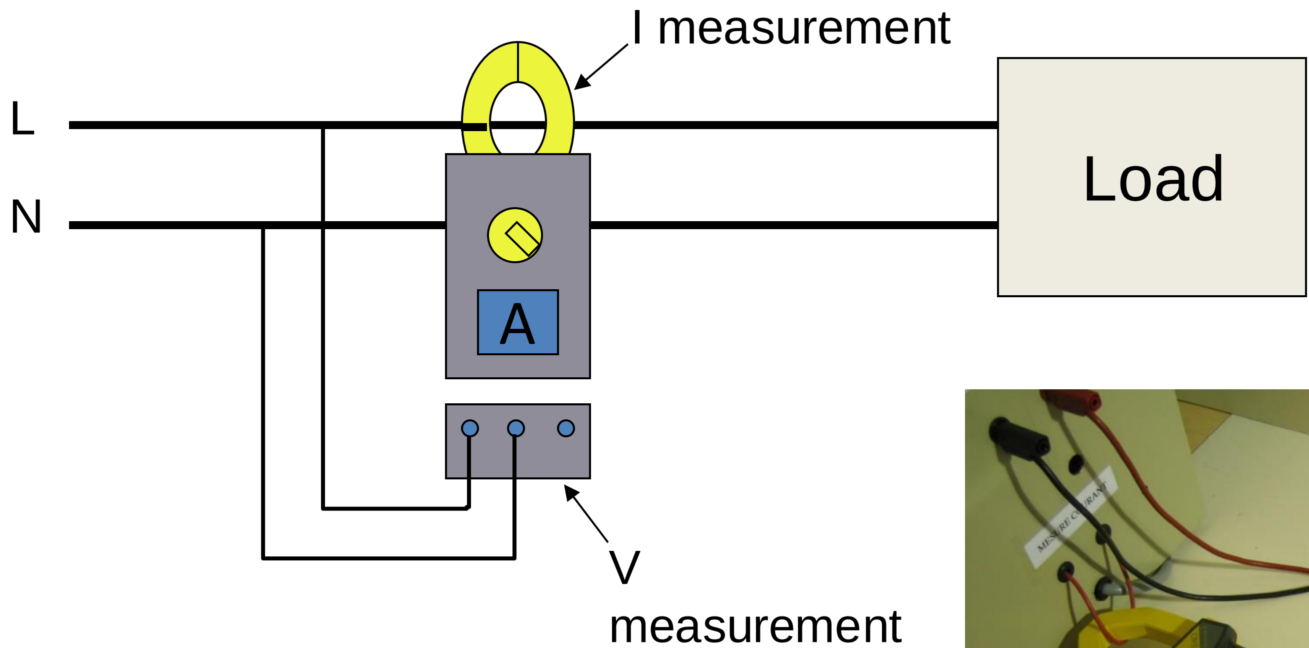
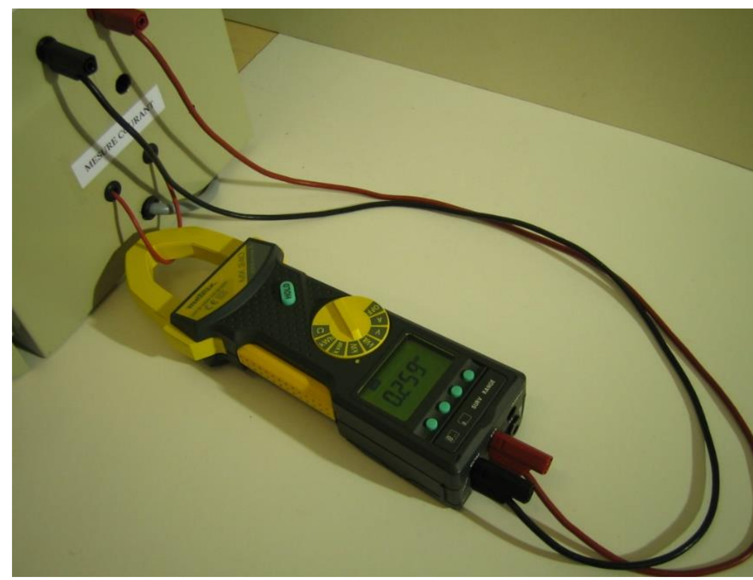


II – Power in sinusoidal regime

Single-phase

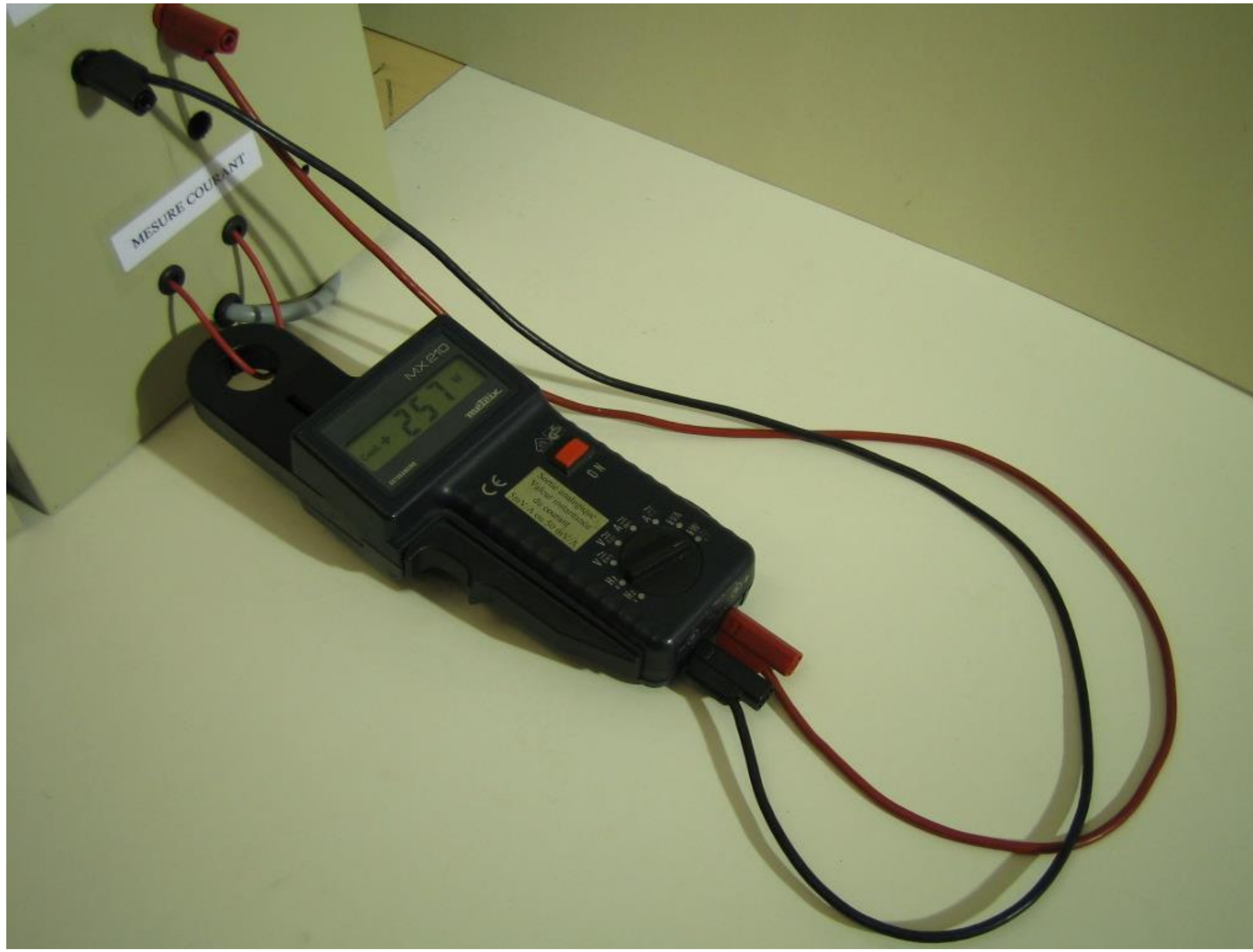
- Measurement of powers

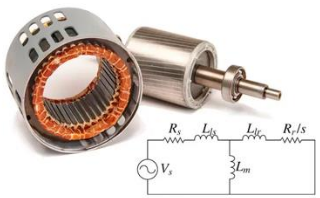




Single-phase

- Measurement of powers

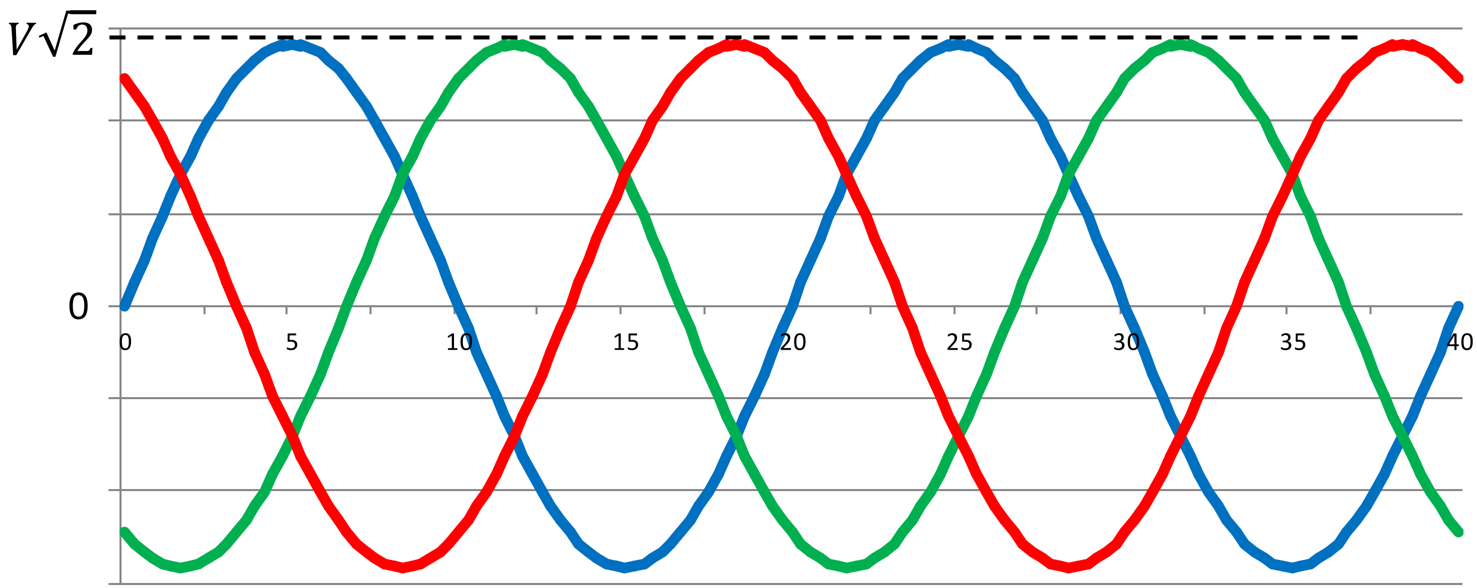




II – Power in sinusoidal regime

3-phase

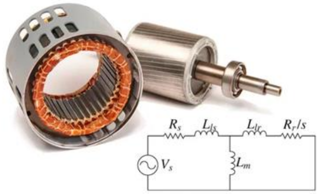
- A direct balanced three-phase voltage system is a set of 3 sinusoidal voltages phase-shifted by 2π/3



$$v_1(t) = V\sqrt{2}\sin(\omega t)$$

$$v_2(t) = V\sqrt{2}\sin(\omega t - \frac{2\pi}{3})$$

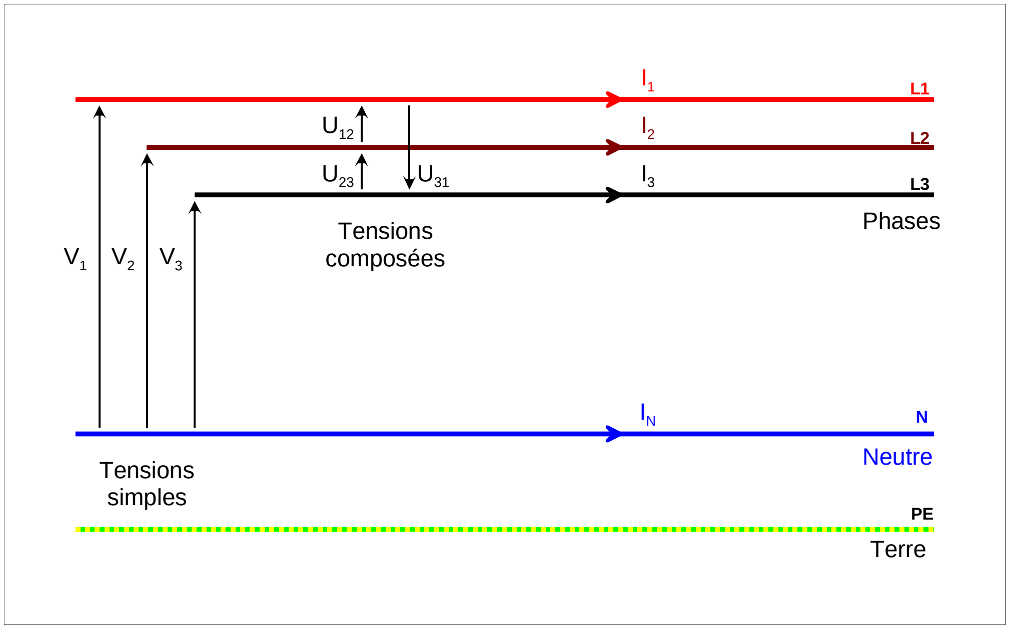
$$v_3(t) = V\sqrt{2}\sin\left(\omega t - \frac{4\pi}{3}\right) = V\sqrt{2}\sin(\omega t + \frac{2\pi}{3})$$



II – Power in sinusoidal regime

3-phase

- Distribution of 3-phase system: BT (low voltage) 230V / 400 V

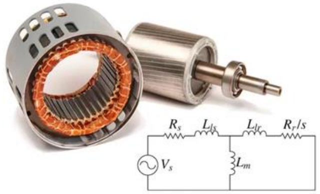


- 4-wire (three phases and neutral)

- Voltages 230 / 400 volts: internationally harmonized

- U = 400V / V= 230V / f = 50Hz
Transformer neutral grounded.

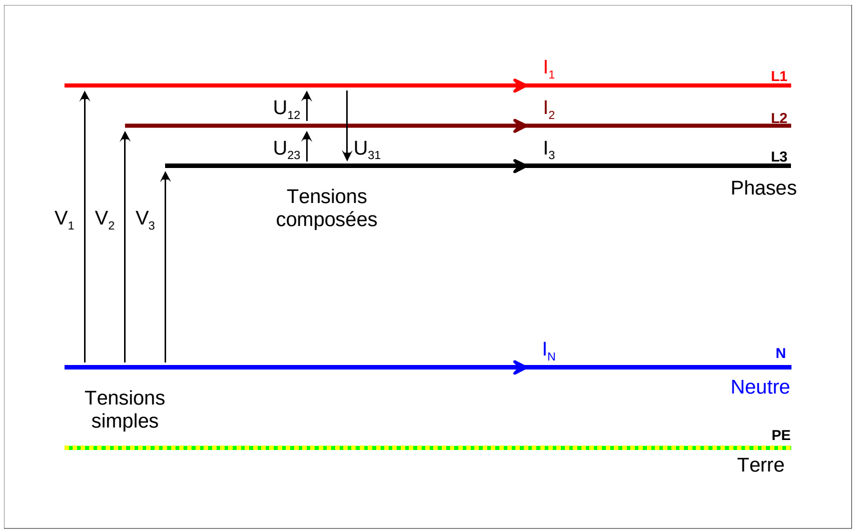
- V is the line voltage = potential difference between a line and the neutral
- U is the phase voltage = potential difference between two phases

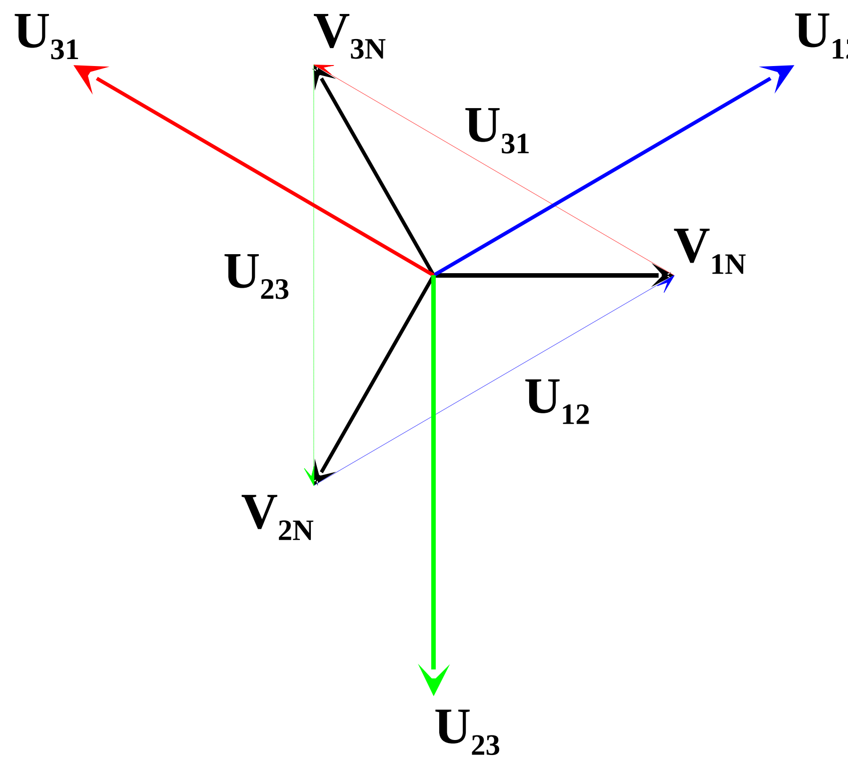


II – Power in sinusoidal regime

3-phase

- Distribution of 3-phase system: direct balanced 3-phase system





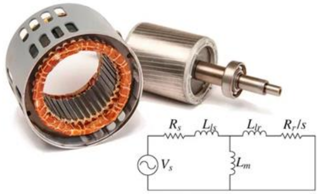
$$\vec{V}_1 + \vec{V}_2 + \vec{V}_3 = 0$$

$$\vec{U}_{12} = \vec{V}_1 - \vec{V}_2$$

$$\vec{U}_{23} = \vec{V}_2 - \vec{V}_3$$

$$\vec{U}_{31} = \vec{V}_3 - \vec{V}_1$$

$$\vec{U}_{12} + \vec{U}_{23} + \vec{U}_{31} = 0$$



II – Power in sinusoidal regime

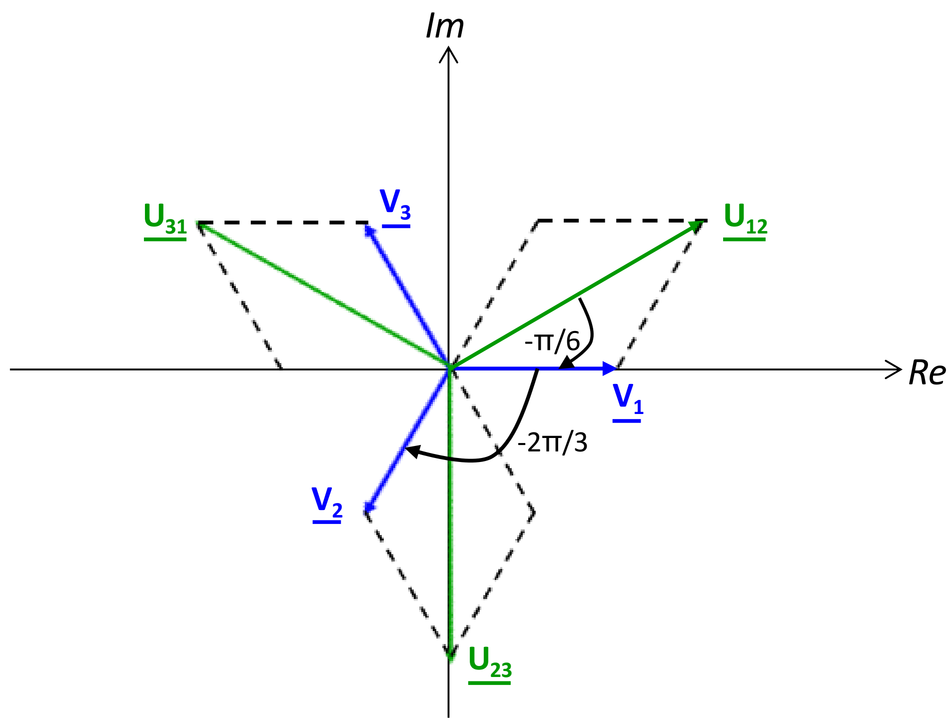
3-phase

- Distribution of 3-phase system: direct balanced 3-phase system

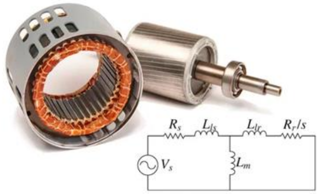
- Relation between U and V

$$\vec{U}_{12} = \vec{V}_1 - \vec{V}_2$$
$$\vec{U}_{23} = \vec{V}_2 - \vec{V}_3$$
$$\vec{U}_{31} = \vec{V}_3 - \vec{V}_1$$

$$U = \sqrt{3}V$$

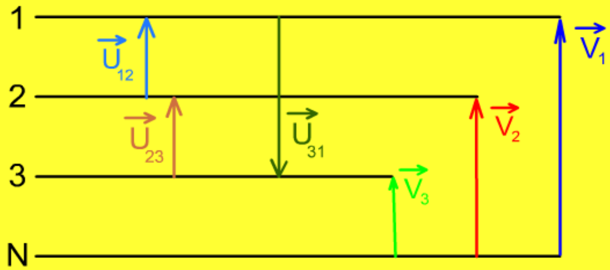


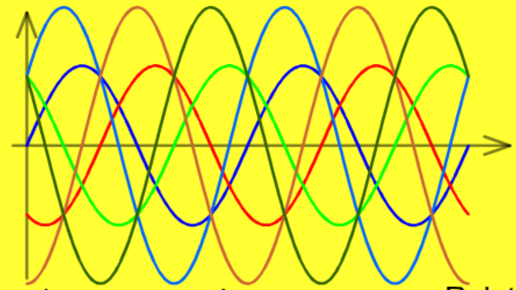
- Balanced: $V_1 = V_2 = V_3 = V$ $\quad U_{12} = U_{23} = U_{31} = U$



II – Power in sinusoidal regime

3-phase: summary





Tensions simples

$v_1(t) = V_{MAX} \sin(\omega t)$

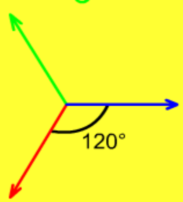
$v_2(t) = V_{MAX} \sin(\omega t - \frac{2\pi}{3})$

$v_2(t) = V_{MAX} \sin(\omega t - \frac{4\pi}{3})$

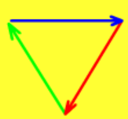
avec

$\omega = 2.\pi.f$

$V_{MAX} = V.\sqrt{2}$



$\vec{V}_1 + \vec{V}_2 + \vec{V}_3 = \vec{0}$



Tensions composées

$u_{12}(t) = U_{MAX} \sin(\omega t + \frac{\pi}{6})$

$u_{23}(t) = U_{MAX} \sin(\omega t - \frac{\pi}{2})$

$u_{31}(t) = U_{MAX} \sin(\omega t - \frac{7\pi}{6})$

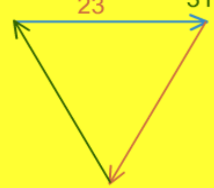
avec

$\omega = 2.\pi.f$

$U_{MAX} = U.\sqrt{2}$

$U = V\sqrt{3}$

$\vec{U}_{12} + \vec{U}_{23} + \vec{U}_{31} = \vec{0}$

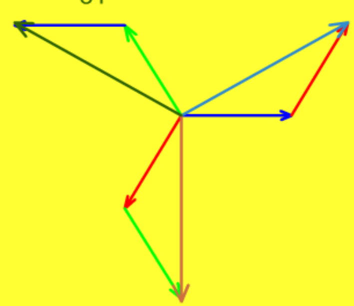


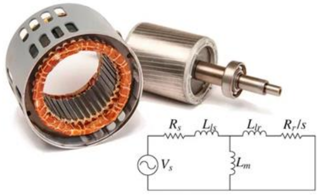
Relations entres tensions simples et composées

$\vec{U}_{12} = \vec{V}_1 - \vec{V}_2$

$\vec{U}_{23} = \vec{V}_2 - \vec{V}_3$

$\vec{U}_{31} = \vec{V}_3 - \vec{V}_1$

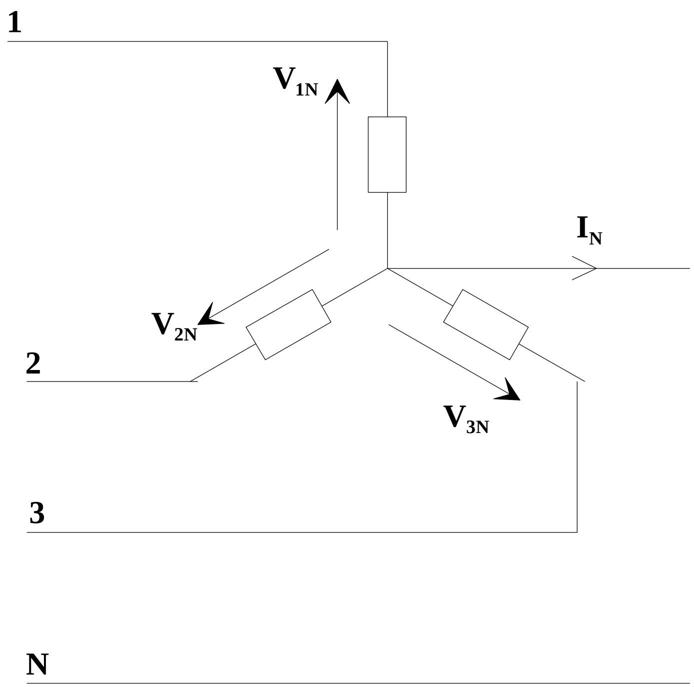




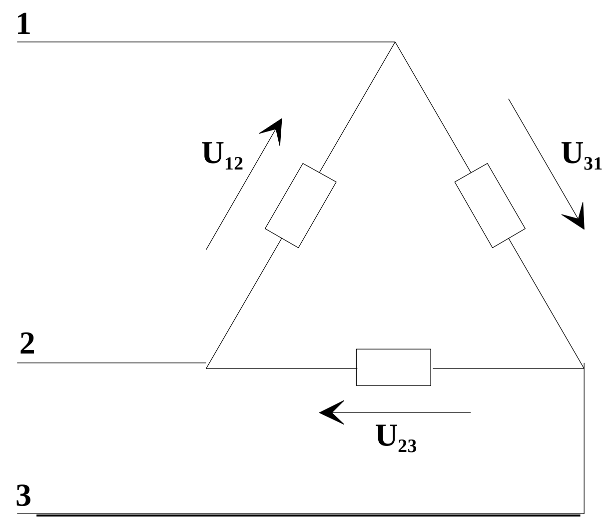
3-phase: connections

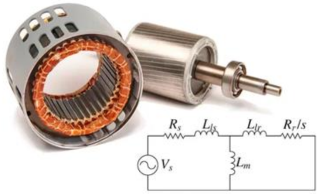
- Both the 3-phase generator and the 3-phase load can be connected according to the following configurations:

Y connection



Delta connection



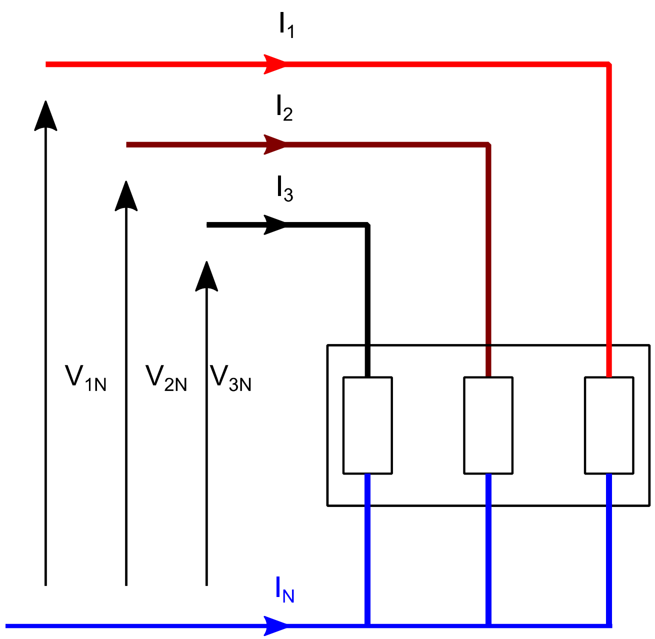


II – Power in sinusoidal regime

3-phase: connections

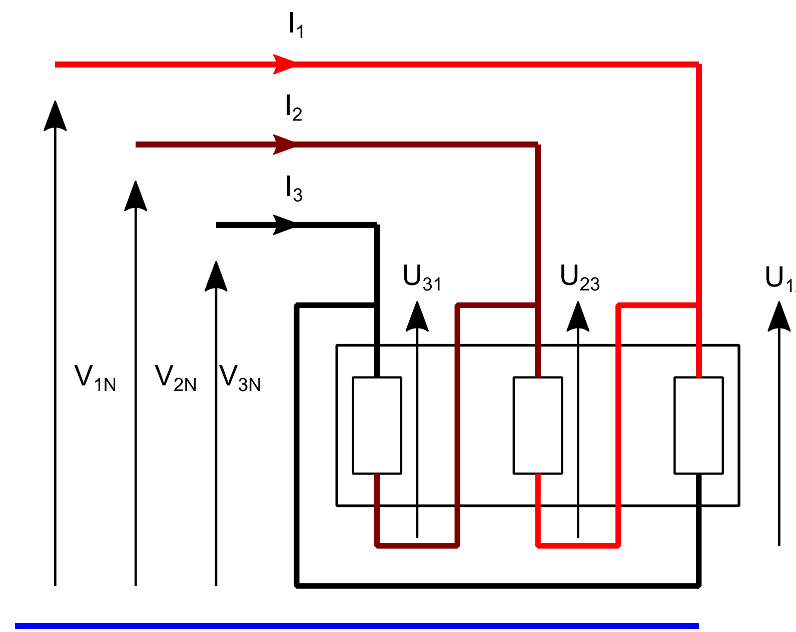
Y connection

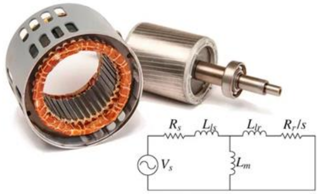
- Each receptor is connected between phase and neutral and subjected to the line voltage



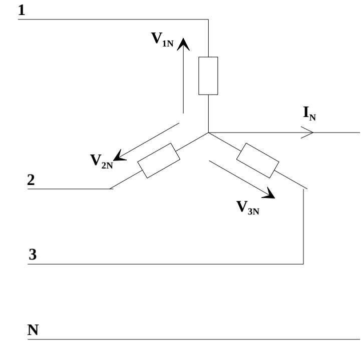
Delta connection

- Each receiver is connected between two phases and subjected to a phase voltage





II – Power in sinusoidal regime



3-phase: Y connection

- Each receptor is connected between phase and neutral.

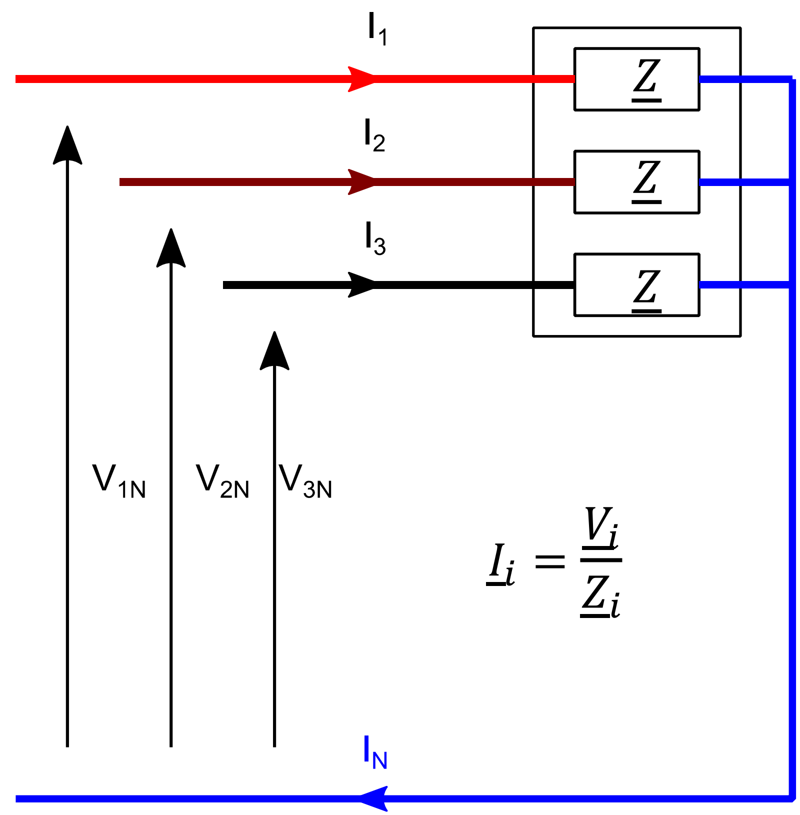
- Each receptor is subjected to the line voltage "V".

- The line current "I" flows through each receptor.

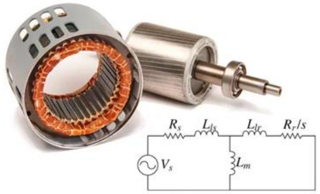
- If all three loads are identical, we speak of a balanced load. $I_1 = I_2 = I_3 = I$

⇒ No current flows in the neutral wire.
⇒ I_N = 0 A.
⇒ Its presence is therefore unnecessary.

$$\vec{I}_1 + \vec{I}_2 + \vec{I}_3 = 0$$

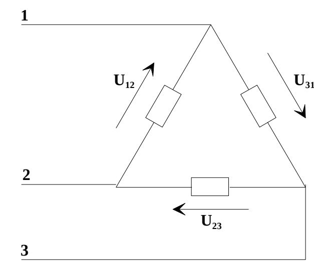


$$\underline{I_i} = \frac{\underline{V_i}}{\underline{Z_i}}$$



II – Power in sinusoidal regime

3-phase: Delta connection



- Each receptor is connected between two phases.

- It is subjected to a phase voltage “U”.

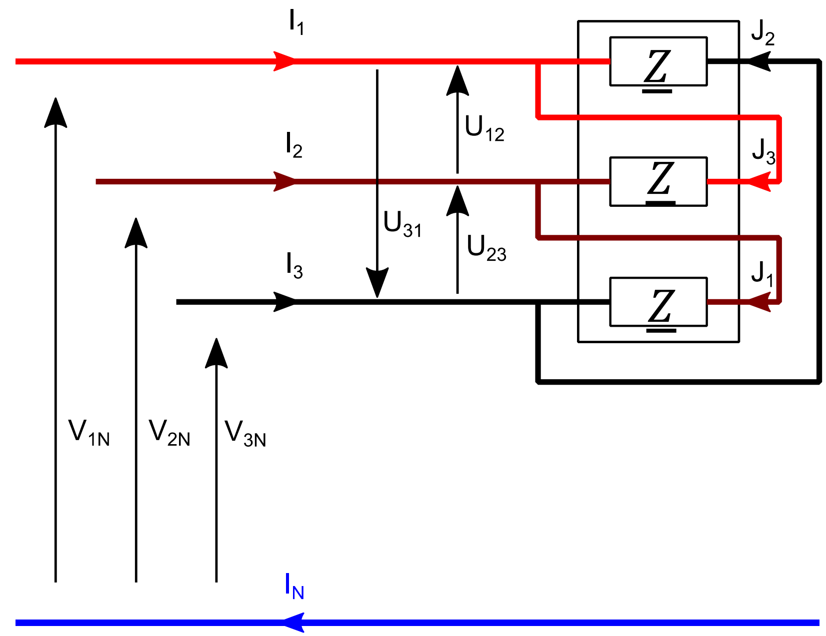
- The phase current “J” flows through the receptor.

- Here, the neutral cannot be wired. Its presence is also not necessary.

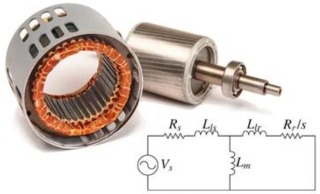
- Kirchoff’s law:

$$\vec{I}_1 = \vec{J}_3 - \vec{J}_2$$
$$\vec{I}_2 = \vec{J}_1 - \vec{J}_3$$
$$\vec{J}_3 = \vec{J}_2 - \vec{V}_1$$

$$I = J\sqrt{3}$$



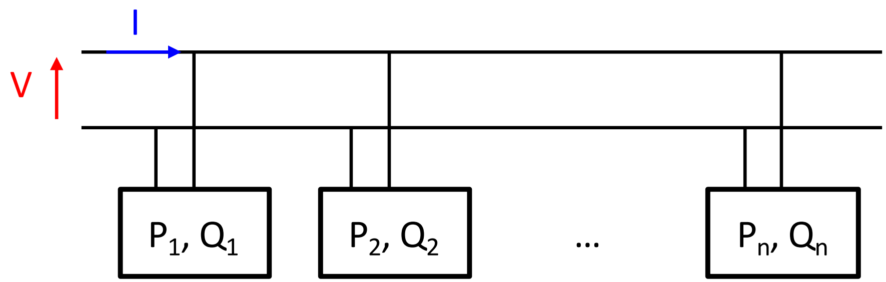
- Balanced load: $J_1 = J_2 = J_3 = J$



II – Power in sinusoidal regime

3-phase: powers

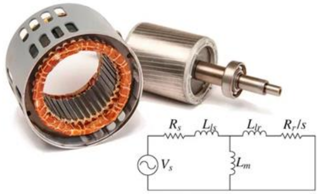
- Boucherot's theorem:



The active power of a system is the sum of the active powers of each element, as are reactive power and complex apparent power. Note that this is not true for real apparent power.

- Active power: $P_{tot} = P_1 + P_2 + \cdots + P_n$
- Reactive power: $Q_{tot} = Q_1 + Q_2 + \cdots + Q_n$
- Complex apparent power: $\underline{S}_{tot} = \underline{S}_1 + \underline{S}_2 + \cdots + \underline{S}_n$
- But: $S_{tot} \neq S_1 + S_2 + \cdots + S_n$

$$S = V.I = \sqrt{P_{tot}^2 + Q_{tot}^2}$$



II – Power in sinusoidal regime

3-phase: powers

- Boucherot's theorem: *general* Y-connection load

$$P = V_1 I_1 cos(\varphi_1) + V_2 I_2 cos(\varphi_2) + V_3 I_3 cos(\varphi_3)$$

$$Q = V_1 I_1 sin(\varphi_1) + V_2 I_2 sin(\varphi_2) + V_3 I_3 sin(\varphi_3)$$

- For the balanced load:

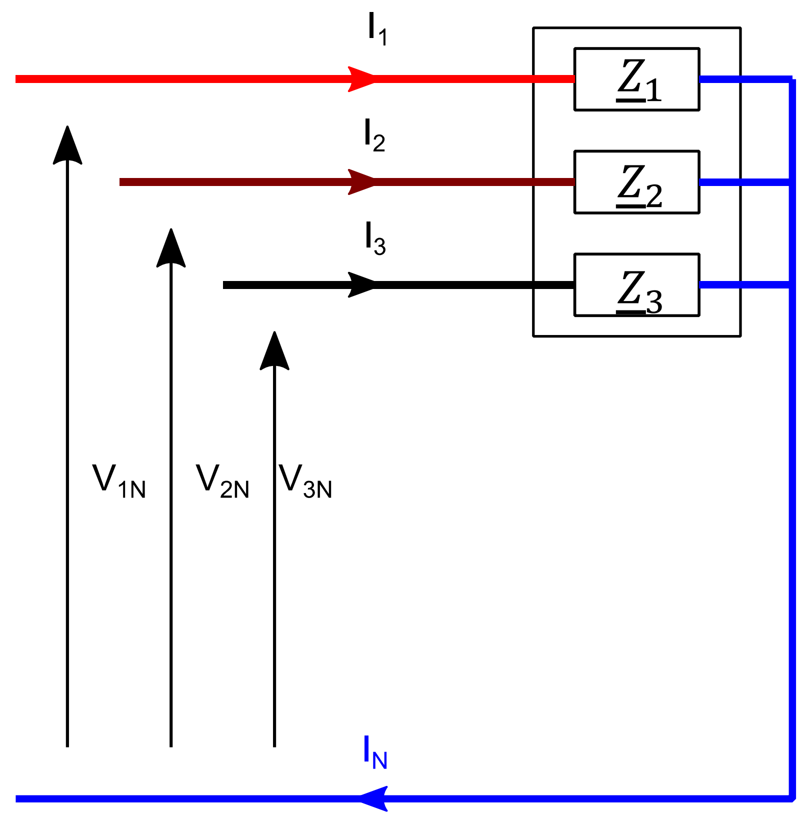
$$P = 3VIcos(\varphi) = \sqrt{3}UIcos(\varphi)$$

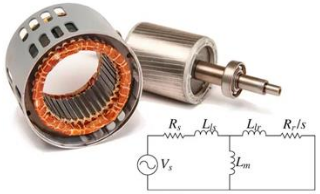
$$Q = 3VIsin(\varphi) = \sqrt{3}UIsin(\varphi)$$

$$S = 3VI = \sqrt{3}UI$$

$$FP = \frac{P}{S}$$

$$S^2 = P^2 + Q^2$$





II – Power in sinusoidal regime

3-phase: powers

- Boucherot's theorem: *general* Delta-connection load

$$P = U_{12}J_2 cos(\varphi_1) + U_{23}J_3 cos(\varphi_2) + U_{31}J_1 cos(\varphi_3)$$

$$Q = U_{12}J_2 sin(\varphi_1) + U_{23}J_3 sin(\varphi_2) + U_{31}J_1 sin(\varphi_3)$$

- For the balanced load:

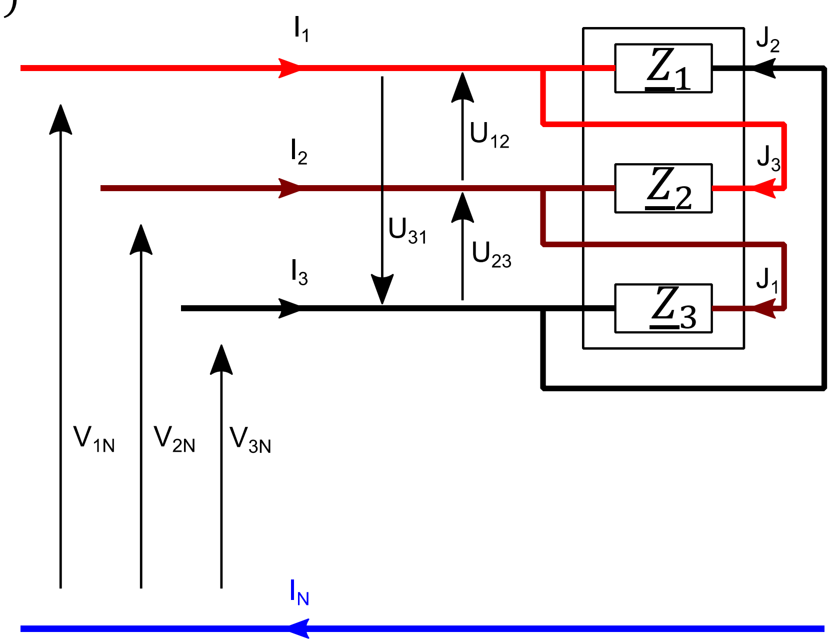
$$P = 3UJcos(\varphi) = \sqrt{3}UIcos(\varphi)$$

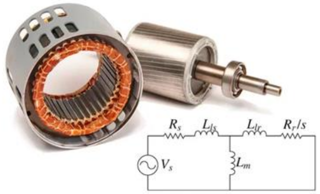
$$Q = 3UJsin(\varphi) = \sqrt{3}UIsin(\varphi)$$

$$S = 3UJ = \sqrt{3}UI$$

$$FP = \frac{P}{S}$$

$$S^2 = P^2 + Q^2$$





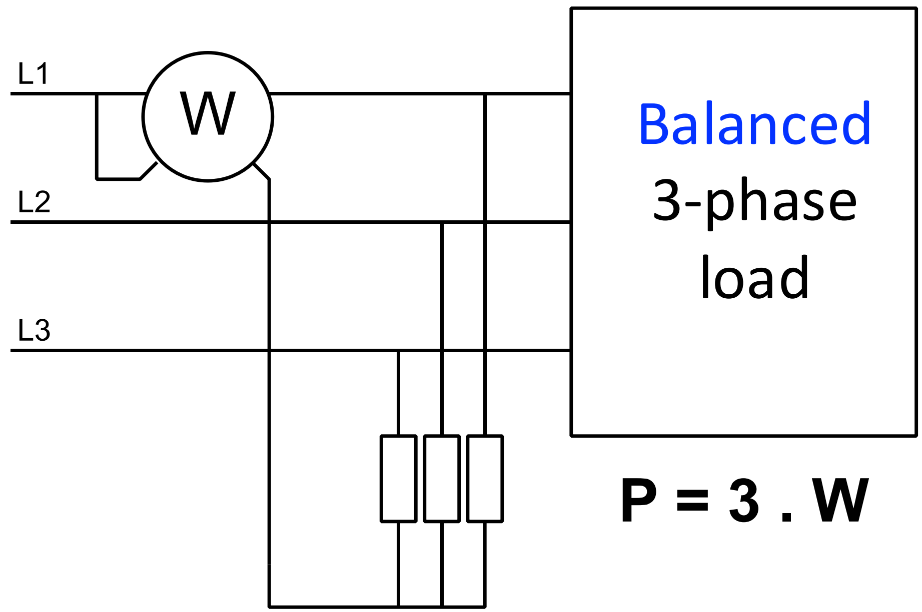
II – Power in sinusoidal regime

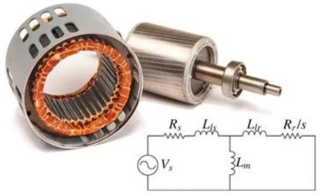
3-phase: power measurement

- Balanced 3-phase load

=> Wattmeter with virtual neutral

$$P = 3.V\,I\;\cos(\varphi)$$





II – Power in sinusoidal regime

3-phase: power measurement

- The 2-wattmeter method:

$$W_1 + W_2 = < \left(v_1(t) - v_3(t)\right) i_1(t) + \left(v_2(t) - v_3(t)\right) i_2(t) >$$