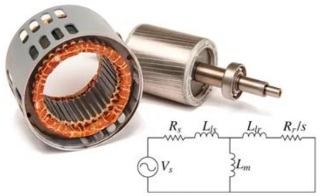


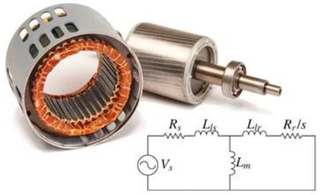
Electrical Engineering

Thomas TILLOCHER

Université d'Orléans
Laboratoire GREMI
thomas.tillocher@univ-orleans.fr

Year 2023/2024

Original material: Emmanuel BEURUAY
English version: Thomas TILLOCHER



Schedule

	M1 AEMS
Lectures	14 x 1,25 h dont 2 DS
Tutorials	8 x 1,25 h
Lab work	1 x 2,50 h + 6 x 3,75 h

Outline

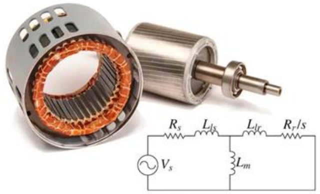
Introduction

I) Reminders (electricity)

II) Power in sinusoidal regime (single-phase and 3-phase)

III) Transformers

IV) Electric motors



Reminders: electromagnetism

- The electrical field: an electrical charge, q_A, placed at any point A in space, acts at any other point M in space, in the form of a vector field called the "electric field $E_A(M)$" expressed in $V.m^{-1}$

- **Electrical field due to charge q_A at point M**

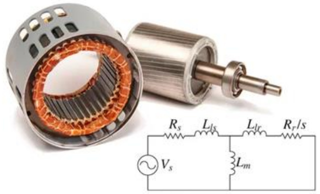
$$\overrightarrow{E_A(M)} = \frac{q_A}{4.\pi.\varepsilon_0.AM^2}.\vec{u}$$

$E_A(M)$ in $V.m^{-1}$, q_A in C, AM in m
ε_0, vacuum permittivity: $\varepsilon_0 = 10^{-9} / 36\pi$ $F.m^{-1}$

- Properties of the electrical field:

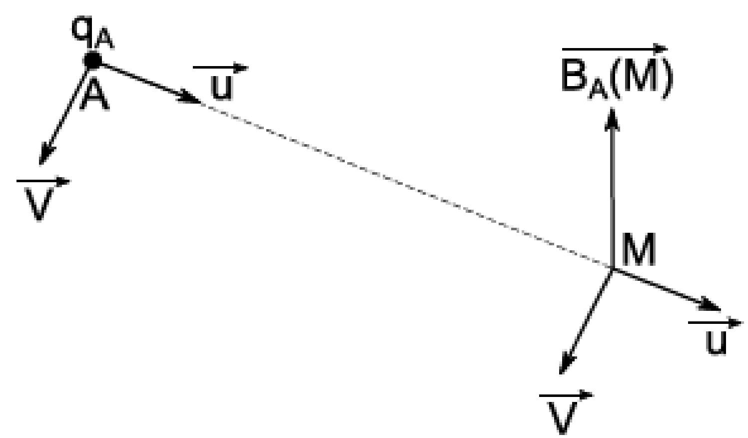
=> Inversely proportional to the square of the distance from its source. It scales with "$1/r^2$"

=> Additive quantity



Reminders: electromagnetism

- The magnetic field: An electrical charge, q_A, located at any point A in space and moving with velocity "V", acts at any other point M in space, in the form of a vector field called the "magnetic field $B_A(M)$" expressed in Tesla (T).



- **Magnetic field in vacuum at point M due to charge displacement q_A**

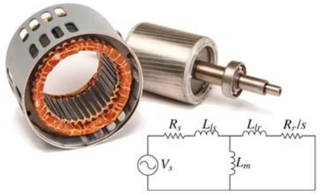
$$\overrightarrow{B_A(M)} = \frac{\mu_0}{4.\pi} . \frac{q_A.\vec{V} \wedge \vec{u}}{AM^2}$$

$B_A(M)$ in Tesla, q_A in C, V in m/s, AM in m
μ_0, vacuum permeability in m.T.A.m^{-1}, $\mu_0 = 4.\pi.10^{-7}$ m.T.A.m^{-1} (or H.m^{-1})

- Properties of the magnetic field:

=> Inversely proportional to the square of the distance from its source. It varies in "$1/r^2$".

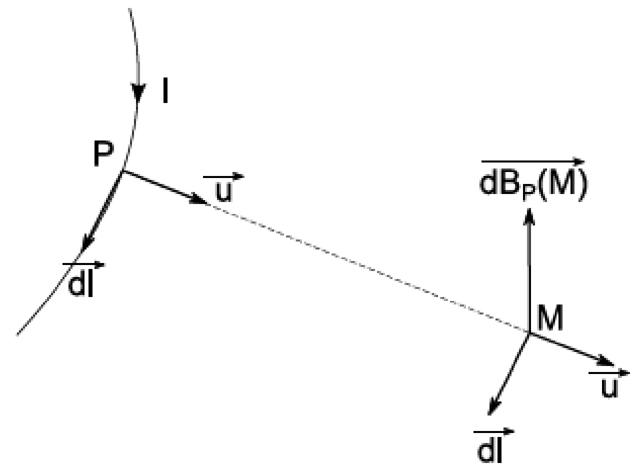
=> Additive quantity



Reminders: electromagnetism

- The Biot and Savart's law: The elementary part dl of an electrical circuit in P through which a current of intensity I flows creates the "magnetic field $dB_P(M)$" at a point M in space

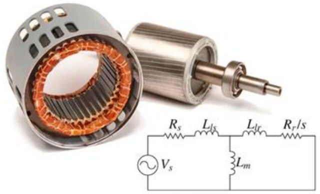
- **Magnetic field at point M due to the current I flowing through the elementary part dl**



$$\overrightarrow{dB_P(M)} = \frac{\mu_0}{4.\pi}.\frac{I.\overrightarrow{dl}\wedge\overrightarrow{PM}}{PM^3}$$

$$\overrightarrow{dB_P(M)} = \frac{\mu_0}{4.\pi}.\frac{I.\overrightarrow{dl}\wedge\vec{u}}{r^2}$$

- Total magnetic field in M created by the electrical circuit : $\overrightarrow{B(M)} = \frac{\mu_0}{4.\pi}.\int_{P\in circuit}\frac{I.\overrightarrow{dl}\wedge\vec{u}}{r^2}$



Reminders: electromagnetism

- The excitation magnetic field $\vec{H}$: dHP(M), is related to the state of magnetic excitation of the medium and is given in $A.m^{-1}$.

- Magnetic excitation field at point M due to the current I flowing through portion dl:

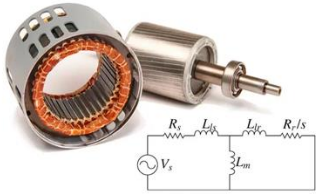
$$\overrightarrow{dH_P(M)} = \frac{1}{4.\pi}.\frac{I.\overrightarrow{dl}\wedge\vec{u}}{r^2}$$

- Total magnetic excitation field at M created by the wire through which a current I flows:

$$\overrightarrow{H(M)} = \frac{1}{4.\pi}.\int_{P\in fil}\frac{I.\overrightarrow{dl}\wedge\vec{u}}{r^2}$$

- If $\vec{H}$ is the excitation magnetic field, $\vec{B}$ is the magnetic induction field: $\vec{B} = \mu_0\mu_r\vec{H}$

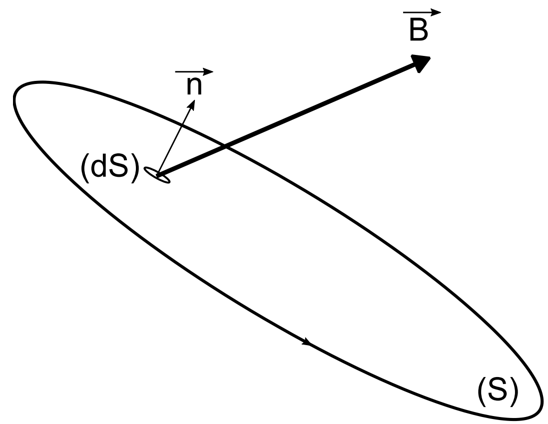
μ_0 vacuum permeability, μ_r the relative permeability



Reminders: electromagnetism

- The magnetic flux: The flux of induction magnetic field B across a closed surface (S) is the quantity ϕ_B given in Weber (Wb)

$$\Phi_B = \oiint_{(S)} \vec{B}.\overrightarrow{n_{ext}}.dS$$



- The magnetic flux F is usually given by the product B.S
- If magnetic leakage is neglected, the flux in a magnetic circuit is conservative

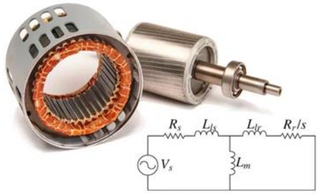
$$\Phi = B.S$$

Analogy with the garden hose :

magnetic flux F → flow

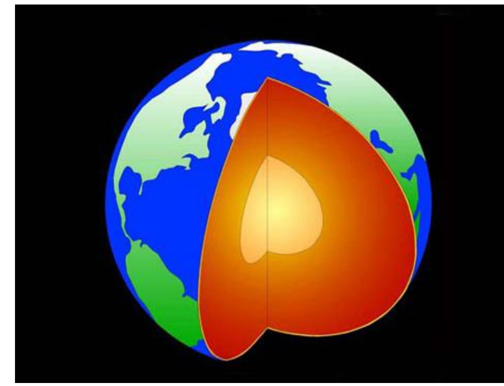
magnetic field B → water speed

solenoid cross-section S → pipe cross-section



Reminders: electromagnetism

- Orders of magnitude

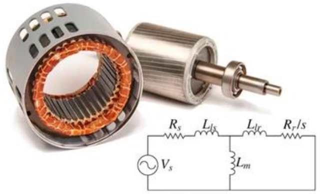


- Earth induction magnetic field: 50.10^{-6} T

In electrical machines:

- Induction magnetic field: 1 T à 1,5 T
- Excitation magnetic field : 1000 à 100000 $A.m^{-1}$
- Magnetic flux: 10^{-5} à 10^{-3} Wb

Transformateur 400 kVA avec accessoires de protection

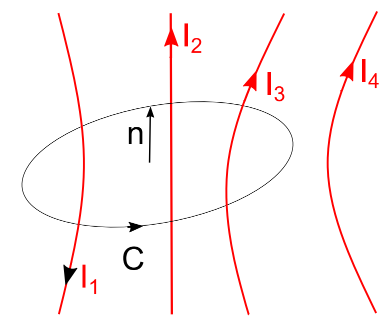


Reminders: electromagnetism

- Ampere's theorem: If (C) is a closed contour of space surrounding N wire conductors through which currents of intensities I_k flow, then the circulation of the magnetic excitation vector H along a closed contour l is equal to the sum of the entwined currents

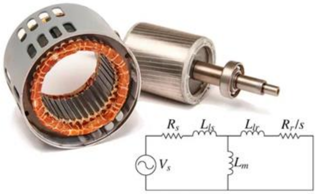
$$\oint_{(C)} \vec{H}.\overrightarrow{d\ell} = \sum_{k=1}^{k=N} i_k \qquad \boldsymbol{H.\ell = N.I}$$

- Example:



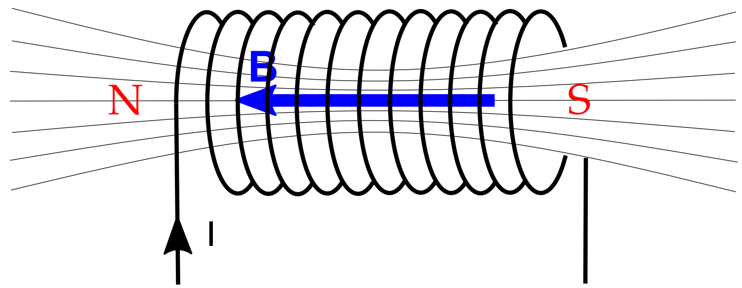
$$\oint_{(C)} \vec{H}.\overrightarrow{d\ell} = -i_1 + i_2 + i_3$$

- i_1 is counted as negative,
- i_2 and i_3 are counted as positive
- i_4 is outside the contour (not taken into account)



Reminders: electromagnetism

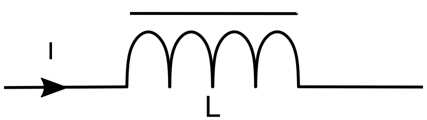
- Example: magnetic field created by a solenoid



- A solenoid is a straight winding with length l greater than its radius r.
- Inside the solenoid, far from its ends, the magnetic field is uniform.
- The field lines are parallel
- They enter at the coil's SOUTH face and exit at its NORTH face (corkscrew rule).

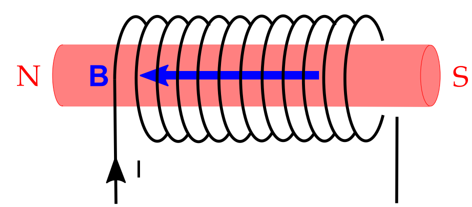
N: number of turns, l : length of solenoid

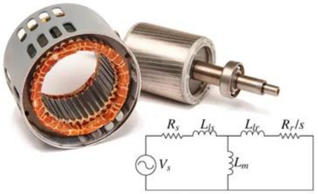
$$H = \frac{N.I}{l} \qquad B = \mu_0 . \frac{NI}{l}$$



$$\vec{B} = \mu \times \vec{H}$$

$$\mu = \mu_0 \times \mu_R$$





Reminders: magnetic materials

- Materials are classified according to their magnetic susceptibility χ

=> χ is related to the relative permeability through: $\mu_r = 1 + \chi$

- Para-magnetic materials: $\chi > 0$, between 10^{-3} and 10^{-7}

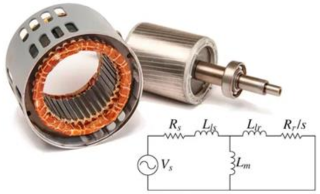
=> These materials are rare and their magnetization is negligible (Al, W, Pt, Sn…)

- dia-magnetic materials: $\chi < 0$, between 10^{-4} and 10^{-6}

=> These materials are common and their magnetization is negligible (non magnetic materials such as Cu, Bi, Au, Ag…)

- Ferro-magnetic materials: $\chi > 0$, between 10^{3} and 10^{6}

=> These are magnetic material of interest for magnetic circuits or transformer core (Fe, Ni, Co)



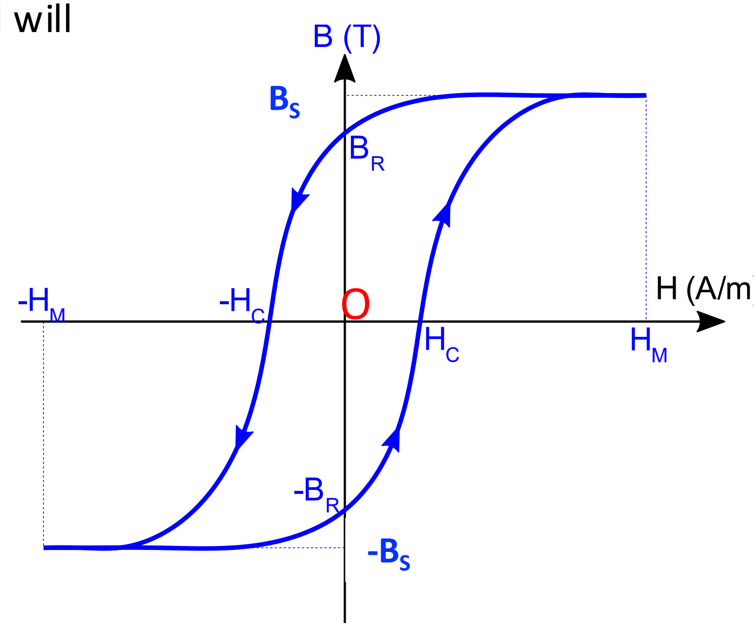
Reminders: magnetic materials

- Magnetic materials are characterized by their hysteresis loop

=> B=f(H) curve showing magnetizing/demagnetizing of the ferromagnetic material

- A ferromagnetic material that has never been magnetized will magnetize starting from O ("first magnetizing curve")

- The loop is run only in the direction of the arrows



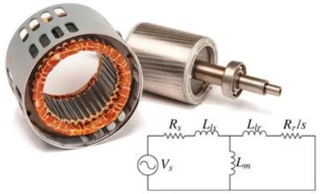
- B_S: saturation induction magnetic field

- B_R: point of retentivity

=> Remanence of residual magnetism in the material

- H_C: point of coercivity

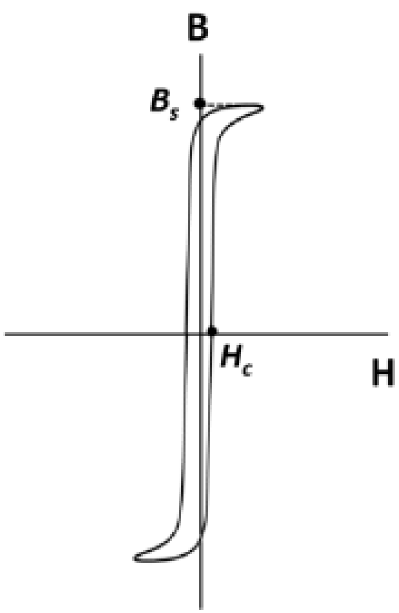
=> Coercive excitation magnetic field required to remove the residual magnetism in the material



Reminders: magnetic materials

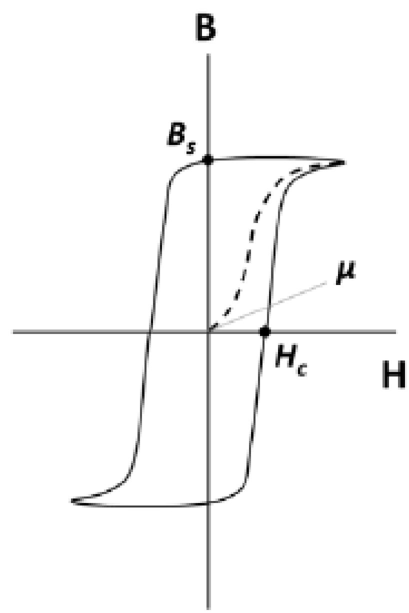
- The shape of the hysteresis loop varies according to the magnetic material

Soft magnetic material

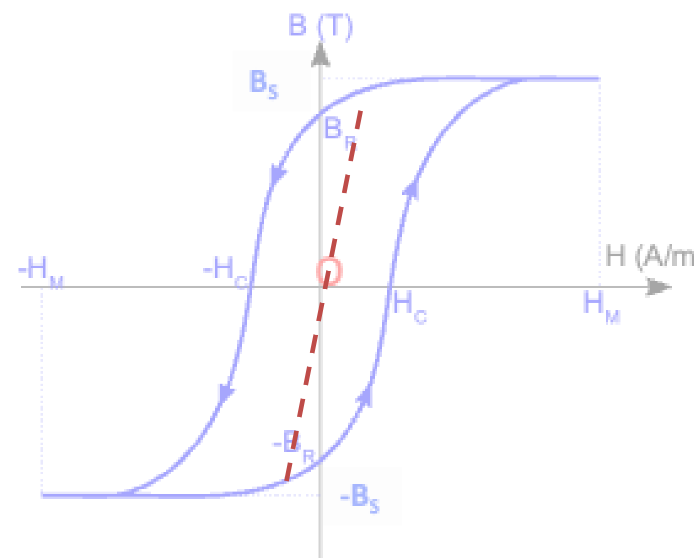


B_S = 1.5 – 2T
B_R< 1T
H_C=1-10 A/m

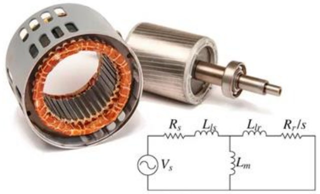
Hard magnetic material



B_R - 1T
H_C=10^3 A/m



Linear approximation outside the saturation region: $B=\mu H$



Magnetic circuits

- Linear homogeneous magnetic circuits:

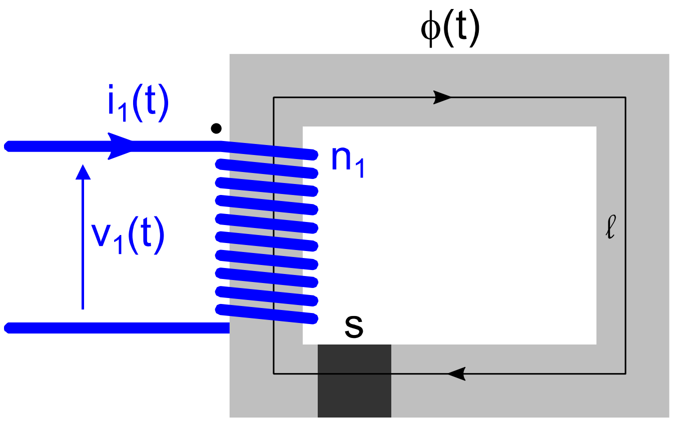
- Materials and geometry are chosen to concentrate flux density as much as possible, thus creating the strongest possible induction => limitation of both mass and size/volume

- Homogeneous = a single magnetic material

- Homogeneous = constant cross section

- Linear = outside the saturation regime

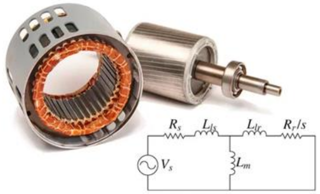
- ℓ (schematic) = mean field line



$$\oint \overrightarrow{H}.\overrightarrow{d\ell} = n.I$$

$$H.\ell = n.I$$

- The nI quantity is also called magnetomotive force



Magnetic circuits

- Hopkinson's relation:

- In a linear homogeneous magnetic circuit, the material exhibits a constant permeability

$$\mu = \mu_0 \mu_r$$

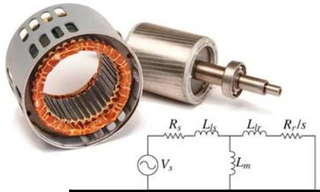
- Therefore: $B = \mu H$

- We have shown previously that $\Phi = B.S$

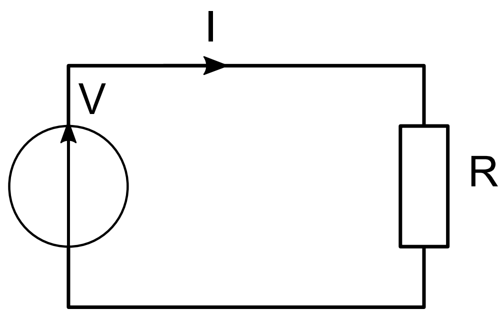
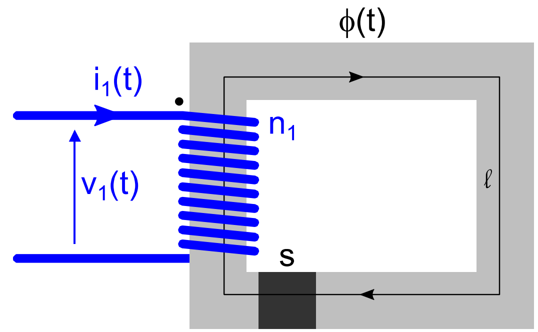
- By considering the Ampere's theorem as well, it can be shown that:

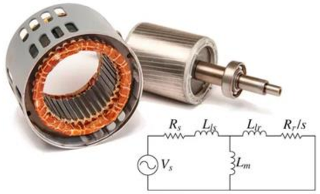
$NI = \Re\Phi$ **Hopkinson's relation**, with the reluctance $\Re = \dfrac{\ell}{\mu S}$ (in H^{-1})

- The reluctance is the opposition that a ferromagnetic material produces to the establishment of a magnetic field



Magnetic circuits: analogy with electrical circuits

Electrical circuit	Magnetic circuit
	
Electromotive force in V : V	Magnetomotive force in A.tr : $\varepsilon = n.I$
Electrical current in A : I	Magnetic flux in Wb : Φ
Electrical resistance in Ω : R $R = \rho \times \frac{\ell}{S}$ • Electrical resistivity in Ω.m : ρ • Conductor length in m : ℓ • Conductor cross section in m² : s	Magnetic reluctance: $\Re$ $\mathcal{R} = \frac{1}{\mu_0 \times \mu_R} \times \frac{\ell}{S}$ • Magnetic permeability in H/m : μ • Circuit length in m : ℓ • Circuit cross section in m² : s
Ohm's law: $V = R.I$	**Hopkinson's law**: $n.I = \mathcal{R}.\Phi$
Electrical field in V/m : E	Excitation magnetic field in A/m : H
Current density in A.m^{-2} : $J = \frac{I}{s} = \sigma.E$	Induction magnetic field in T: $B = \frac{\Phi}{s} = \mu.H$



Magnetic circuits

- Lenz's law and Faraday's law:

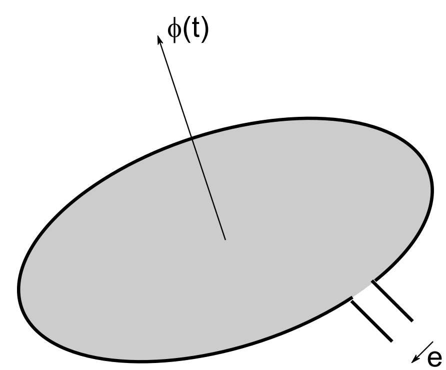
- Lenz's law (qualitative law): Induced currents and fields oppose the causes that gave rise to them

=> The induced field and current oppose the change in flux through the circuit

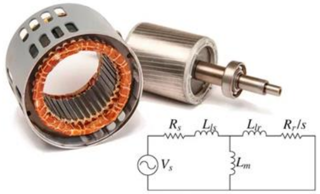
- Faraday's law (quantitative law): Any flux variation produces an induced electromotive force across a circuit

=> For a coil, an electromotive force is produced at across each turn of the winding

$$e(t) = -\frac{d\phi(t)}{dt}$$ (receptor convention)



For n turns: $$e(t) = -n \times \frac{d\phi(t)}{dt}$$

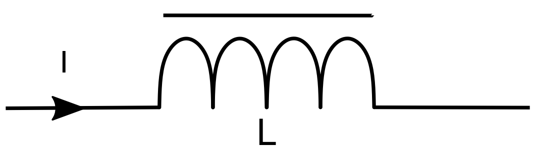


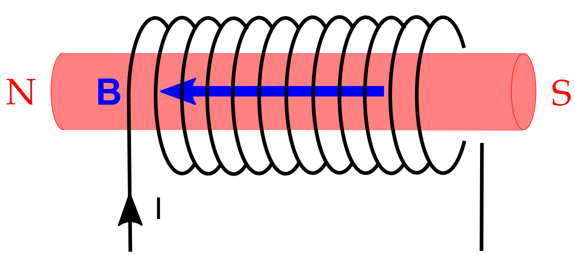
Magnetic circuits

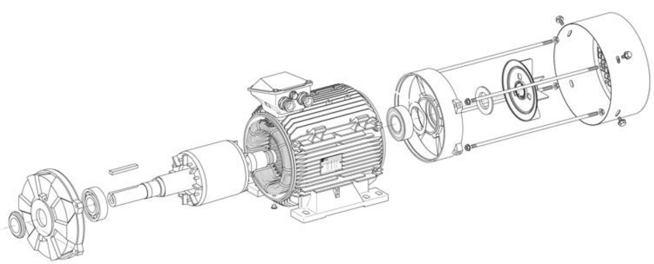
- <u>The iron-core coil:</u>

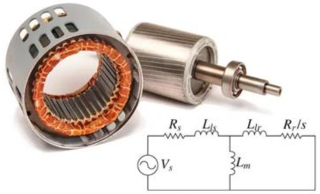
  - A winding of copper wire wrapped on a magnetic circuit of ferromagnetic material forms an iron-core coil

    => transformers, electromagnets, motors









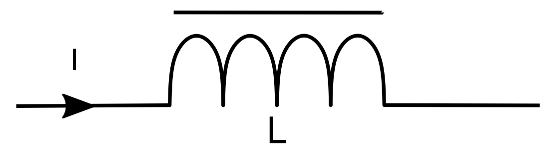
Magnetic circuits

- The iron-core coil:

  - Consider an iron-core coil with N turns

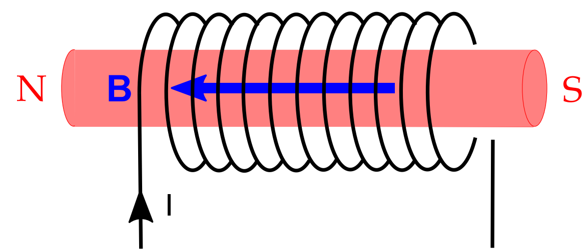
  => Each turn is crossed by the flux ϕ created in the material => Total flux $\phi_T = N\phi$

  - By considering both Hopkinson's relation and Faraday's law:



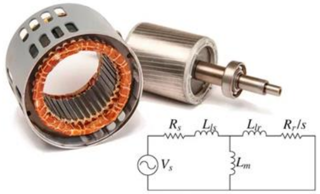
$$\boldsymbol{Ni = \Re\Phi} \qquad \boldsymbol{e(t) = -\frac{d\phi_T(t)}{dt} = -N\frac{d\phi(t)}{dt}}$$

  - We introduce the inductance L such that: $\phi_T = Li$



  - With L (in H): $L = \frac{N^2}{\Re} = \frac{N^2\mu S}{\ell}$

  - We retrieve the expression: $\boldsymbol{e(t) = -L\frac{di(t)}{dt}}$

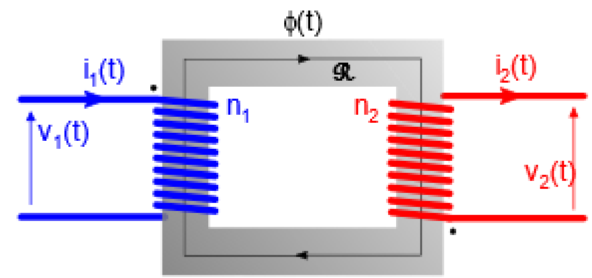


Magnetic circuits

- Mutual inductance: (case of the homogeneous linear magnetic circuit)

- Mutual inductance occurs when the magnetic circuit has at least two windings

=> Each current has an influence on the flux flowing in the circuit.



- Flux created by coil 1 (current i_1) and flowing through coil 2

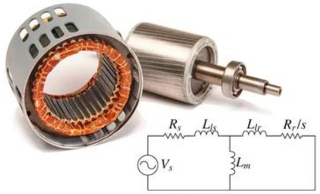
$$\phi_{1\to 2} = \frac{N_1 i_1}{\Re} \longrightarrow \phi_{T2} = N_2 \phi_{1\to 2}$$

- By analogy with the definition of the inductance: $\phi_{T2} = N_2 \phi_{1\to 2} = M_{12} i_1$

- With M (in H): $M_{12} = \frac{N_1 N_2}{\Re}$ $\longrightarrow$ $e_2(t) = -M_{12}\frac{di_1(t)}{dt}$

- Voltage across winding k among n other windings: mutual inductances + self-inductance

$$-e_k(t) = M_{1k}\frac{di_1(t)}{dt} + M_{2k}\frac{di_2(t)}{dt} + \cdots + M_{nk}\frac{di_n(t)}{dt} + L\frac{di_k(t)}{dt}$$



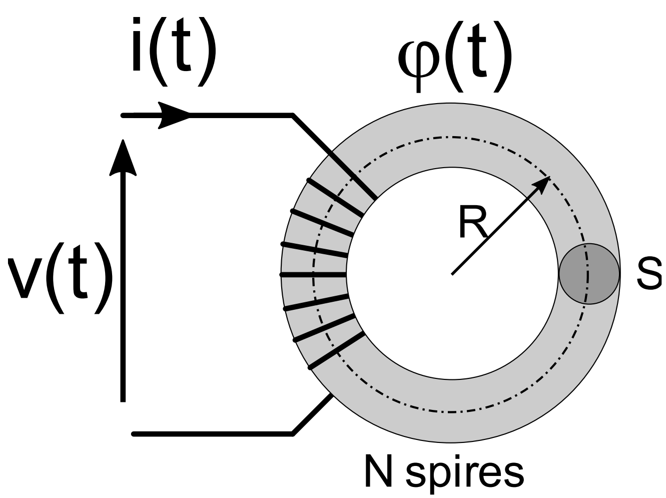
Magnetic circuits

- Boucherot's formula : Ideal coil (no losses)

- The winding is subjected to a sinusoidal voltage

$$v(t) = V\sqrt{2}.\cos(\omega t)$$

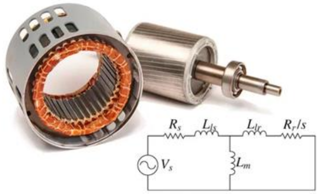
=> Assumption of forced flux



$$\frac{d\varphi(t)}{dt} = \frac{V\sqrt{2}}{N}.\cos(\omega t)$$

$$\varphi(t) = \frac{V\sqrt{2}}{N}.\int_0^t \cos(\omega t).dt$$

$$\varphi(t) = \frac{V\sqrt{2}}{N\omega}.\sin(\omega t) + \varphi(0)$$

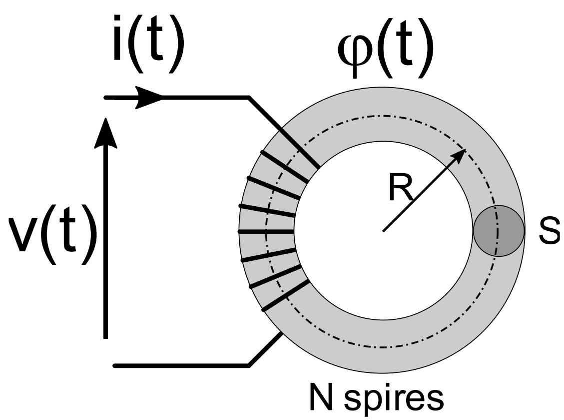


Magnetic circuits

- Boucherot's formula : Ideal coil (no losses)

- ϕ(0)=0: no permanent magnet, no remanent flux, no second DC winding).

$$\varphi(t) = \frac{V\sqrt{2}}{N\omega}.\sin(\omega t) + \varphi(0)$$



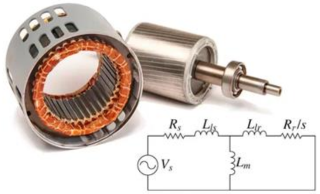
$$\varphi(t) = \frac{V\sqrt{2}}{N\omega}.\sin(\omega t)$$

$$\boldsymbol{\varphi(t) = \frac{V\sqrt{2}}{N\omega}.\cos(\omega t - {}^{\pi}/_{2})}$$

- The flux lags the current

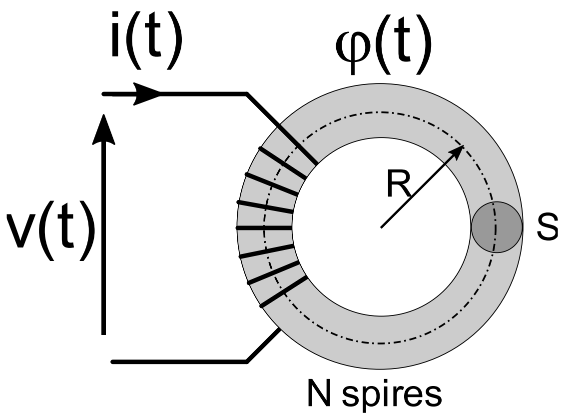
- The relation Nϕ=Li (Hopkinson's law) gives the current

$$i(t) = \frac{V\sqrt{2}}{L\omega}.\cos(\omega t - {}^{\pi}/_{2})$$



Magnetic circuits

- <u>Boucherot's formula</u> : Ideal coil (no losses)



$$\begin{cases} \varphi(t) = \dfrac{V\sqrt{2}}{N\omega}.\cos(\omega t - {}^{\pi}/_{2}) \\ \varphi(t) = \Phi_M.\cos(\omega t - {}^{\pi}/_{2}) \end{cases}$$

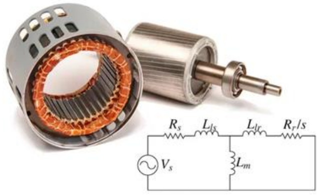
$$\frac{V\sqrt{2}}{N\omega} = \Phi_M$$

$$V = \frac{N.\omega.\Phi_M}{\sqrt{2}}$$

$$V = \frac{N.(2.\pi.f).S.B_M}{\sqrt{2}}$$

$$V = \frac{2.\pi}{\sqrt{2}}.S.N.B_M.f$$

$$\boldsymbol{V = 4{,}44.S.N.B_M.f}$$

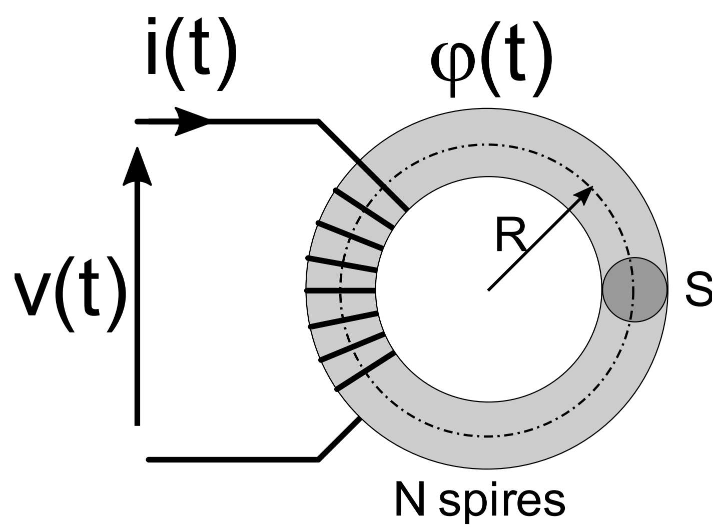


Magnetic circuits

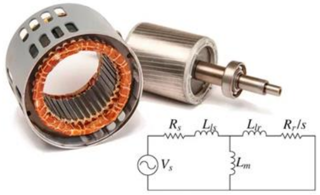
- Boucherot's formula : Ideal coil (no losses)

$$V = 4,44.S.N.B_M.f$$

If the flux ϕ of a coil is due to the current **i** flowing through it (Hopkinson's law), its maximum value ϕ_M depends only on the RMS value of the voltage **V** (at constant frequency).



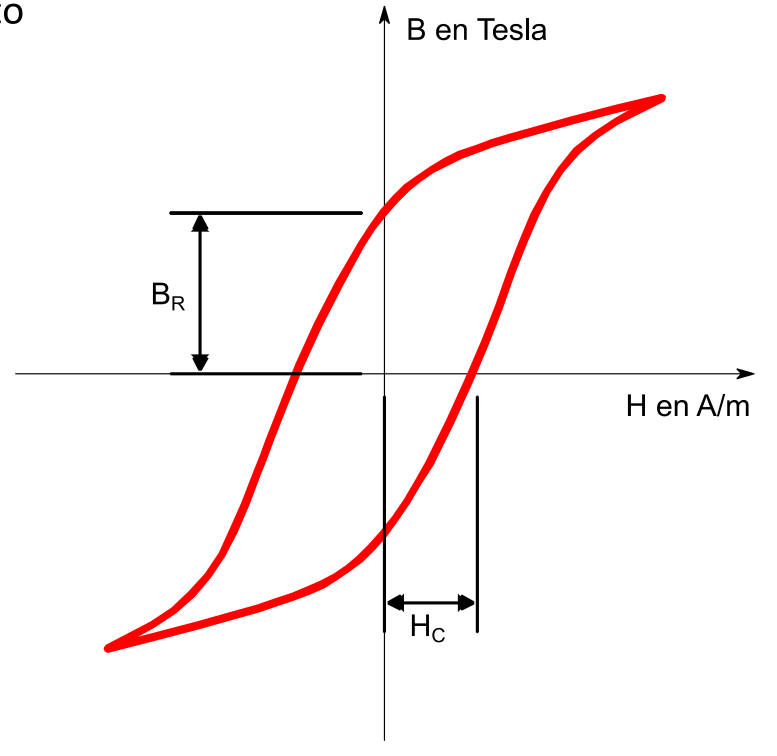
The voltage imposes a flux and the winding draws the current accordingly



Magnetic circuits

- Defects of the iron-core coil: hysteresis losses

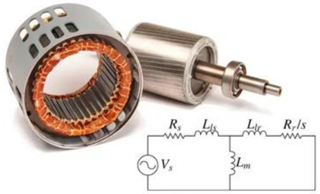
- Hysteresis losses correspond to the power required to magnetize and demagnetize the material over its hysteresis loop.



- Empirical formula:

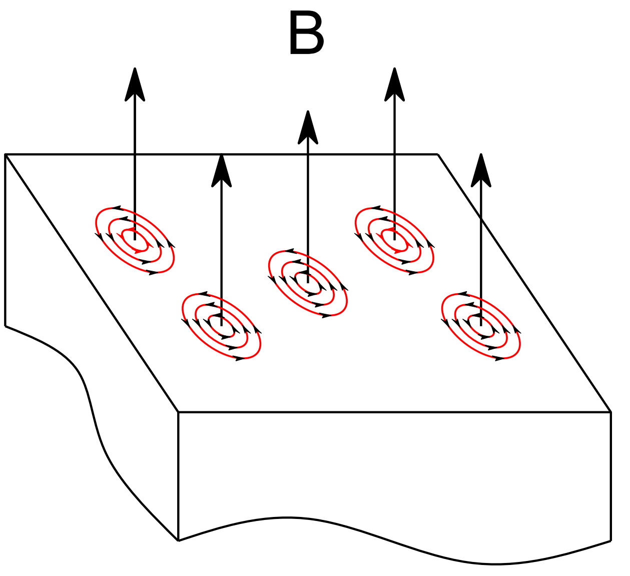
$$P_H = K_H \, l \, S \, f \, B_{max}^n$$

- K_H is a material-related constant
- n is the Steinmetz coefficient (around 1.8)
- l is the mean field line, f the frequency, B_{max} the maximum induction field



Magnetic circuits

- Defects of the iron-core coil: Eddy currents



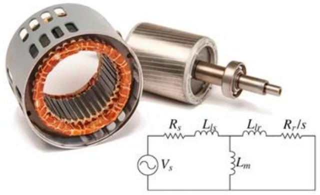
- Currents induced in the magnetic material in which they flow freely

- These currents cause losses in the form of power dissipated by Joule effect

- Empirical formula: $P_H = K_{FC}\, l\, S\, f^2\, B_{max}^2$

- K_{FC} is a material-related constant
- d is the thickness of the foil in the case of a laminated material
- l is the mean field line, f the frequency, B_{max} the maximum induction field

- Defects of the iron-core coil: **IRON LOSSES P_F** $\boldsymbol{P_F = P_H + P_{FC}}$



Magnetic circuits

- Defects of the iron-core coil: Leak inductance

- Magnetic materials are never perfect, and never channel all the field lines

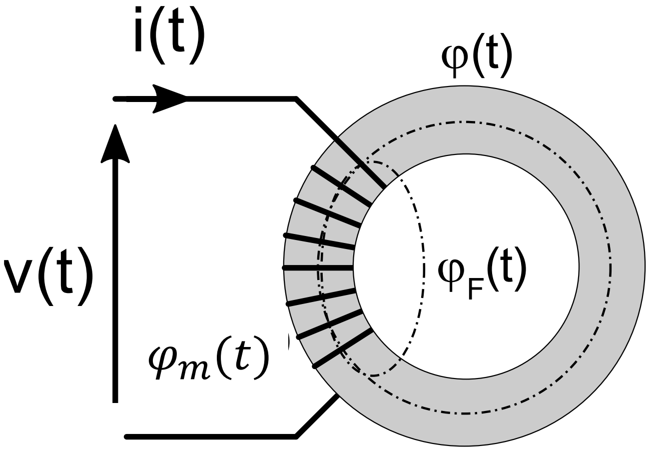
=> Part of the magnetic flux propagates in the air via less reluctant paths

=> This corresponds to magnetic leaks (flux outside the magnetic circuit)

- The magnetic field channeled in the magnetic circuit is called "magnetizing flux"

$$\phi = \phi_m + \phi_f$$

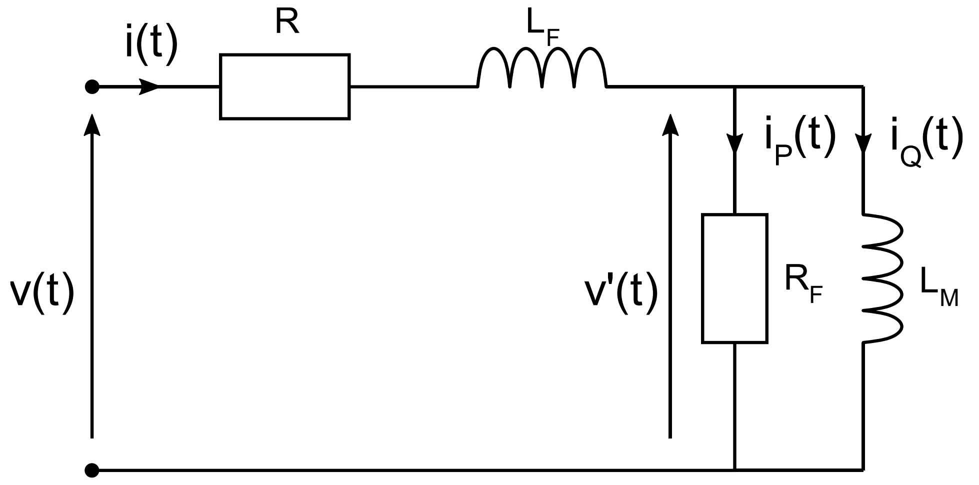
$$v(t) = -N\frac{d\phi}{dt} = -(L_m\frac{di}{dt} + L_f\frac{di}{dt})$$



- L_m is the magnetizing inductance, L_f is the leak inductance

Magnetic circuits

- Linear model of the iron-core coil:



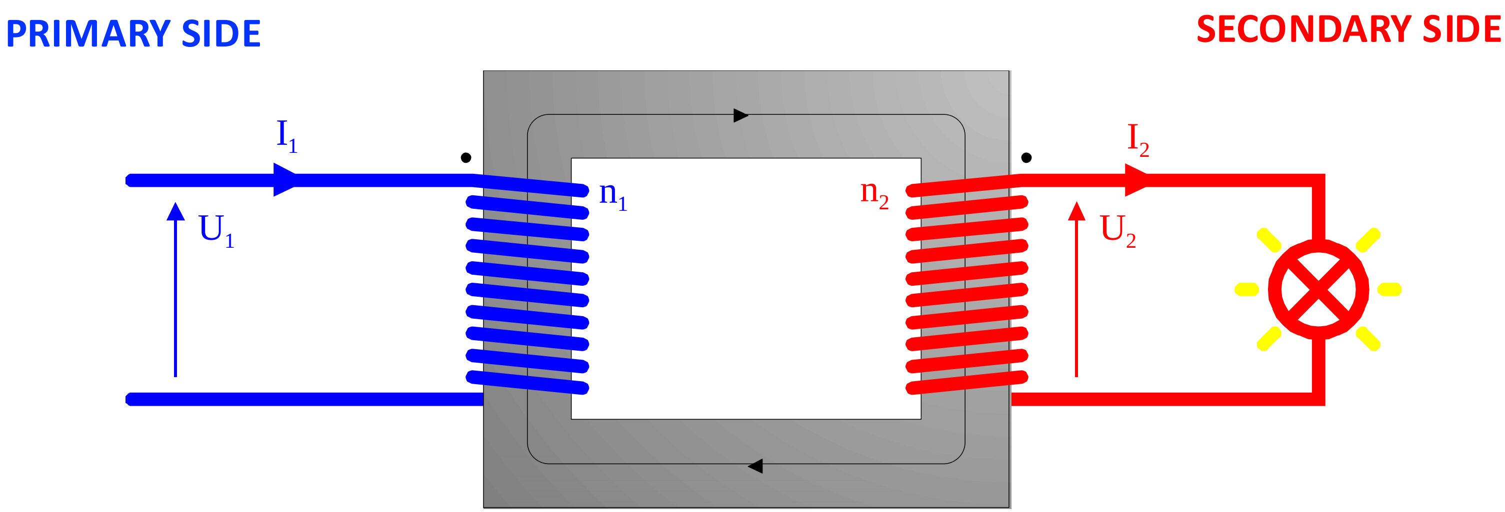
- R is the resistance of the coil N turns
- L_F is the leak inductance, L_M is the magnetizing inductance
- R_F is the resistor modelling the iron losses

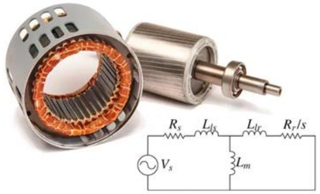
Single-phase transformer

- The transformer enables a change in the RMS value of an AC voltage to be achieved with high efficiency => step-up transformer or step-down transformer

- A single-phase transformer consists of two windings wound on the same magnetic circuit

=> Usually, the two windings have different numbers of turns





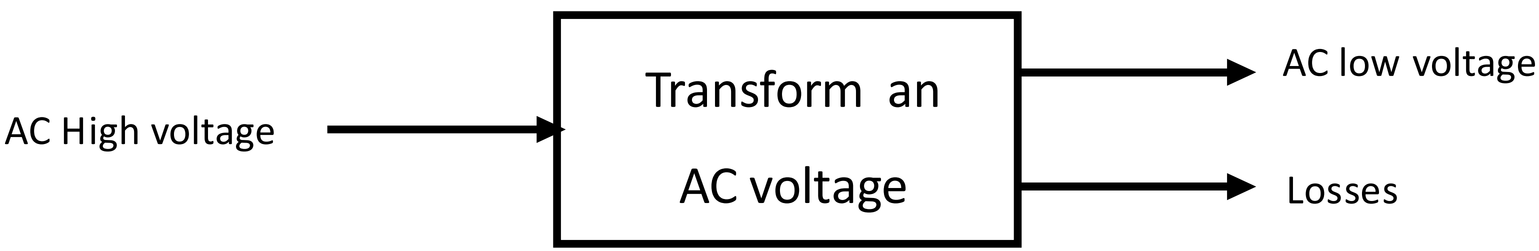
III - Transformers

Single-phase transformer

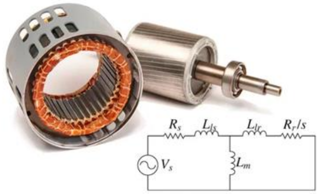
- Functional approach: More precisely, The transformer is a static machine that allows sinusoidal quantities (voltages, currents) to be modified without changing their frequency.

=> **Voltage adaptation**: step-down or step-up transformer

=> **Galvanic insulation**: insulation transformer

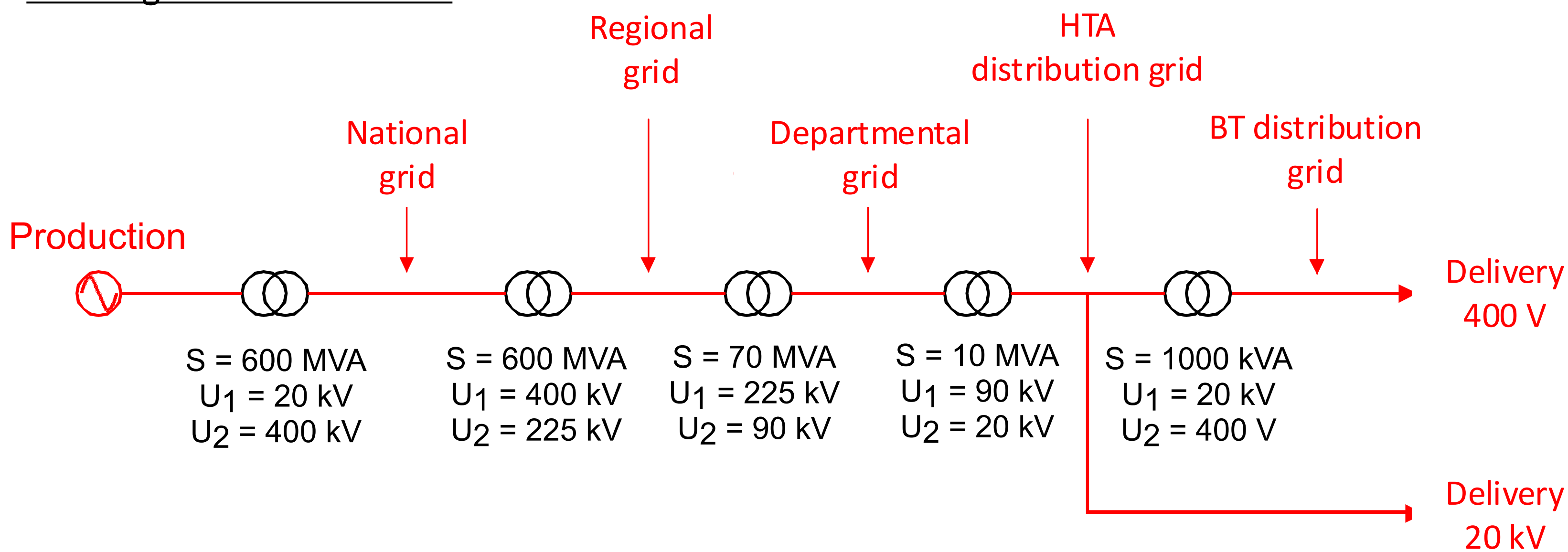


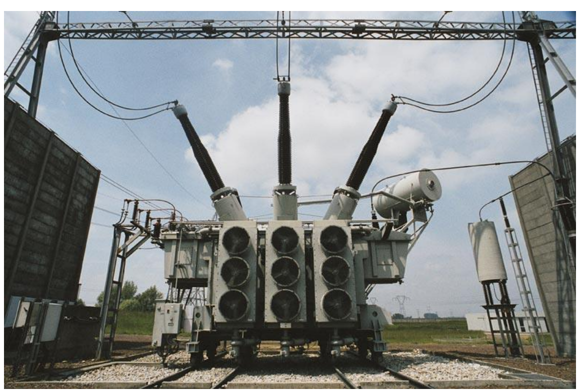
Transformer

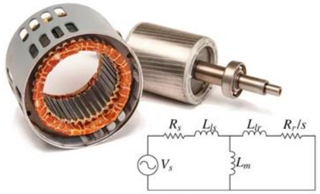


Single-phase transformer

- Power grid base structure:





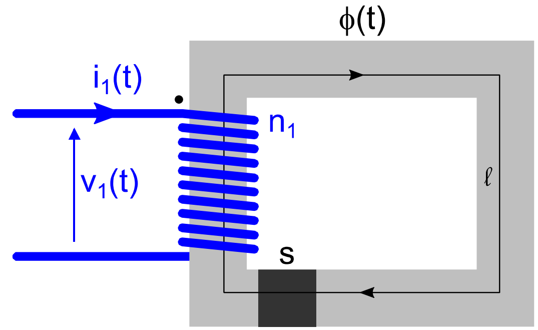


Single-phase transformer

- Principle:

- Hopkinson's relation in the case of a single winding at the primary side:

$$n_1 . i_1 = \mathcal{R} . \Phi$$

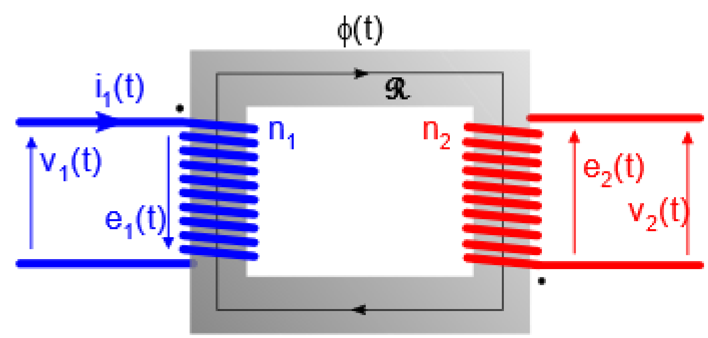


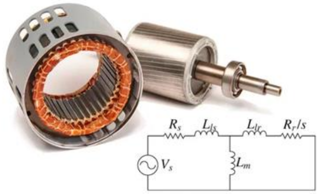
- For two windings and i2 =0 (no load)

$$e_1 = -v_1 = -n_1 \frac{d\varphi}{dt}$$

$$v_1 = n_1 \frac{d\varphi}{dt}$$

$$e_2 = -n_2 \frac{d\varphi}{dt} = v_2$$



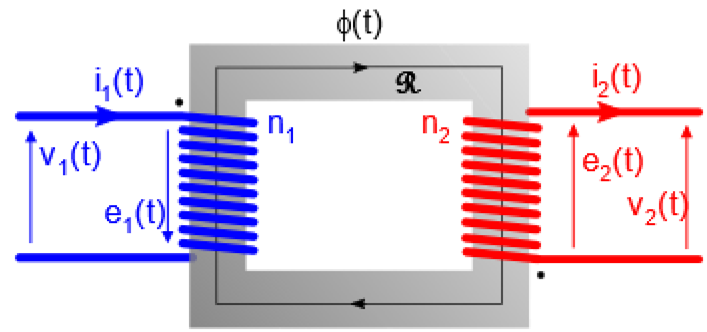


Single-phase transformer

- Principle:

- For two windings and i2 ≠ 0 (presence of a load)

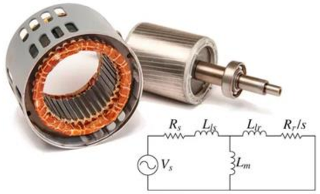
$$n_1 . i_1 + n_2 . i_2 = \mathcal{R}\phi$$



$$e_1 = -v_1 = -n_1 \frac{d\varphi}{dt}$$

$$v_1 = n_1 \frac{d\varphi}{dt}$$

$$e_2 = -n_2 \frac{d\varphi}{dt} = v_2$$



Single-phase transformer

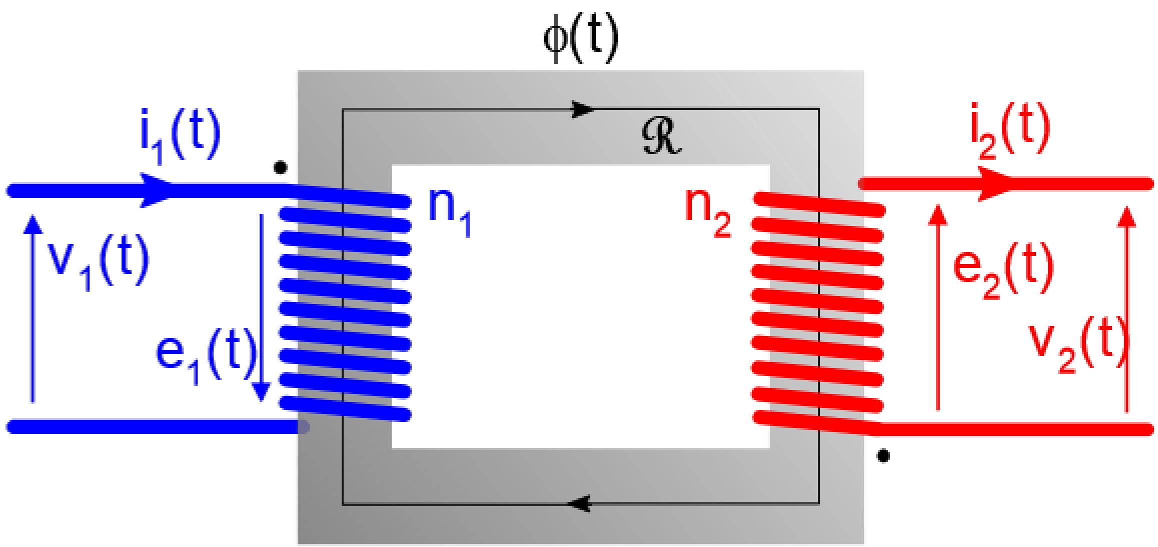
- Principle: receptor convention/generator convention

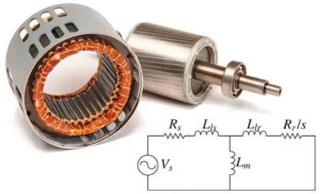
- Seen from the primary grid, the transformer is a receptor

=> Receptor convention at the primary side

- Seen from the load, the transformer is a generator

=> Generator convention at the secondary side





Single-phase transformer

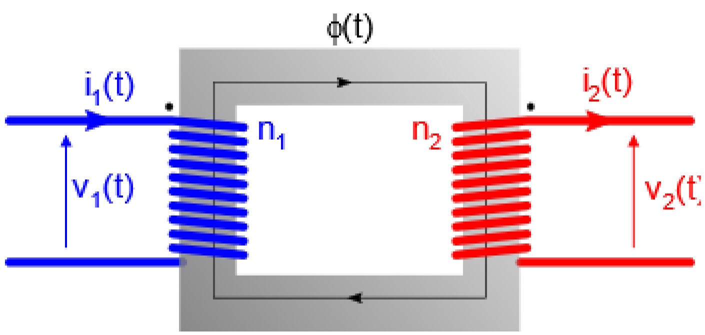
- Principle: homonymous terminals

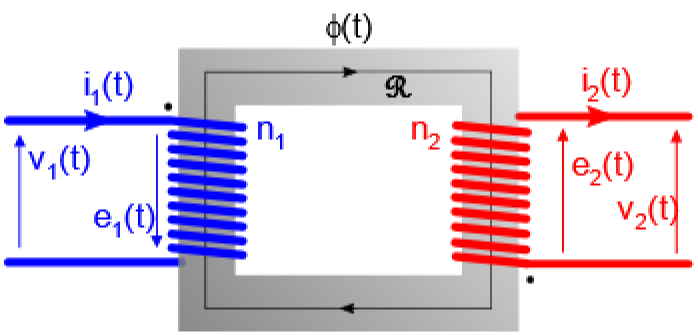
- The winding direction is marked with dots (●). Terminals marked in this way are called “homonymous terminals”. They correspond to points of the same instantaneous polarity.

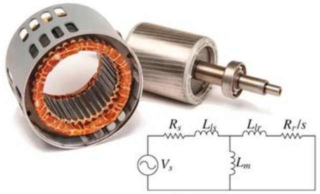
- Sign conventions for magnetic and electrical quantities:

=> A positive current entering through a homonymous terminal creates a positive flux in the magnetic circuit

=> According to Hopkinson's law, the magnetomotive force is preceded by the sign + if the current orientation arrow enters through a homologous terminal, and by the sign - otherwise.







Single-phase transformer

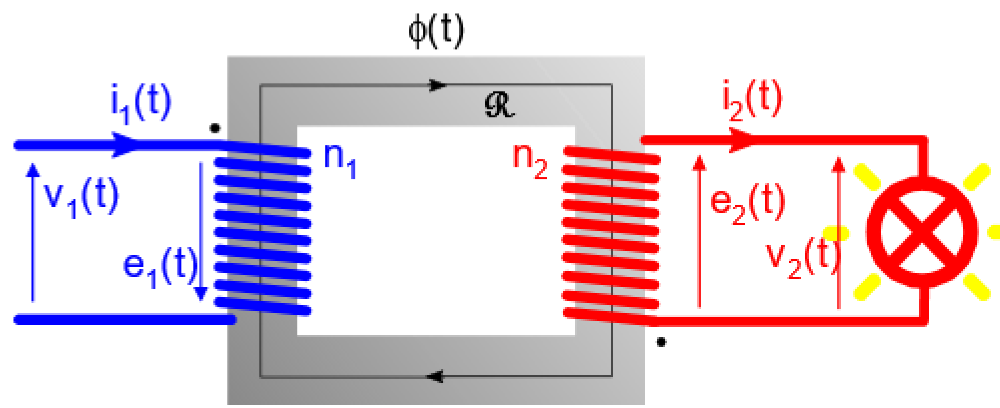
- Principle: the ideal transformer - voltages

- Ideal windings = no voltage drop, no losses

- Ideal magnetic circuit = zero reluctance

$$v_1 = -e_1 = n_1 \frac{d\varphi}{dt}$$

$$v_2 = e_2 = -n_2 \frac{d\varphi}{dt}$$

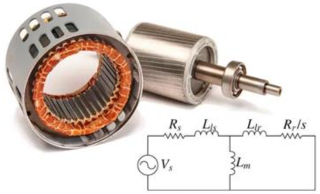


- The **transformation ratio** is given by:

$$m = \frac{n_2}{n_1} = -\frac{v_2}{v_1}$$

m > 1: Step-up transformer

m < 1: Step-down transformer



Single-phase transformer

- Principle: the ideal transformer - currents

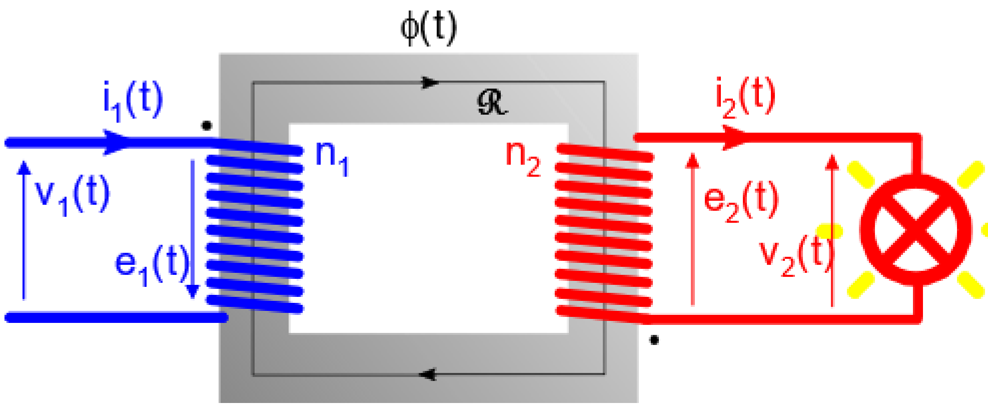
- Ideal windings = no voltage drop, no losses

- Ideal magnetic circuit = zero reluctance

- Hopkinson's relation:

$$n_1.i_1 + n_2.i_2 = \mathcal{R}\phi$$

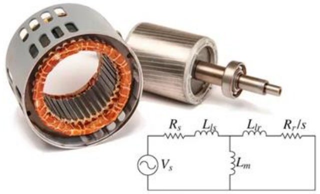
$$n_1.i_1 + n_2.i_2 = \boldsymbol{\mathcal{R}\phi = 0}$$



$$\frac{i_2}{i_1} = -\frac{n_1}{n_2}$$

$$\frac{i_2}{i_1} = -\frac{1}{m}$$

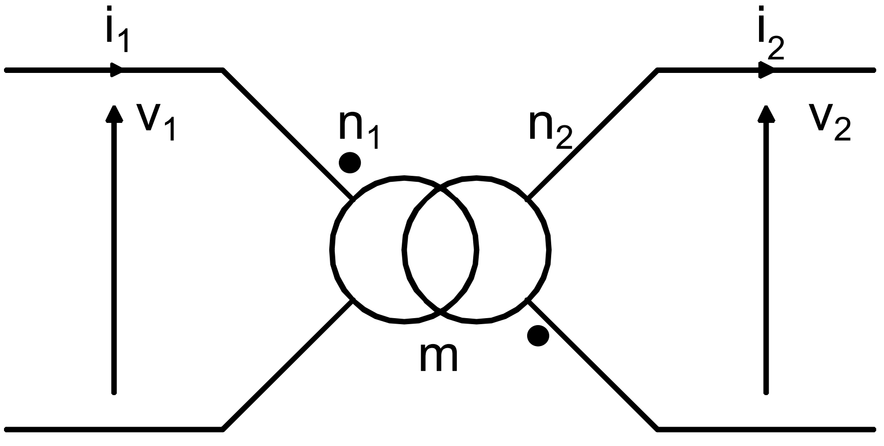
$$i_1 = -m.i_2$$



Single-phase transformer

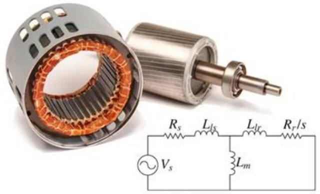
- Principle: the ideal transformer - symbol

$$m = \frac{n_2}{n_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$



$$\eta = \frac{P_2}{P_1} = \frac{V_2 . I_2}{V_1 . I_1} = 1$$

- Conservation of power, for the ideal transformer only



Single-phase transformer

- Principle: winding imperfection

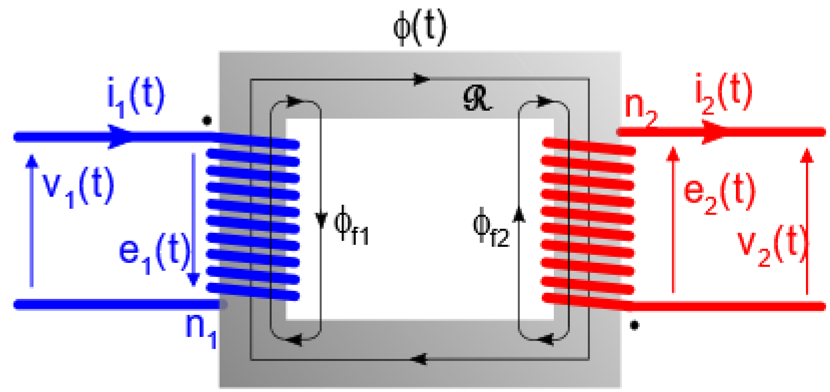
- Joule losses (or copper losses)

$$P_J = r_1.I_1^2 + r_2.I_2^2$$

$$R = \rho.\frac{\ell}{s} \qquad \rho_\Theta = \rho_0(1 + a\Theta)$$

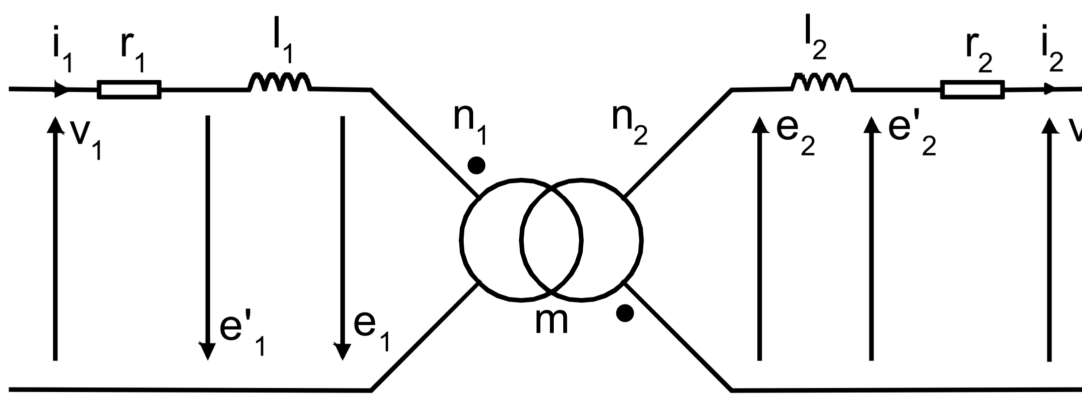
Copper (Cu) : $\rho_0 = 1{,}6.10^{-8}\,\Omega.m$, a = 0,39
Aluminum (Al) : $\rho_0 = 2{,}42.10^{-8}\,\Omega.m$, a = 0,43

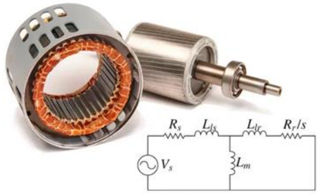
$$v_1 = -e_1' + r_1.i_1$$
$$v_2 = e_2' - r_2.i_2$$



- Magnetic leaks

$$\phi_T = \phi_{f1} + \phi_{f2} + \phi$$
$$e_1' = n_1.\frac{d\varphi}{dt} + n_1.\frac{d\varphi_{f1}}{dt}$$
$$e_1' = e_1 + l_1.\frac{di_1}{dt}$$
$$e_2' = e_2 + l_2.\frac{di_2}{dt}$$





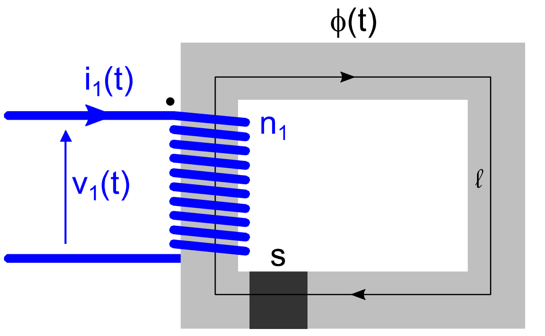
Single-phase transformer

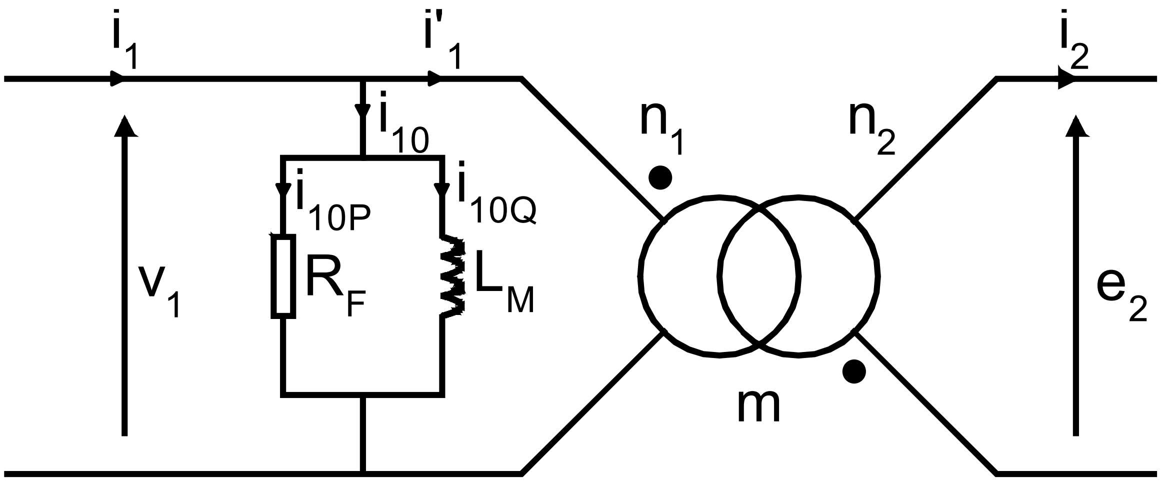
- Principle: magnetic circuit imperfection

- Finite permeability and non zero reluctance of the magnetic circuit

=> A primary current is consumed at no load

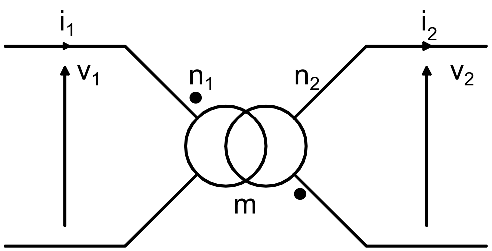
$$\mathcal{R}\phi = n_1 . i_{10}$$

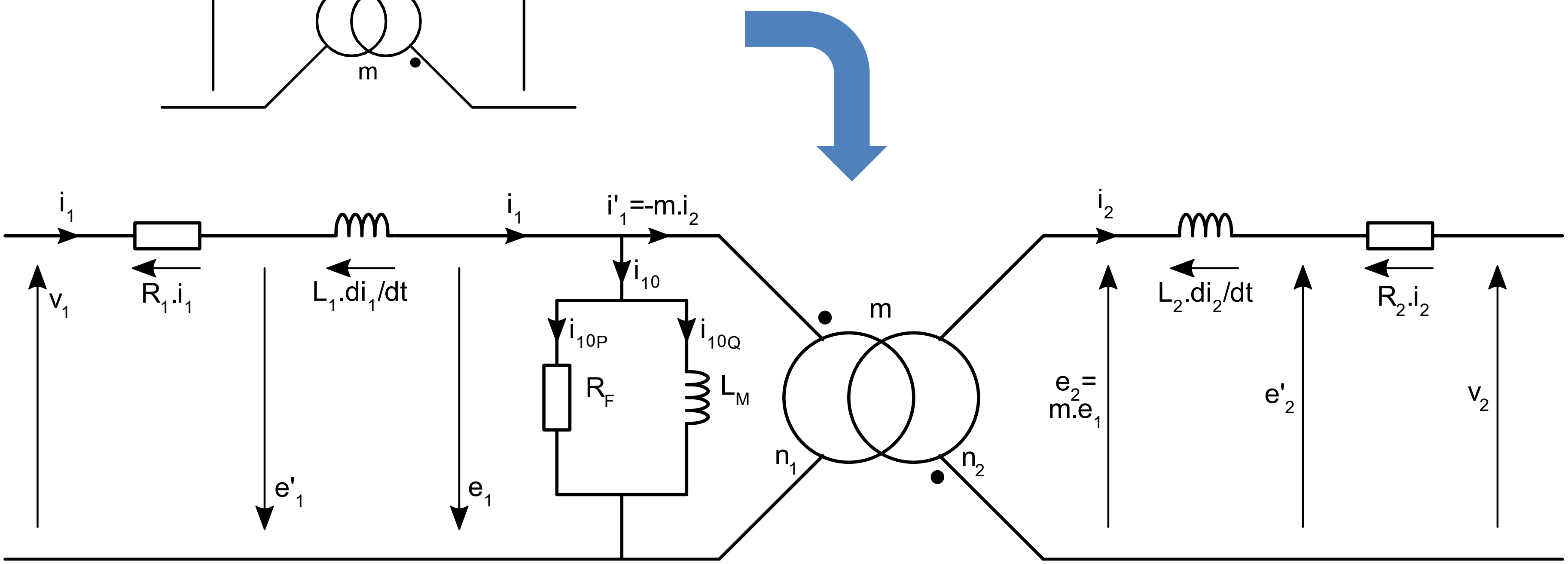




Single-phase transformer

- Equivalent model of the real transformer



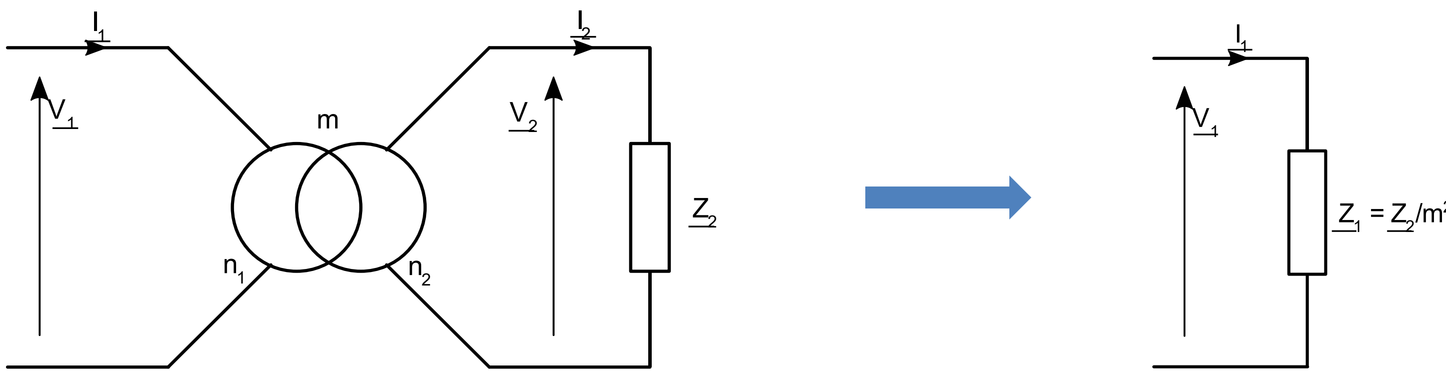


Single-phase transformer

- Equivalent model of the real transformer: Impedance transfer

- A transformer, in which an impedance Z_2 is connected to the secondary side is equivalent to an impedance Z_1 connected to the primary side with:

$$\boldsymbol{Z_1 = \frac{Z_2}{m^2}}$$

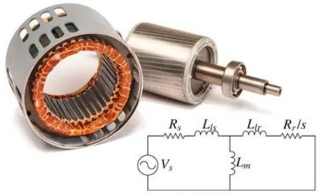


- We remind: $I_1 = m.I_2$ et $V_1 = \frac{V_2}{m}$

- We write: $Z_2 = \frac{V_2}{I_2}$ et $Z_1 = \frac{V_1}{I_1}$

$$Z_1 = \frac{V_1}{I_1} = \frac{V_2/m}{m.I_2} = \frac{1}{m^2}.\frac{V_2}{I_2} = \frac{Z_2}{m^2}$$

$$Z_1 = \frac{Z_2}{m^2}$$

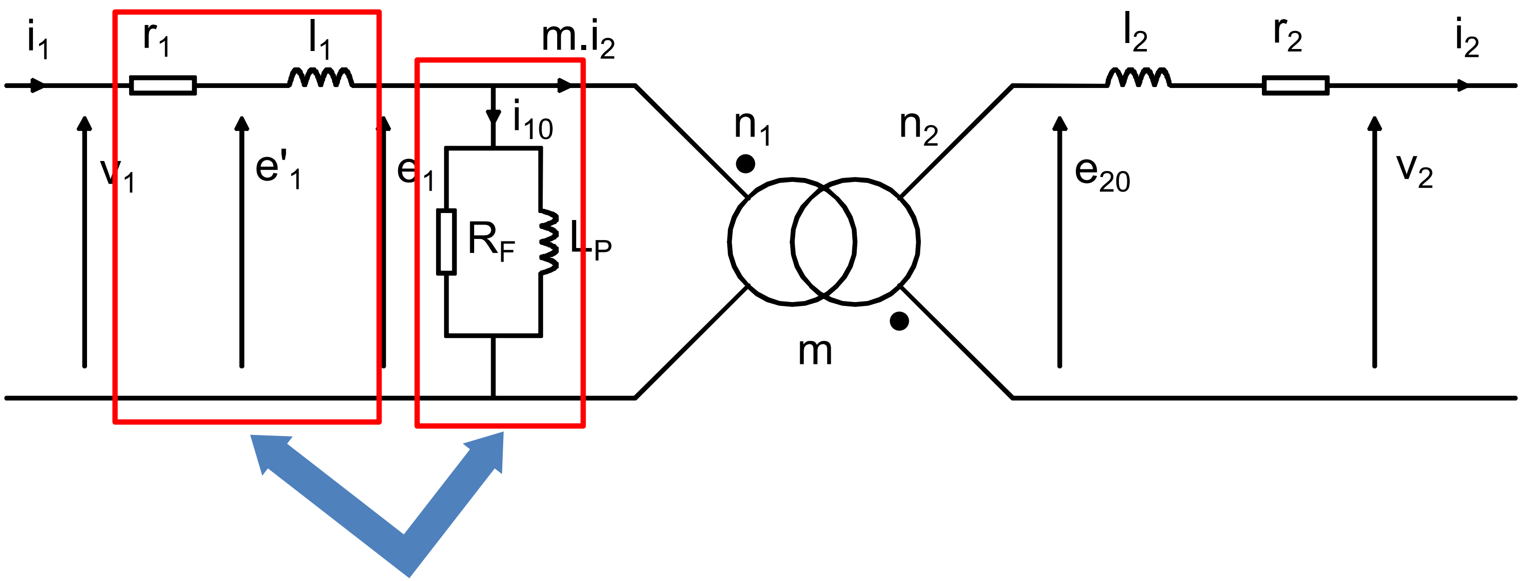


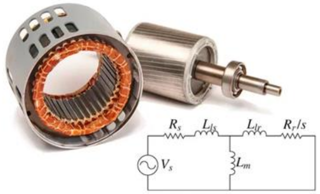
Single-phase transformer

- Equivalent model of the real transformer: Kapp's assumption

- The voltage drop across R_1 and L_1 is small compared to the voltages e_1 and V_1

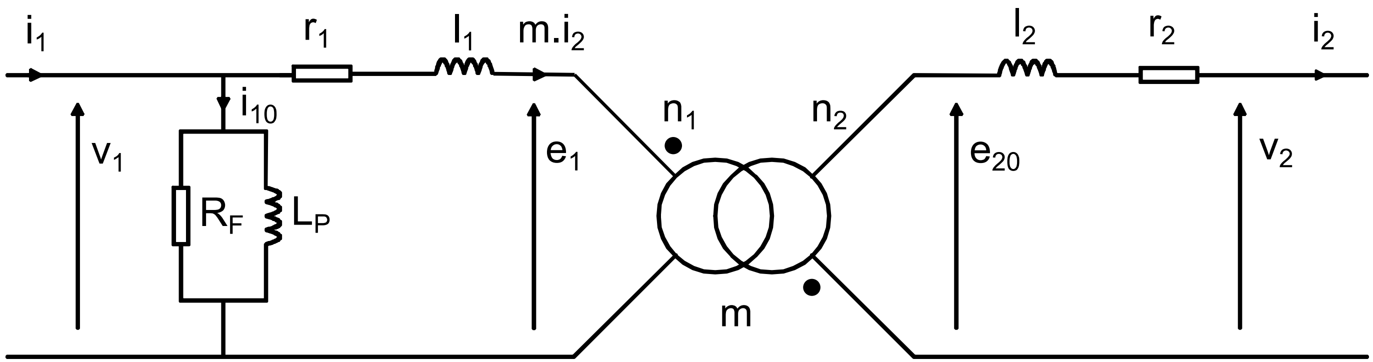
=> R_1/L_1 and R_F/L_p can be swapped





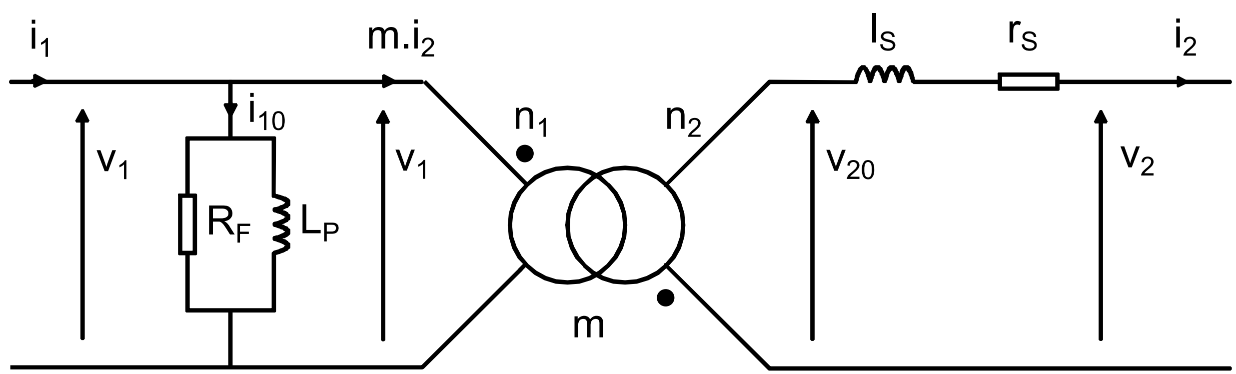
Single-phase transformer

- Equivalent model of the real transformer: Kapp's assumption

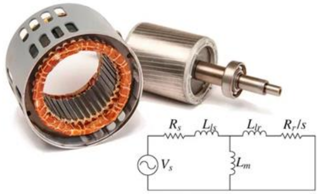


- Kapp's equivalent secondary model (R_1 and L_1 are transferred to the secondary side)

$$r_S = r_2 + m^2 . r_1$$
$$l_S = l_2 + m^2 . l_1$$
$$m = \frac{n_2}{n_1} = \frac{V_{20}}{V_1} \cong \frac{I_1}{I_2}$$



$$\underline{V_2} = -m.\underline{V_1} + R_s.\underline{I_2} + jX_s.\underline{I_2}$$



Single-phase transformer

- Equivalent model of the real transformer: Kapp's assumption

- Kapp's equivalent primary model (R_2 and L_2 are transferred to the primary side)

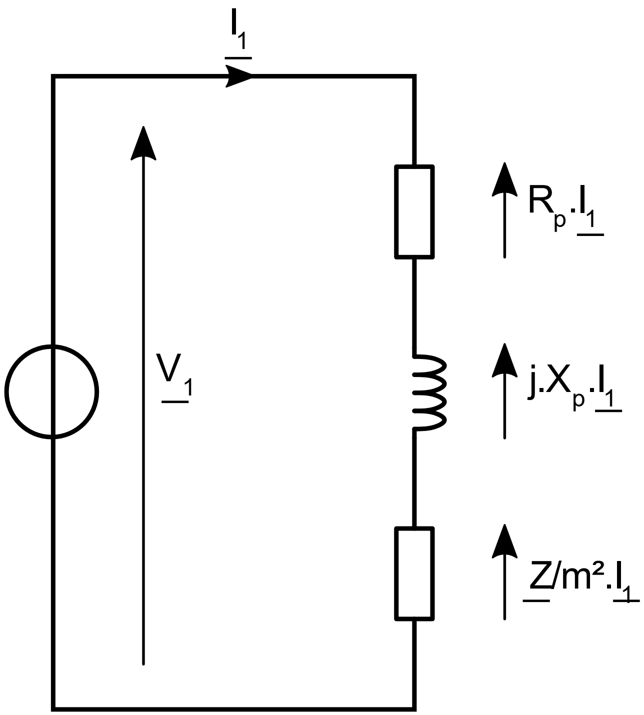
$R_P = R_1 + \frac{R_2}{m^2}$: total winding resistance transferred to the primary side

$X_P = X_1 + \frac{X_2}{m^2}$: total winding reactance transferred to the primary side

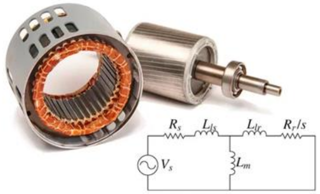
$\underline{Z_p} = R_p + jX_p$: total winding impedance transferred to the primary side

$\frac{\underline{Z}}{m^2}$: secondary side load transferred to the primary side.

$$\underline{V_1} = \frac{\underline{Z}}{m^2} \cdot \underline{I_1} + \underline{Z_P} \cdot \underline{I_1}$$



(this model will be useful for the induction motor)



Single-phase transformer

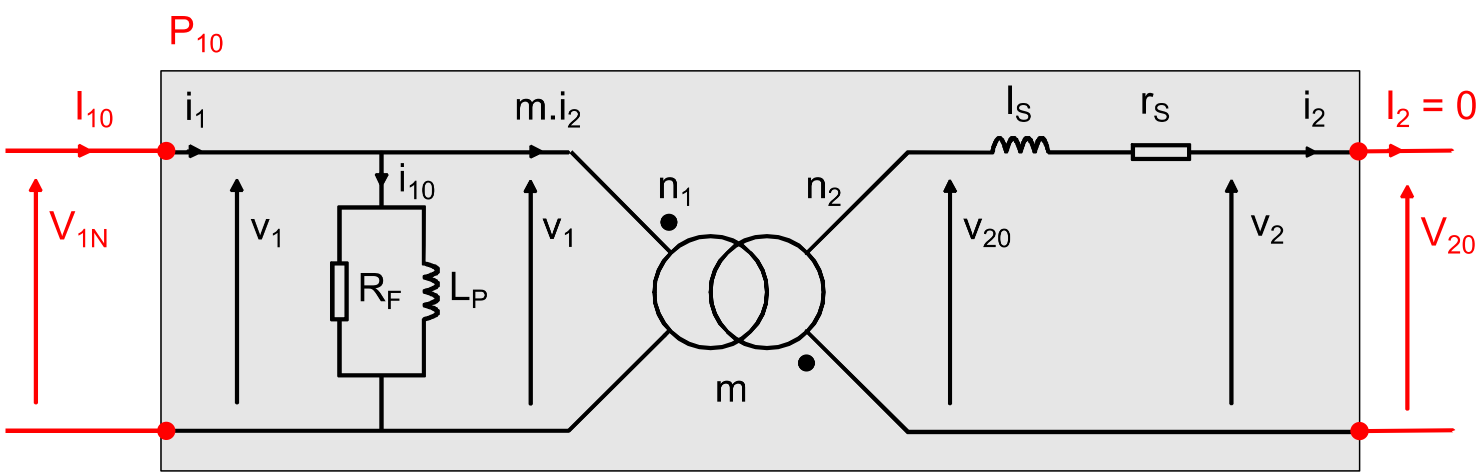
- Determining the elements of the Kapp secondary equivalent model:

- Test at no load:

- conditions: Transformer supplied at rated voltage (V_{1N}) + open secondary side (I_2 = 0)

- Measured quantities: V_{1N}, I_{10}, V_{20}, P_{10} (iron losses) and Q_{10}

$$R_F = \frac{V_{1N}^2}{P_{10}} \qquad X_P = L_P.\omega = \frac{V_{1N}^2}{Q_{10}} \text{ with } Q_{10} = \sqrt{(V_{1N}.I_{10})^2 - P_{10}^2} \qquad m = \frac{V_{20}}{V_1}$$



Single-phase transformer

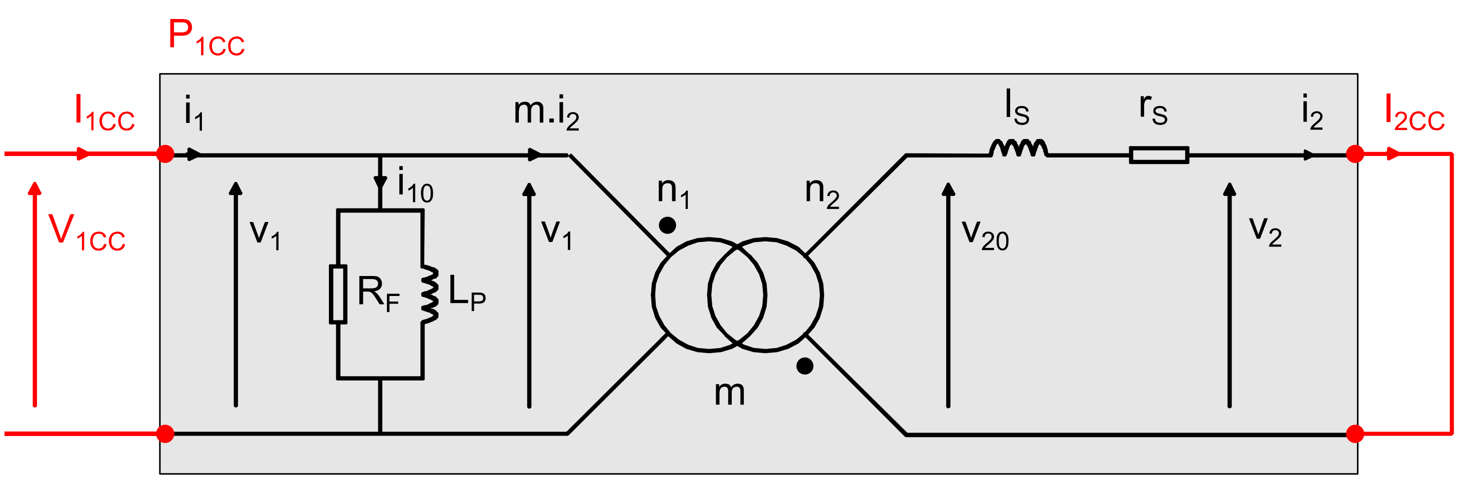
- Determining the elements of the Kapp secondary equivalent model:

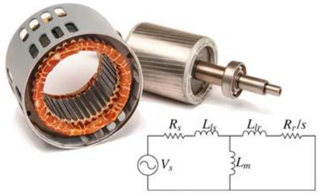
- Short-circuit test: (short circuit at the secondary side)

- conditions: Transformer supplied at reduced voltage (V_{1CC}) + rated secondary side current ($I_{2CC} = I_{2N}$)

- Measured quantities: V_{1CC}, I_{1CC}, I_{2CC}, P_{1CC} (Joule losses) and Q_{1CC}

$$R_S = \frac{P_{1CC}}{I_{2CC}^2} \qquad X_S = L_S.\omega = \frac{Q_{1CC}}{I_{2CC}^2} \quad \text{with } Q_{1CC} = \sqrt{(V_{1CC}.I_{1CC})^2 - P_{1CC}^2}$$

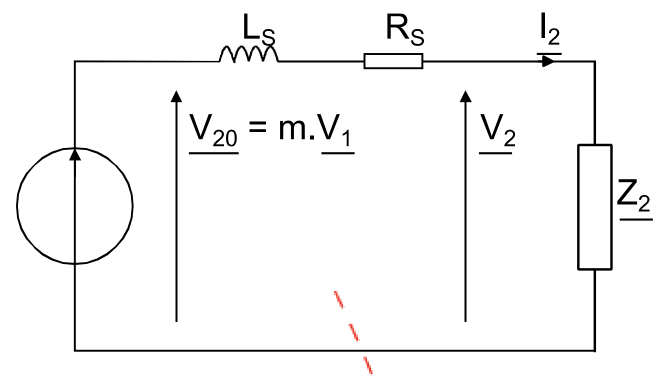




Single-phase transformer

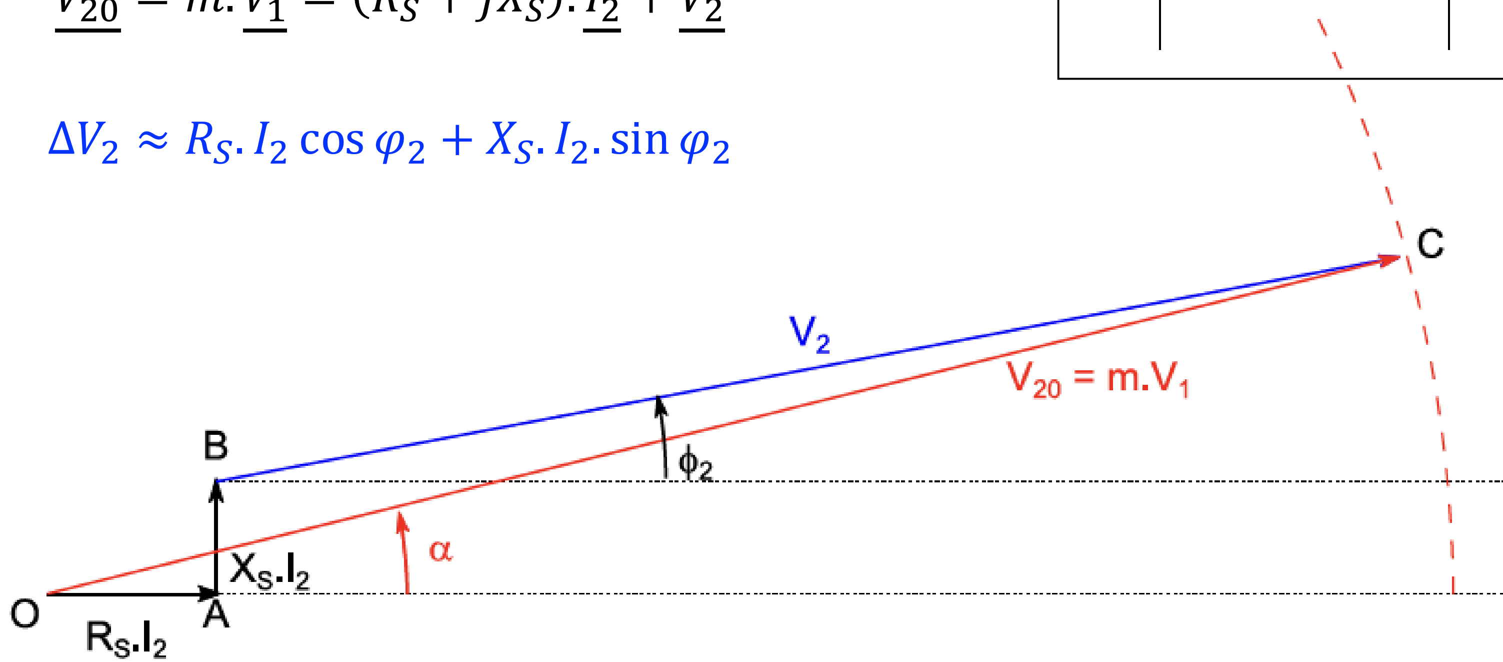
- Voltage drop at the secondary side of the transformer:

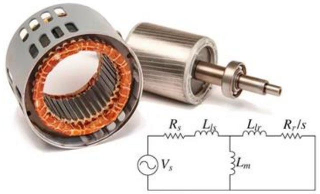
- Kapp's triangle: case of the inductive load ($\phi_2 > 0$)



$$\underline{V_{20}} = m.\underline{V_1} = (R_S + jX_S).\underline{I_2} + \underline{V_2}$$

$$\Delta V_2 \approx R_S.I_2 \cos\varphi_2 + X_S.I_2.\sin\varphi_2$$

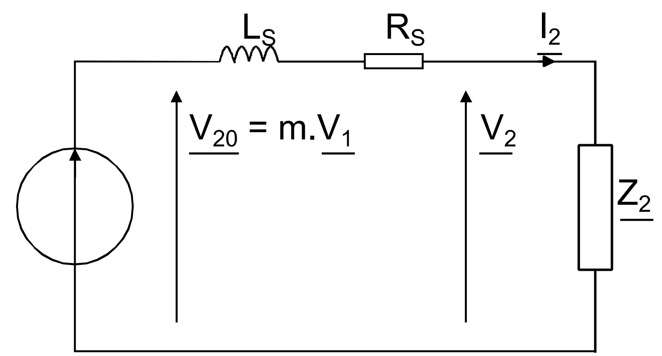




Single-phase transformer

- Efficiency:

$$\eta = \frac{V_2 . I_2 . \cos\varphi_2}{V_2 . I_2 . \cos\varphi_2 + R_S . I_2^2 + P_F}$$



- P_F is obtained from the test at no load (P_{10})

- Optimum current obtained from

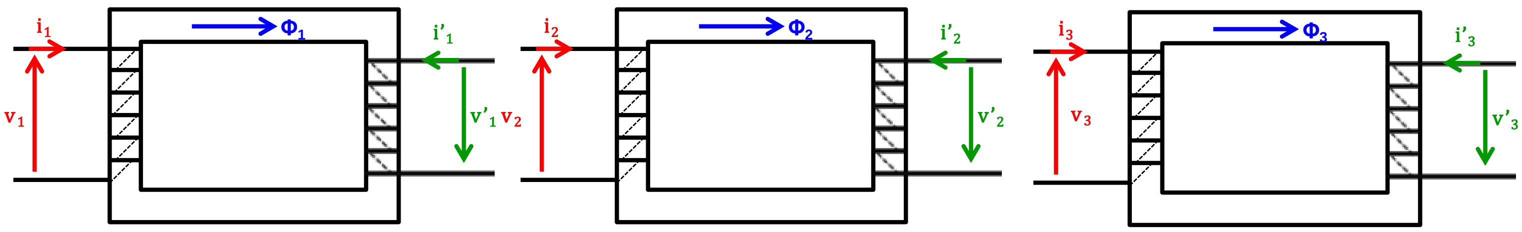
$$\eta = \frac{V_2 . \cos\varphi_2}{V_2 . \cos\varphi_2 + R_S . I_2 + \frac{P_{Fer}}{I_2}}$$

- The maximum efficiency is given for $R_S . I_{2Opt} = \frac{P_{Fer}}{I_{2Opt}}$ and $I_{2Opt} = \sqrt{\frac{P_{Fer}}{R_S}}$

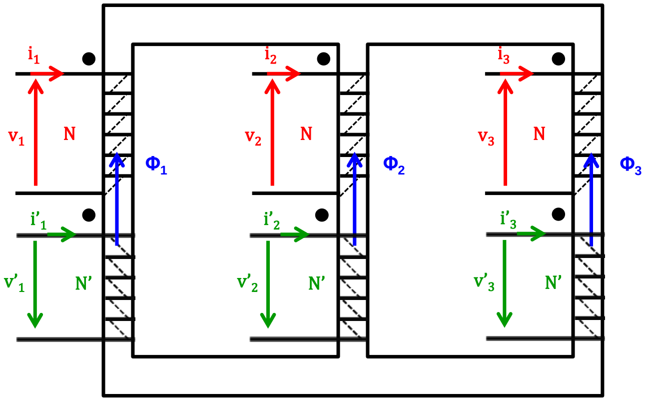
Three-phase transformer

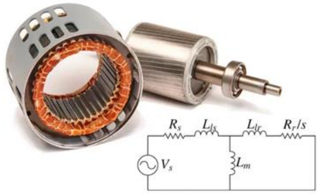
- Construction and connection:

- At first glance, the three-phase transformer can be considered as the combination of 3 single-phase transformers



- Or it can be integrated on a single magnetic circuit comprising 3 columns, each carrying the primary winding and the secondary winding





<u>Three-phase transformer</u>

- <u>Construction and connection</u>:

- Both the primary and the secondary sides of the transformer need to be connected (delta or Y)

- The nature of these couplings is designated by letters, using upper case letters for the high voltage side and lower case letters for the secondary side

1st letter (upper case): connection on H.T. side

- **Y: «** star »

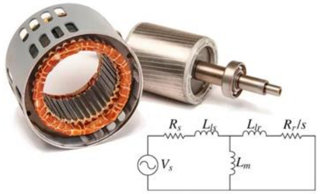
- **D** or **Δ:** delta

2nd letter (lower case): coupling on B.T. side

- y: star

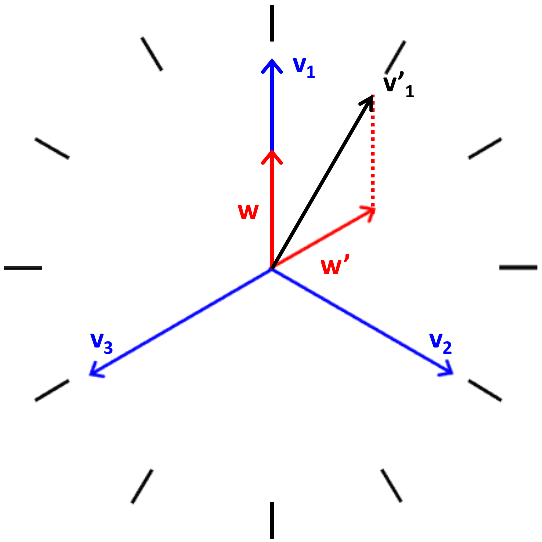
- d**:** delta

Add letter “ N ou **n”** if neutral is out.



Three-phase transformer

- Hourly index: Phase shift angle between primary and secondary voltages expressed in hours



- Marked from 0 to 11, each hour angle is always a multiple of 30°:
=> 0 for 0
=> 1 for 30
=> 2 for 60°, and
=> 6 for 180
=> ...

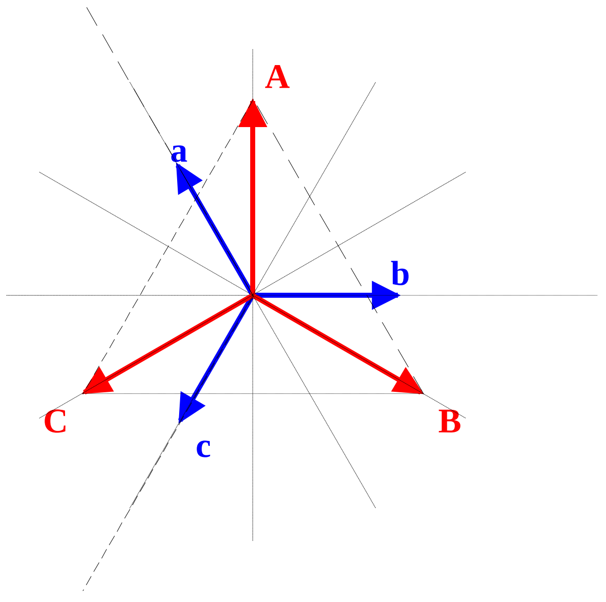
- That's why a specific representation has been chosen:
=> On the same Fresnel diagram, we plot two vectors representing two homologous voltages, one on the primary side, the other on the secondary side.

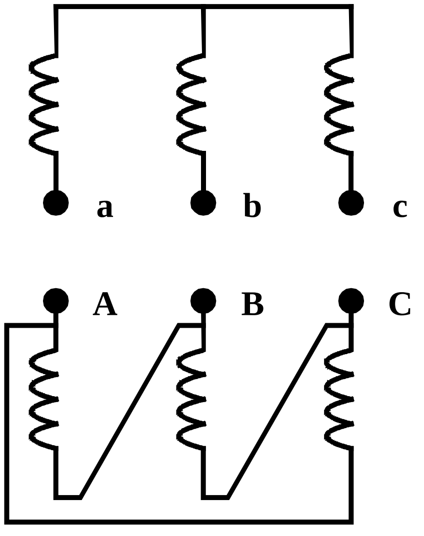
=> The voltage on the primary side is shown vertically, pointing upwards.

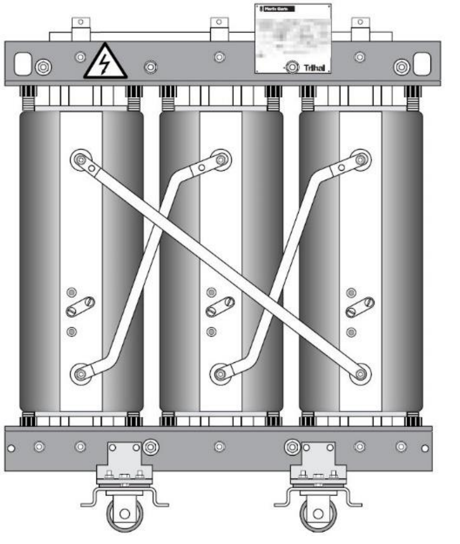
If we consider these two vectors as the two hands of a watch, the time indicated by the watch is by definition the transformer hourly index

Three-phase transformer

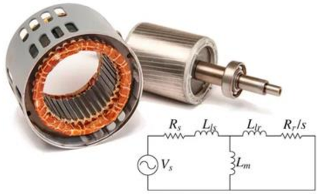
- Example of Dyn 11 connection:





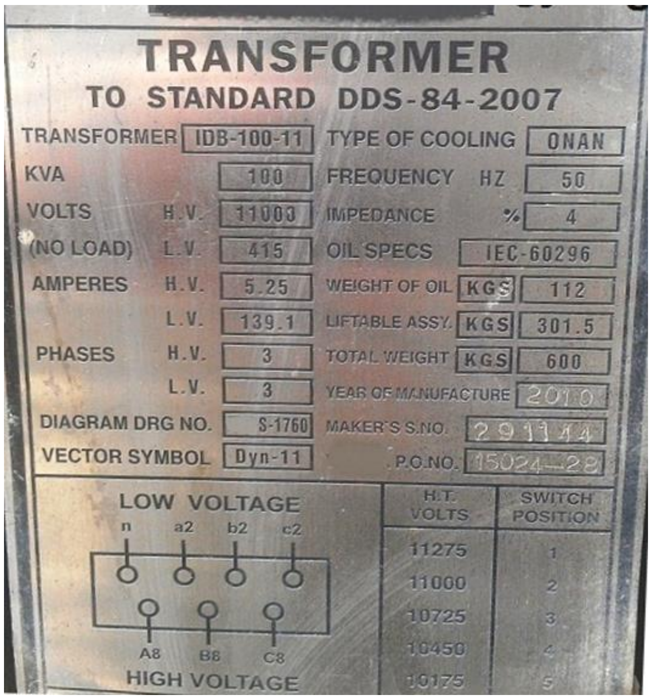


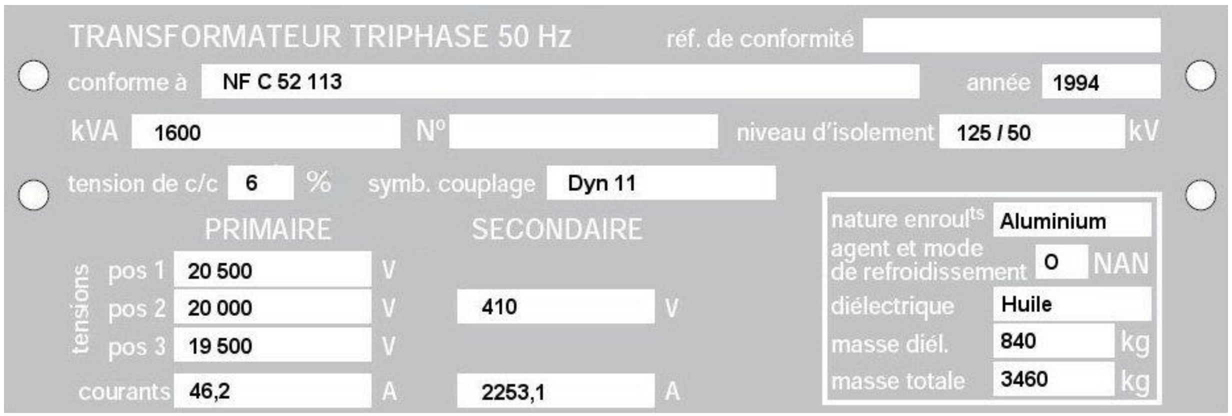
- High-voltage side => delta coupling.
- Low-voltage side => Y connection with neutral out.
- 330° phase shift (11x30) between primary and secondary.

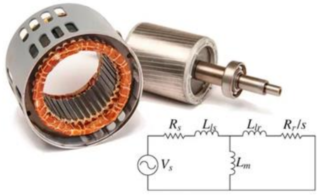


Three-phase transformer

- Transformer name plate: gives all the rating







Three-phase transformer

- Equivalent model of the real transformer:

- Each transformer column can be modeled separately by a single-phase equivalent diagram at the secondary (see before)

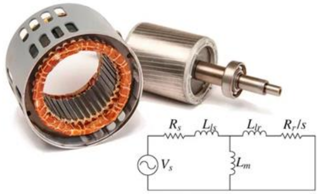
- Each element of the secondary model can be determined by the same tests as those of the single-phase transformer

- Test at no load:

$$R_F = \frac{3V_N^2}{P_0} \qquad X_P = L_P.\omega = \frac{3V_N^2}{Q_0} \quad \text{with } Q_0 = \sqrt{(V_N.I_0)^2 - P_0^2} \qquad m = \frac{V'_N}{V_N}$$

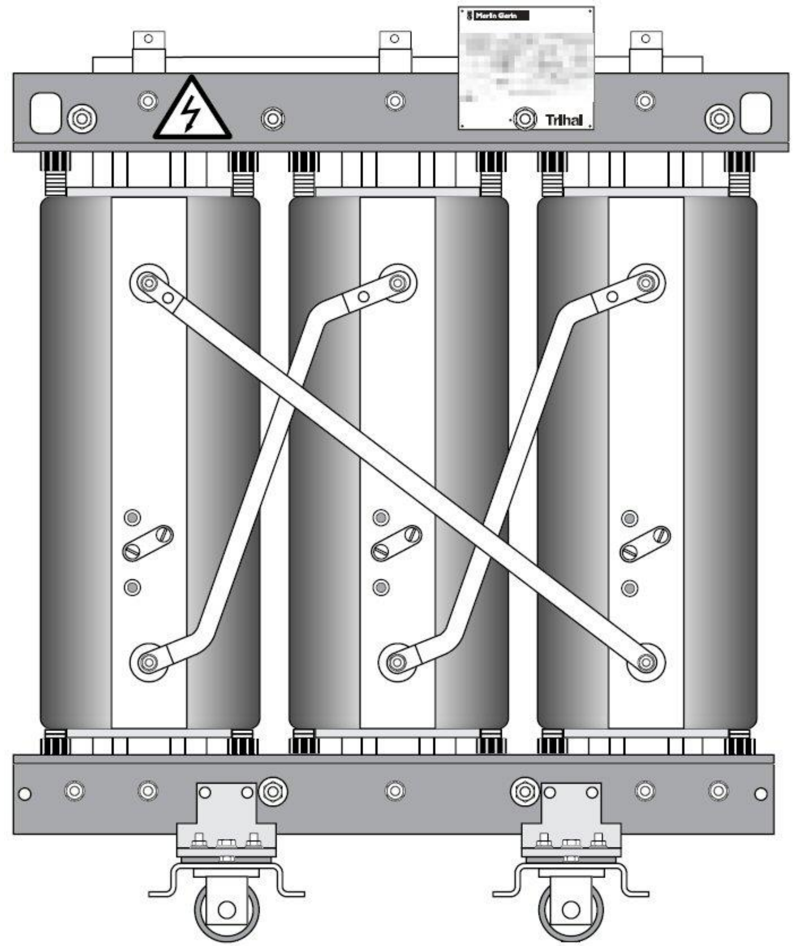
- Short-circuit test: (short circuit at the secondary side)

$$R_S = \frac{P_{CC}}{3I'^2_{CC}} \qquad X_S = L_S.\omega = \frac{Q_{CC}}{3I'^2_{CC}} \quad \text{with } Q_{CC} = \sqrt{(V_{CC}.I_{CC})^2 - P_{CC}^2}$$



Three-phase transformer

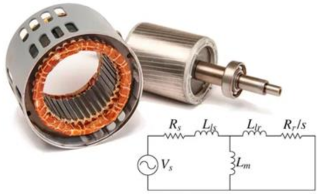
- <u>HTA/BT distribution transformers</u>: Dry transformer



- Active parts coated in protective resins (often epoxies) and mounted on a support frame in the open air.

- Good ventilation of the device and the room is required

- Dust removal from ambient air recommended



Three-phase transformer

- HTA/BT distribution transformers: Immersed transformer

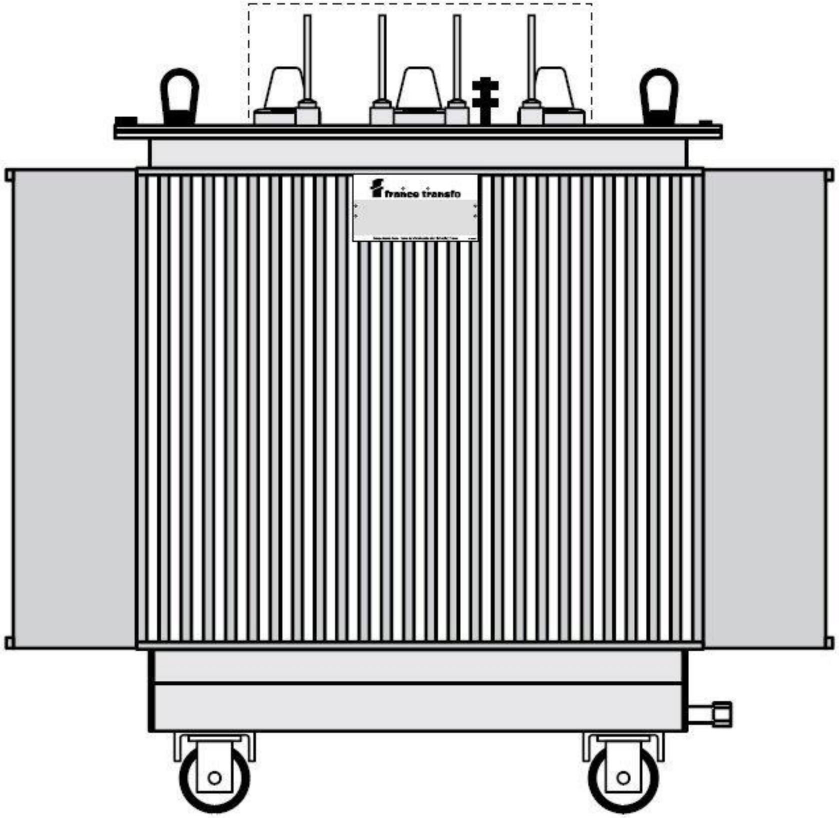
- Sealed with Total/Integral filling

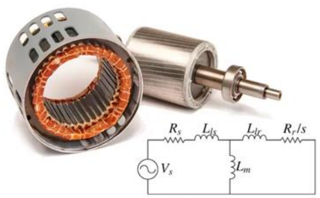
- Hermetically sealed transformer

- Flexible tank

- Flexible tank

- Accordion-folded walls to absorb changes in dielectric volume as it heats up





III - Transformers

Three-phase transformer

- HTA/BT distribution transformers: Dry and Immersed transformers

Transformateur de type enrobé sous enveloppe

Transformateur 400 kVA avec accessoires de protection