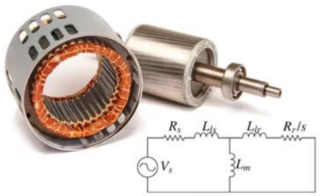
Polytech Orléans – M1 AESM

Electrical Engineering



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Year 2023/2024

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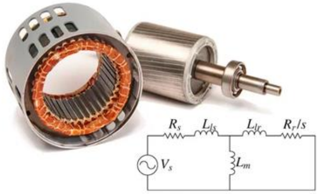
English version: Thomas TILLOCHER



Introduction

Schedule

	M1 AEMS
Lectures	14 x 1,25 h dont 2 DS
Tutorials	8 x 1,25 h
Lab work	1 x 2,50 h + 6 x 3,75 h

Outline

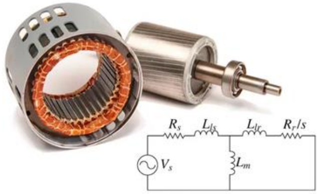
Introduction

I) Reminders (electricity)

II) Power in sinusoidal regime (single-phase and 3-phase)

III) Transformers

IV) Electric motors



Introduction

- 3 main families:

DC Motor

DCM

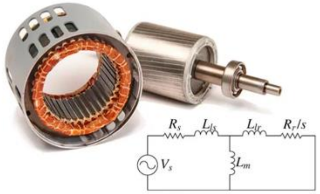
Synchronous machine

SM

Induction machine

/Asynchronous machine

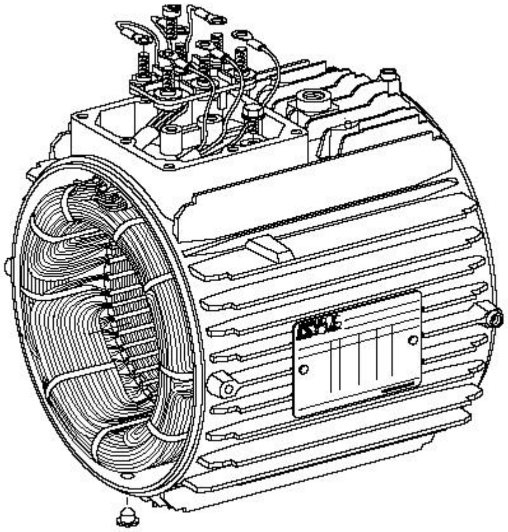
ASM



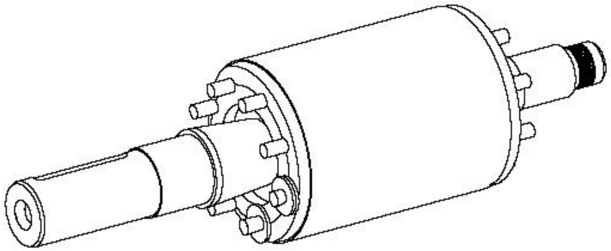
Introduction

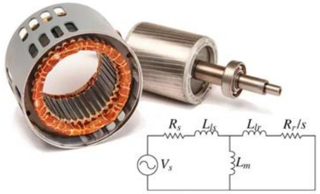
- Machine construction = 2 distinct parts for all 3 types of motors

Stator



Rotor

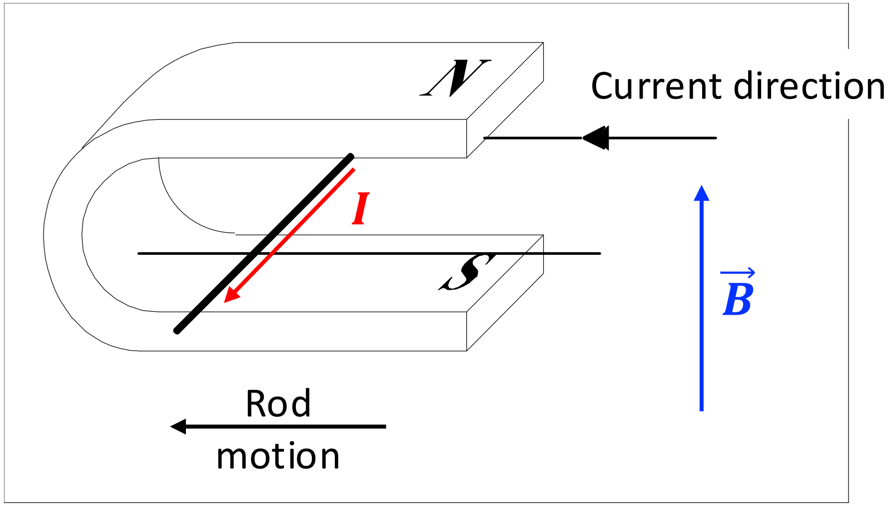




The DC motor

- Laplace's force:

"A moving conductor placed in a magnetic field is set in motion when a current flows through it"



$$\vec{F} = I d\vec{l} \wedge \vec{B}$$

$$F = B \times I \times l$$

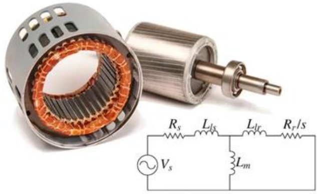
B : induction magnetic field (T)

I : current intensity (A)

l : length (m)

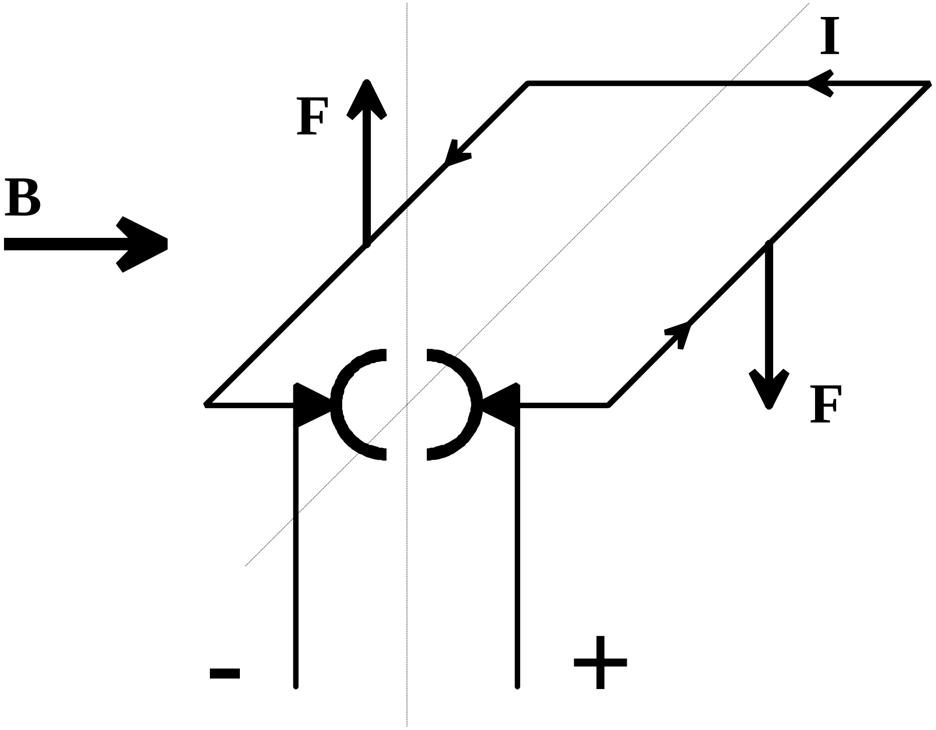
Right hand rule :

Thumb => Thrust
Index => Intensity
Major => Magnetism



The DC motor

- First approach of the principle: Moving frame with current flowing through it



- Two diametrically opposed conductors:

⇒ current flowing through them in opposite directions

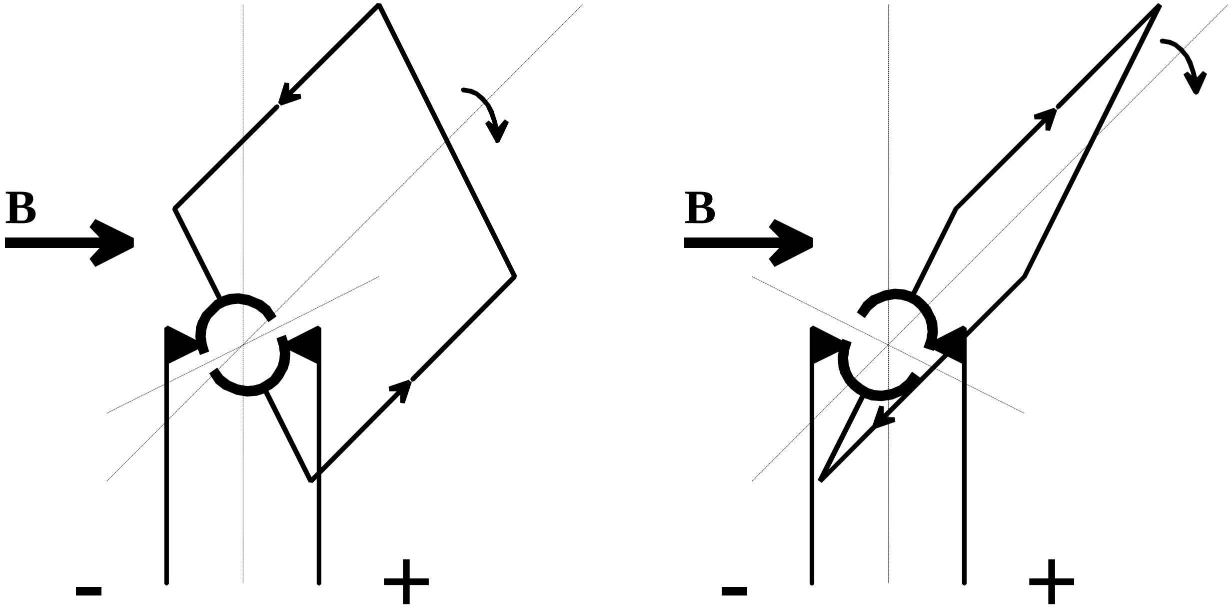
⇒ placed in a magnetic field

- Will be subjected to two forces in opposite directions:

=> Turn in the same direction.

The DC motor

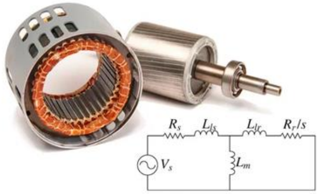
- First approach of the principle: Current reversal



- When the two conductors reach the "neutral" line, the direction of the forces is reversed.

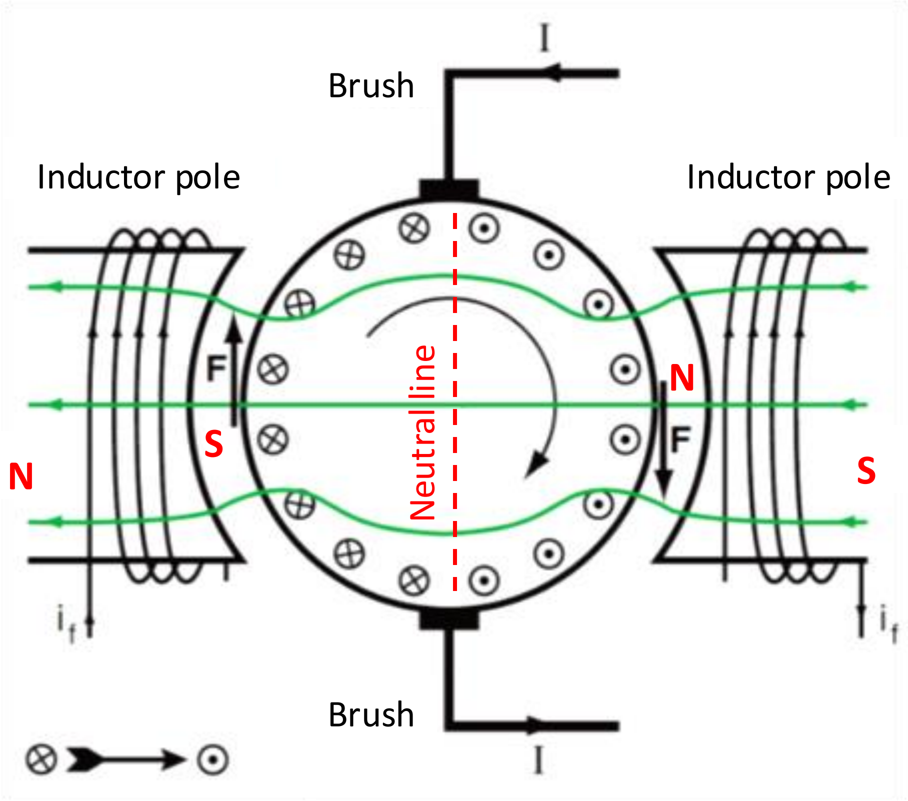
⇒ the direction of the current in each conductor must change

- Thanks to the collector, even though the voltage is DC, the current in the winding made up of the two conductors will reverse under the switching axis, and rotation will continue.



The DC motor

- Construction: basic schematic



The motor is made up of two main parts:

- The stator (fixed)

=> Acts as an electromagnet

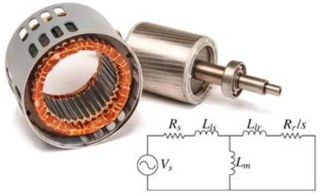
=> Usually called "inductor",

- The rotor (moving),

=> Rotating part where Laplace forces are applied

=> Also acts as an electromagnet

=> Usually called "armature".



The DC motor

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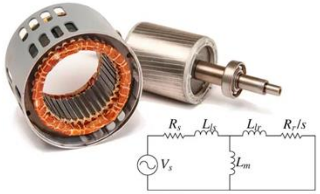
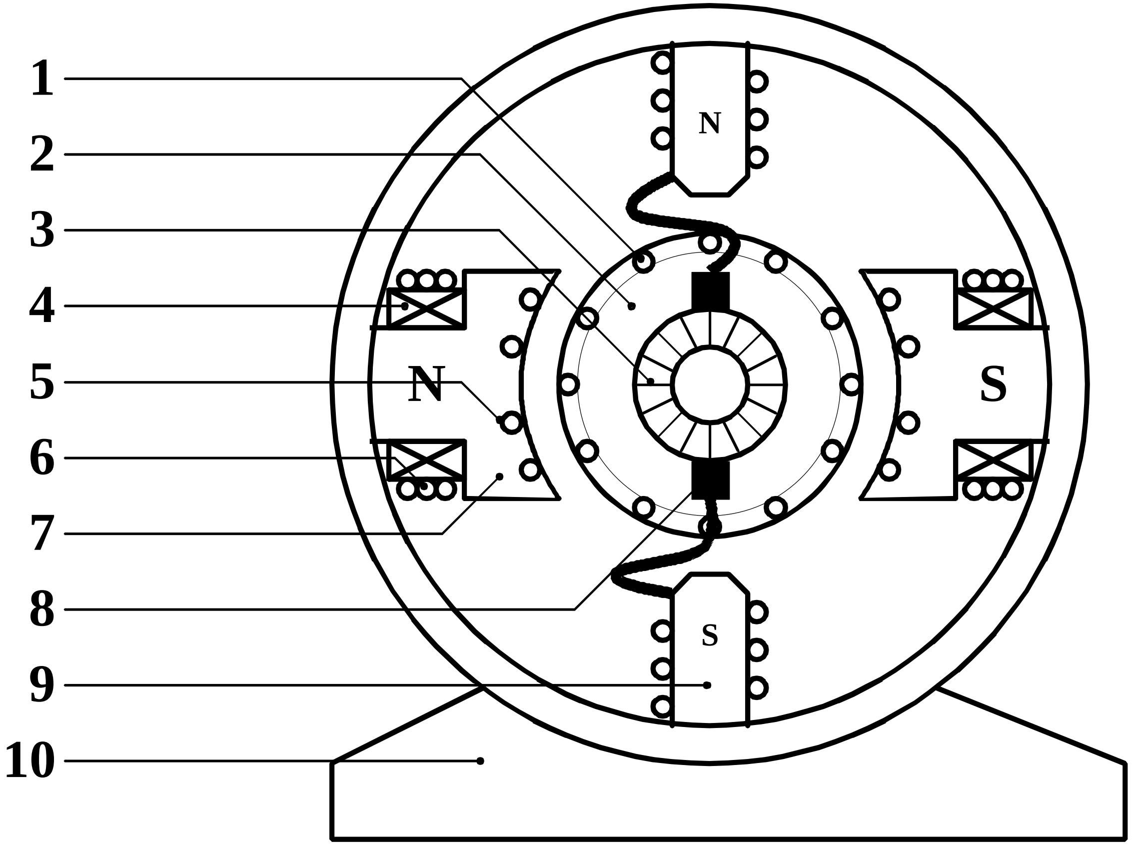
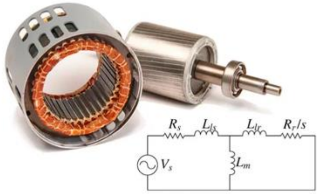


The DC motor

- Construction:



1) Armature windings
2) Rotor
3) Collector/commutator
4) Inductor (excitation) windings
5) Compensation poles
6) Stabilization windings
7) Pole shoe
8) Brushes
9) Switching poles
10) Stator frame (ferromagnetic)



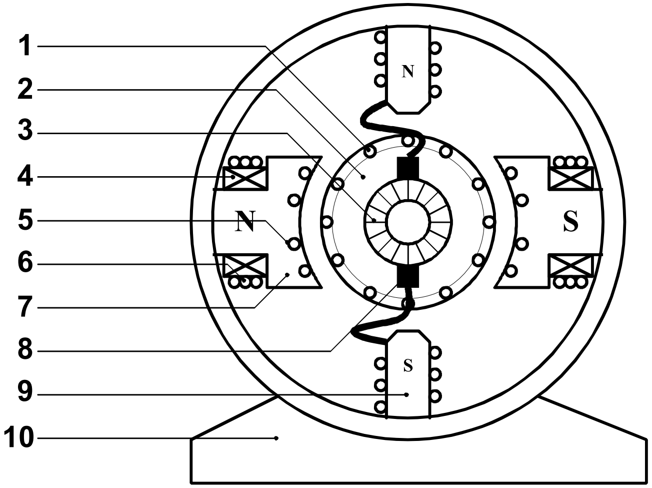
The DC motor

- The inductor (4):

⇒ Coils wound around polar cores arranged around the periphery of the stator

⇒ Excitation current I_e flows through it, producing a magnetic flux ϕ.

⇒ On small machines, the inductor is replaced by permanent magnets.



- Compensation poles (5):

⇒ Compensate for the armature magnetic reaction

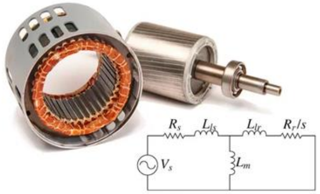
⇒ Placed in the notches of the pole shoes

⇒ The current flowing in the armature flows through them

⇒ Ensures compensation for any load.

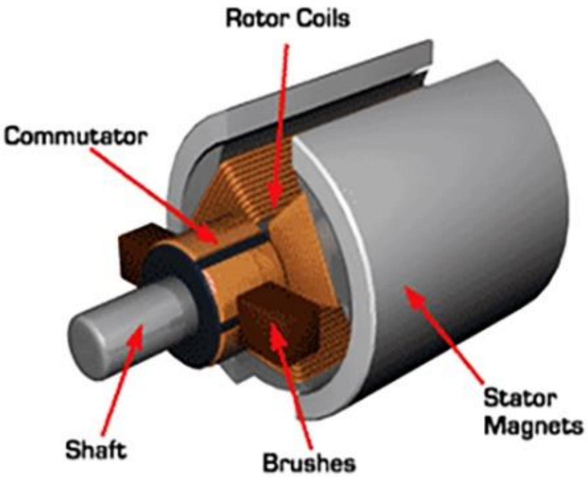
- The switching poles (9):

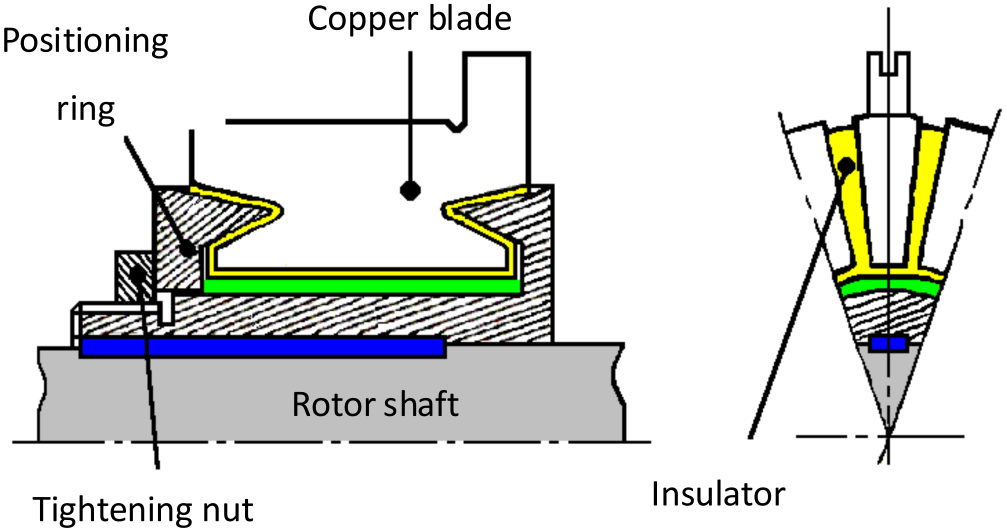
⇒ Ease current switching in conductors by separating the neutral line from the switching axis (prevent shortcut)



The DC motor

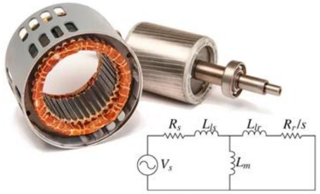
- Collector and brushes (8):





⇒ The commutator is a set of copper blades, laterally insulated from each other, and arranged in a cylinder at the end of the rotor.

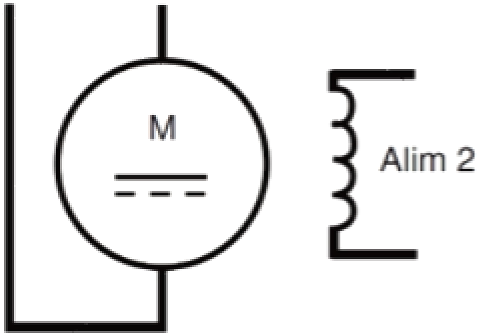
⇒ The brushes, carried by the stator, rub against the commutator blades. The commutator assembly reverses the direction of current in the rotor conductors as they cross the machine neutral line.



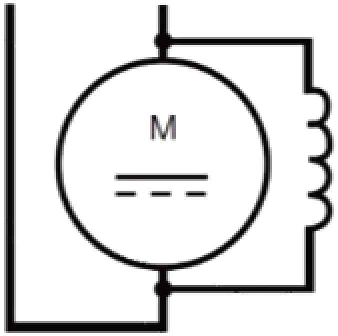
The DC motor

- Connections of the DC motor:

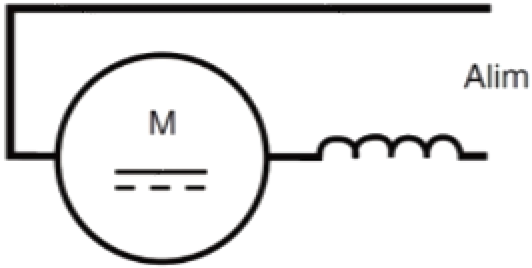
Separately excited DC motor



Shunt excited DC motor

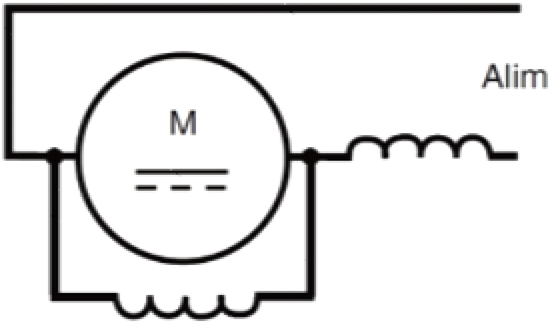


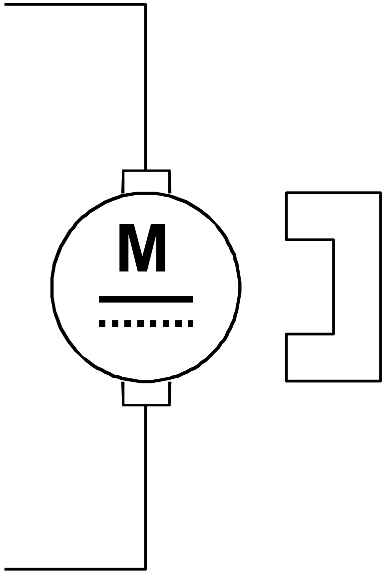
Series excited DC motor



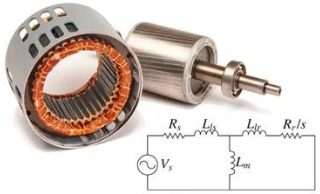
=> High torque at start

Compound excited DC motor



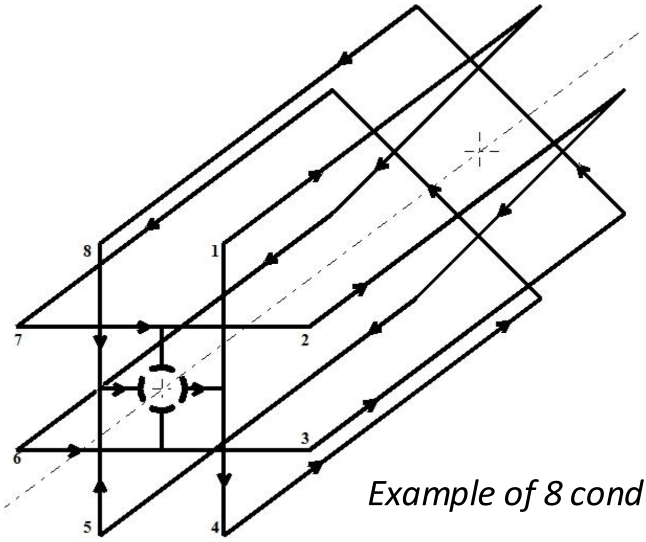


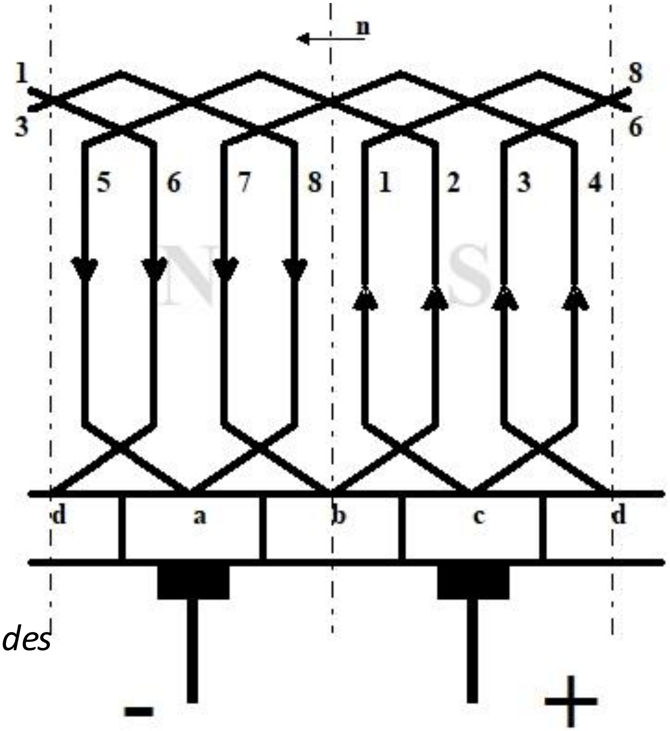
Magnet excited DC motor



The DC motor

- The electric circuit at the armature (1):



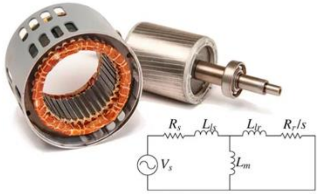


Example of 8 conductors, 4 blades

- Windings:

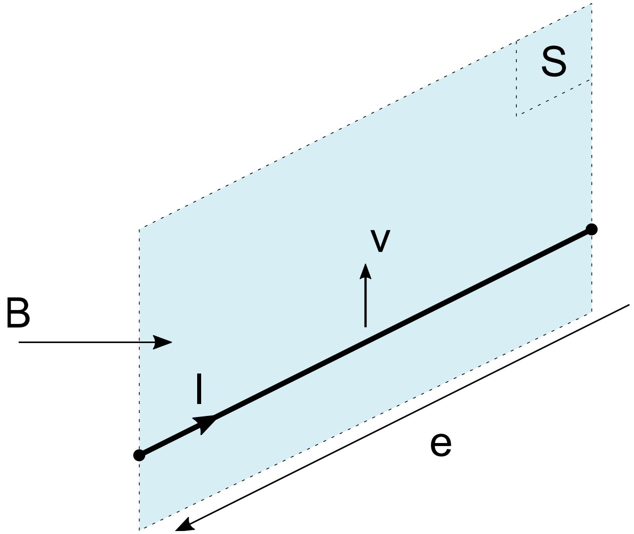
⇒ Each individual wire is called a “conductor”, two conductors form a “turn”, turns are grouped by “section” (one section is between 2 blades) and sections by coil.

⇒ The two halves of a section are located in almost diametrically opposed slots. The section input and output conductors are soldered to two adjacent switch blades.



The DC motor

- Electromechanic relations: counter electromotive force



- The machine rotates at a speed “n” expressed in rpm

“Φ ” is the flux produced by the inductor in Wb.

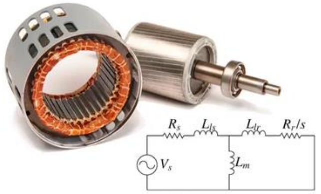
- By considering “p” the number of pole pairs in the machine stator:

=> During one revolution, an active armature conductor crosses “p” times the “+ Φ” flux and “p” times the “- Φ” flux.

- The flux variation seen by a conductor is written: $\Delta\Phi = +p.(+\phi) - p.(-\phi) = 2.p.\phi$

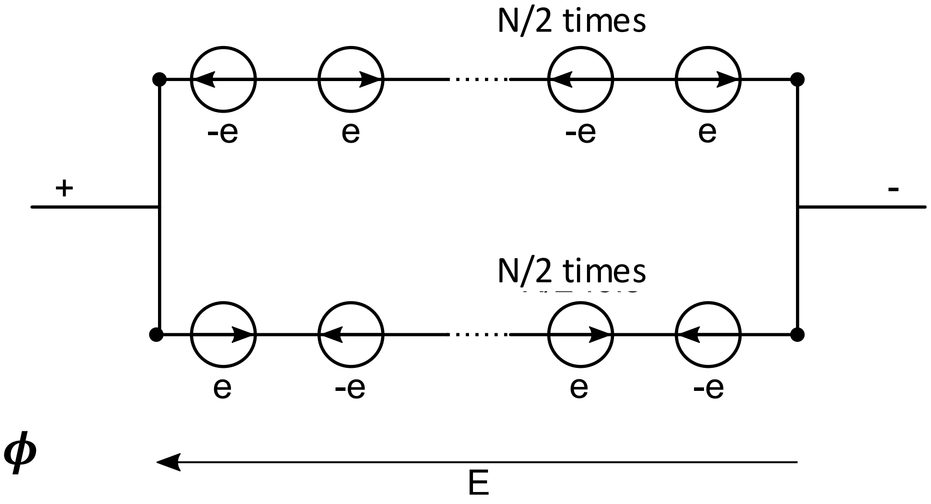
- At each conductor is created a counter electromotive force e:

$$e = \frac{\Delta\phi}{\Delta t} = \frac{2.p.\phi}{1/n} = 2p.n.\phi$$



The DC motor

- Electromechanic relations: counter electromotive force



- The armature has "N" active conductors and "a" winding paths

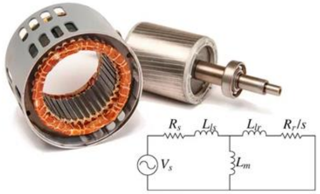
- The armature's counter-electromotive force is written:

$$E = \frac{N}{2a}.e \qquad E = \frac{N}{2a}.2p.n.\phi \qquad \boldsymbol{E = \frac{p}{a}.N.n.\phi}$$

- Usually written: $\boldsymbol{E = k.n.\phi}$ with k=(p/a)N a constant related to the construction of the machine

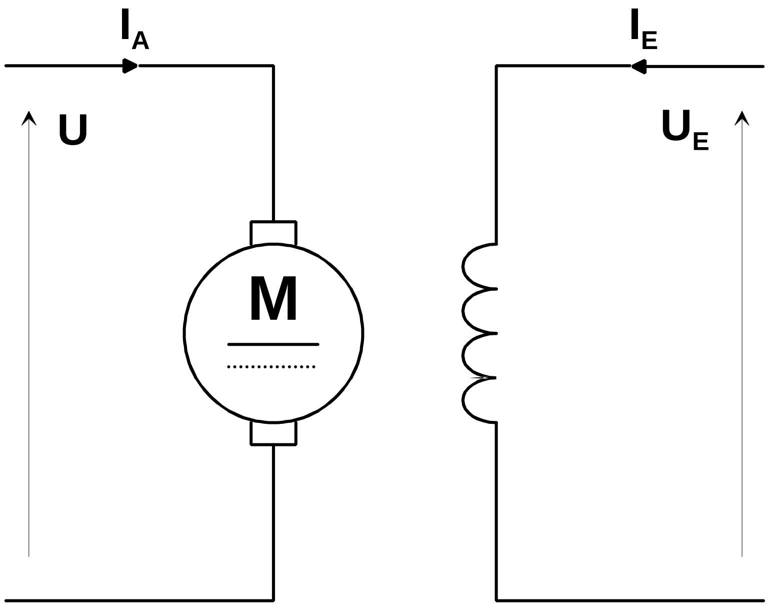
- Or also: $\boldsymbol{E = k_n.n}$ if Φ remains constant, permanent magnet motor or i_E =Cste

k_n: speed constant, given by the manufacturer, in $V.tr^{-1}.s$ or $V.tr^{-1}.min$



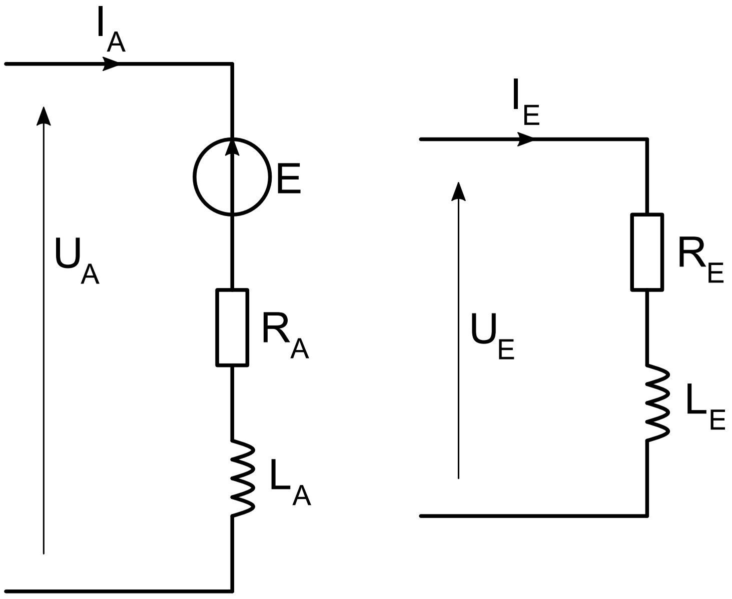
The DC motor

- Equivalent circuit of the DC motor:



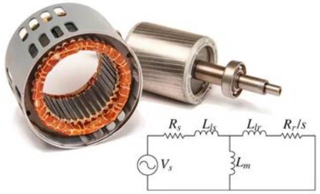
Armature

Inductor



Armature

Inductor



The DC motor

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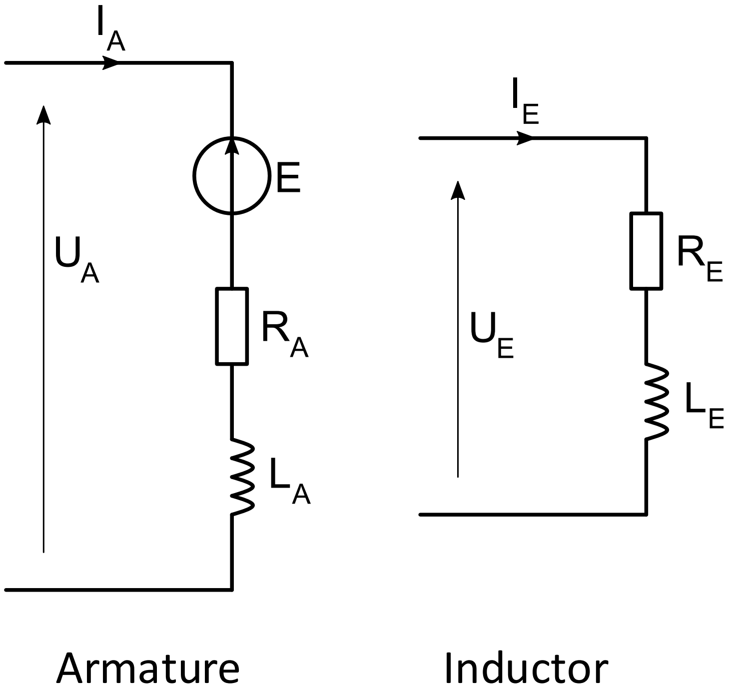


- Armature: Each active conductor is made of copper, has a length and a cross-section, and therefore has a resistance to current flow

=> The total armature resistance is called R_A

- The equivalent models of both armature and inductor windings actually consist of a resistor R and an inductance L with no steady-state effect

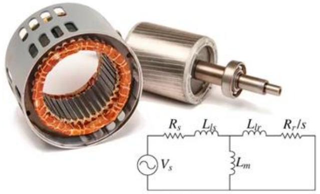
$\Rightarrow$ I_A = Cste and $L.dI_A/dt = 0$

$\Rightarrow$ L usually discarded for this reason

- Expression of the voltage at the terminals of the motor armature:

$$U_A = E + R_A.I_A$$

- U_A: supply voltage in Volts
- E :.c.e.m.f in Volts
- R_A: armature resistance in Ohms
- I_A : armature current in Amperes



The DC motor

- Characteristic equations:

- Expression of rotation speed:

$$n = \frac{E}{k.\Phi} = \frac{U_A - R.I_A}{k.\Phi} \cong \frac{U}{k.\Phi}$$

=> If the flux disappears, the motor goes into overdrive

=> The speed is proportional to voltage if the flux is constant

$$n \cong \frac{U}{k_n.}$$

- Power absorbed by the armature:

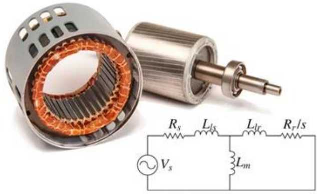
$$\mathrm{P_A = U_A.I_A}$$

- Power absorbed by the inductor:

$$\mathrm{P_E = U_E.I_E}$$

- Total electrical power supplied to the motor:

$$\mathrm{P_{AbsTot} = U_A.I_A + U_E.I_E}$$



The DC motor

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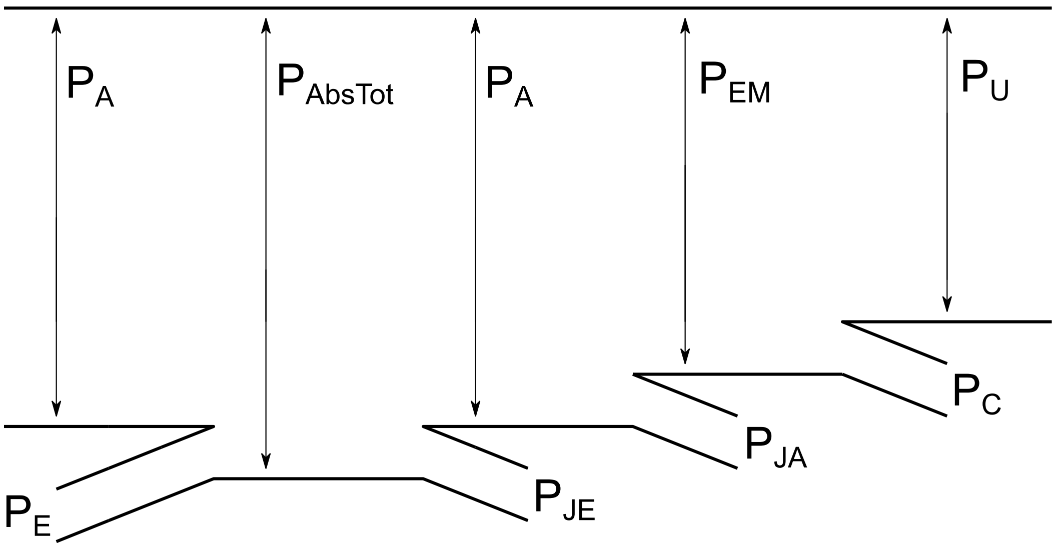


- Inductor Joule losses:

$$\mathbf{P_{JE} = U_E . I_E}$$

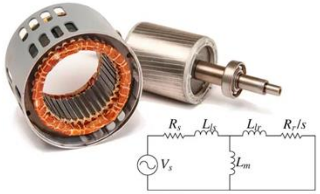
- Armature Joule losses:

$$\mathbf{P_{JA} = R_A . I_A}^2$$

- Useful electrical power / electromagnetic power: $\mathbf{P_{EM} = E . I_A = U_A . I_A - P_{JA}}$

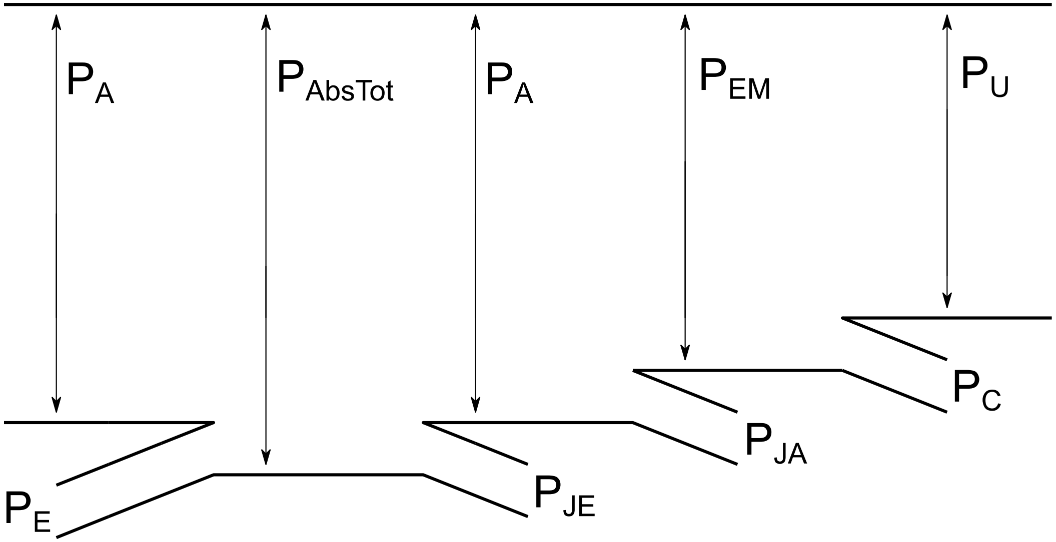
- Constant losses: iron losses + mechanical losses: $\mathbf{P_C = P_f + P_m = P_0 - R_A . I_A}^2 \mathbf{= C_P . \Omega}$

=> Can be represented by a loss torque (Cp)



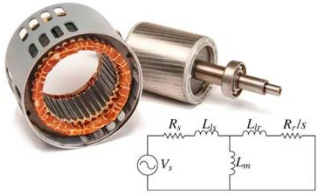
The DC motor

- Characteristic equations:



- Useful power: $\mathbf{P_U = U_A . I_A - P_{JA} - P_C = C_U . \Omega}$

- Efficiency: $\boldsymbol{\eta = \frac{P_U}{P_{AbsTot}}}$



The DC motor

- Expressions of torques:

- Electromagnetic torque:

$$\left.\begin{aligned}
\boldsymbol{C_{EM}} &= \frac{\boldsymbol{E.I_A}}{\boldsymbol{\Omega}} = \frac{\boldsymbol{E.I_A}}{2.\boldsymbol{\pi.n}} \\
\boldsymbol{C_{EM}} &= \frac{1}{2.\boldsymbol{\pi}}.\frac{\boldsymbol{p}}{\boldsymbol{a}}.\boldsymbol{N.\Phi.I_A} = \boldsymbol{k.\Phi.I_A}
\end{aligned}\right\} \quad \boldsymbol{C_{EM} = k_C.I_A}$$

=> If the flux is constant, C_{EM} is proportional to I_A (k_C: torque constant, in $N.m.A^{-1}$ (manufacturer's data))

- Loss torque:

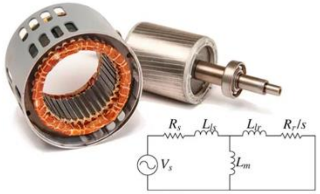
$$\boldsymbol{C_P} = \frac{\boldsymbol{P_0 - R_A.I_{A0}}^2}{2.\boldsymbol{\pi.n}}$$

=> Due to iron losses in the rotor (hysteresis + eddy currents) and to mechanical losses (bearing friction, brush friction on the collector). It can be estimated from a no-load test

- Useful torque:

$$\boldsymbol{C_U = C_{EM} - C_P} = \frac{\boldsymbol{P_U}}{2.\boldsymbol{\pi.n}}$$

=> Also called driving torque



The DC motor:summary

- Mechanical equations:

- Fundamental principle of dynamics applied to a rotating solid: $$J\frac{d\Omega}{dt} = C_M - C_R$$

=> C_M is the driving torque, C_R the resistant torque and J the moment of inertia

- With the electromagnetic torque: $$\boldsymbol{C_{EM} = k.\Phi.I_A}$$

- Electrical equations:

- Counter electromotive force: $$\boldsymbol{E = k.\Phi.\Omega}$$

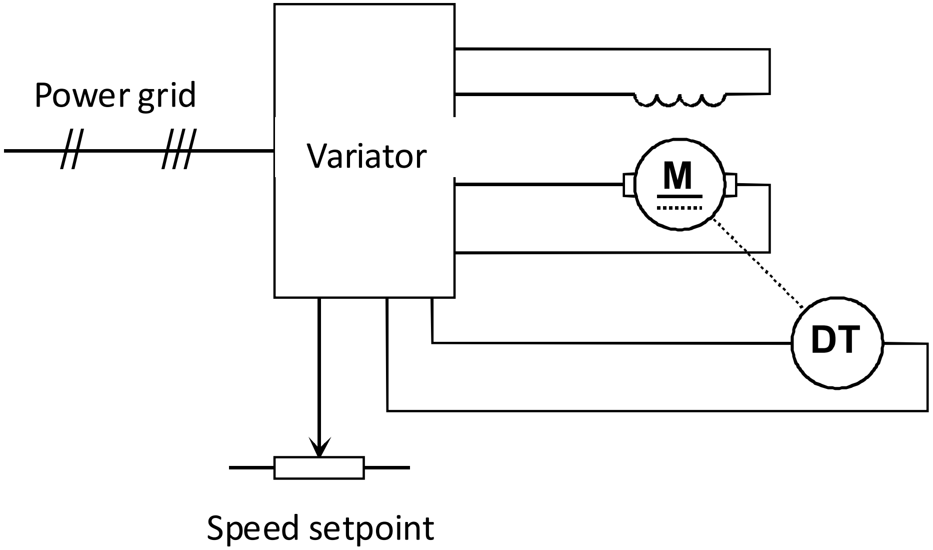
- Voltage at the terminals of the armature: $$\boldsymbol{U - E = R.i_A + L.\frac{di_A}{dt}} \qquad U = E + R.I_A$$

The DC motor

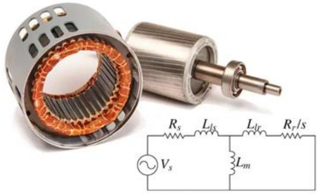
- Speed variation:

- Industrial variable speed drive consist of an all-thyristor, single-phase or three-phase, head-to-tail double bridge supplying the armature, enabling operation in all four quadrants, and a mixed bridge for the inductor.

=> Speed varied by playing on the armature voltage



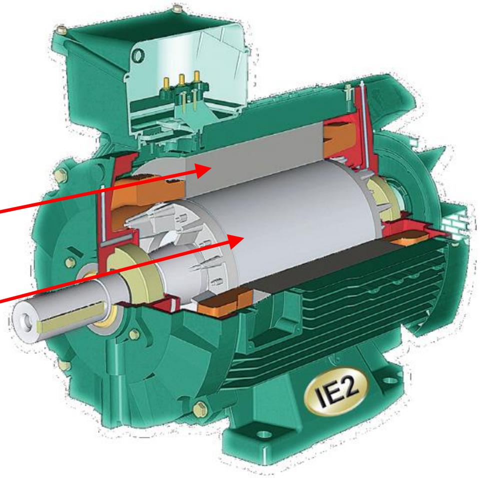
- Small permanent-magnet motor use a switch-mode power supply.



3-phase motors

- Construction:

3-phase
STATOR
+
ROTOR

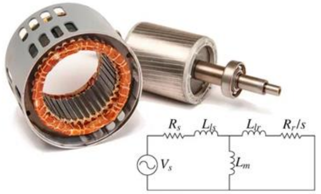


DC rotor

=> Synchronous machine

3-phase rotor

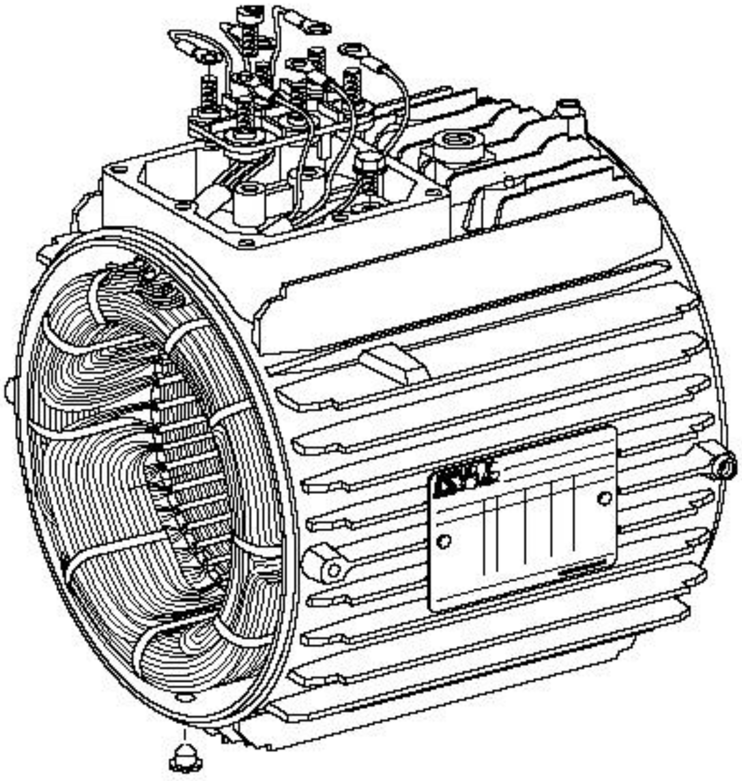
=> Induction/Asynchronous machine



3-phase motors

- The 3-phase stator:

- The stator is the same for both the synchronous and the asynchronous machines

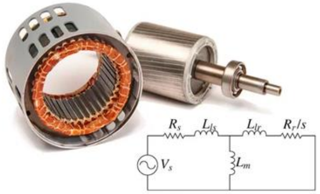


- Consists of an aluminum frame onto which a crown of notched steel sheets is fixed.

- Windings of appropriate cross-sections are distributed in these slots, forming a winding assembly with as many circuits as supply phases (3).

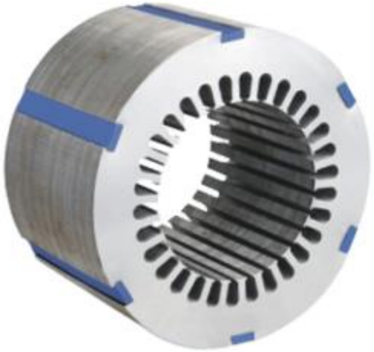
⇒ Spatial distribution of the sinusoidal field

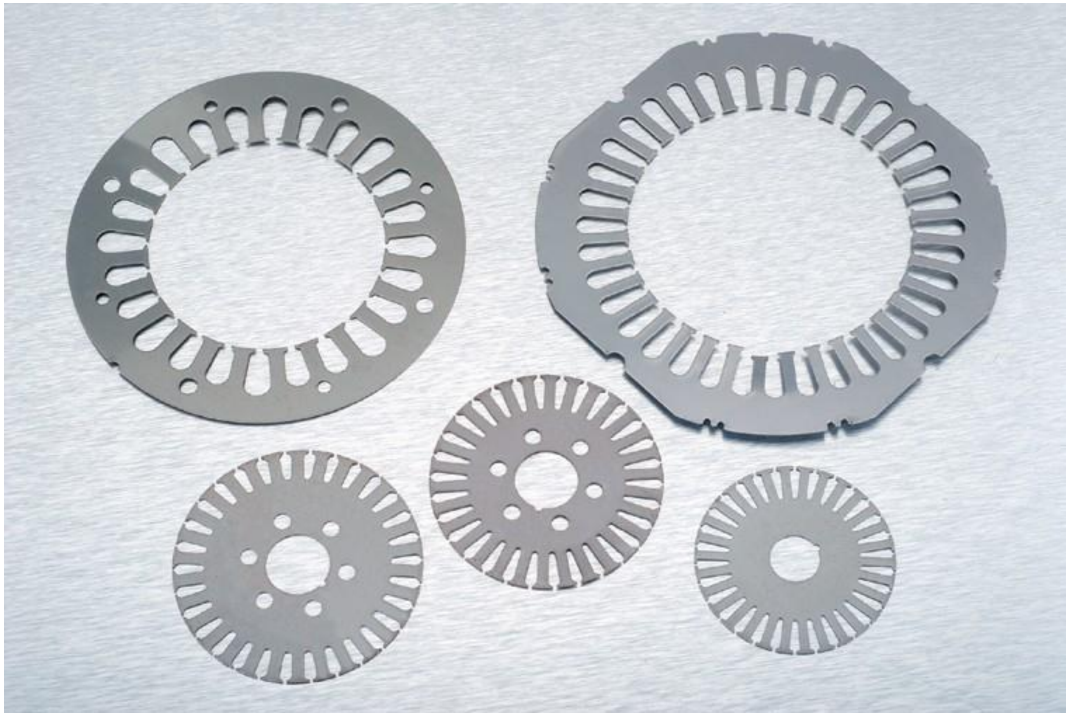
⇒ Creation of a rotating magnetic field



3-phase motors

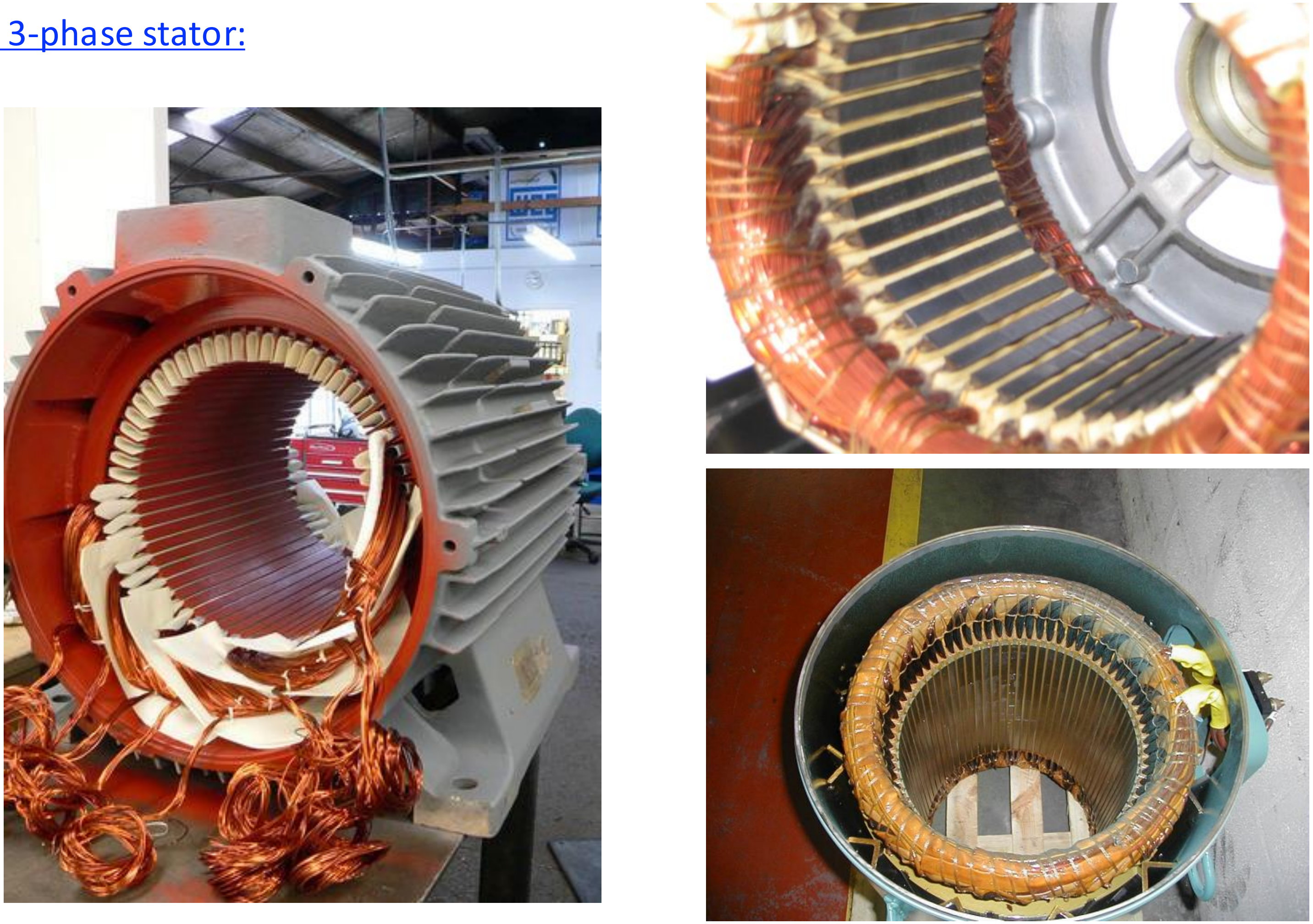
- The 3-phase stator:

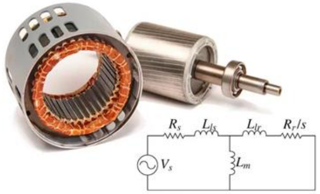




3-phase motors

- The 3-phase stator:





3-phase motors

- Stator: field distribution

- Field created in an air gap by one or several turns

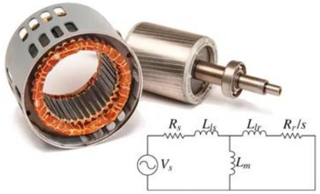
=> N turns winding arranged in slots on the surface of an air gap, without leakage, through which flows a direct current I

=> The iron frame channels the field lines. In the air gap, there is a B field that is assumed to be radial.

=> Conventions: North pole axis = angular abscissa

=> Conventions: Field counted positively when oriented towards the outside of the machine (north pole) and negatively towards the inside (south pole)

=> Conventions: Unsaturated magnetic material

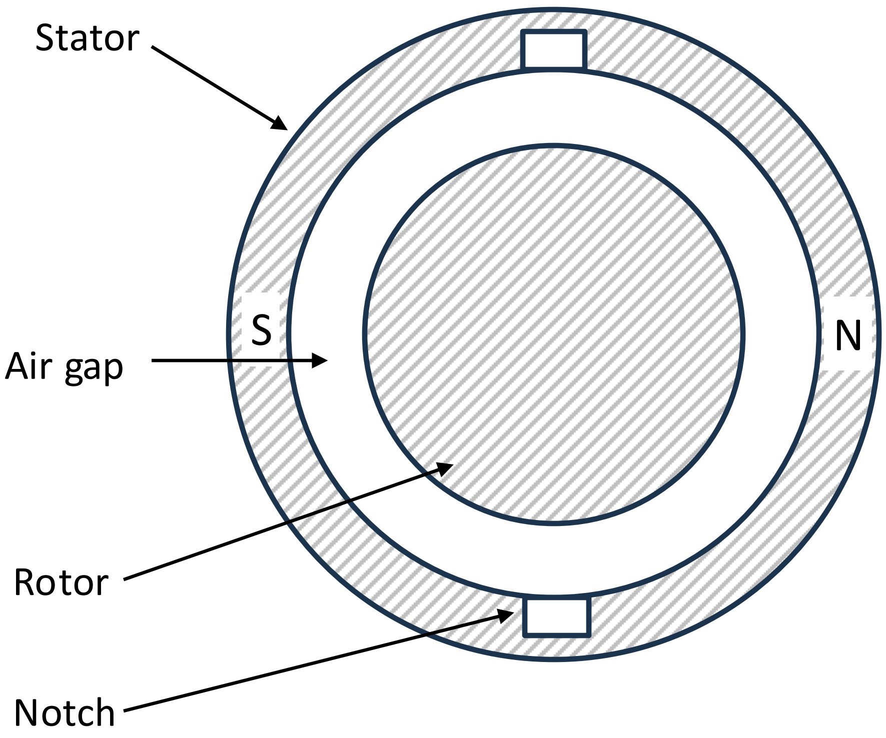


IV - Motors

3-phase motors

- Stator: field distribution

- Field created in an air gap by one or several turns: case of the bipolar stator + 2 notches



- Ampere's theorem:

$$H_{iron} l_{iron} + H_{air} l_{air} = \sum i_{interlaced}$$

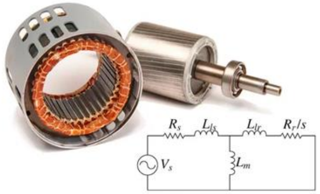
- Assumption of perfect magnetic material

=> $\mu_r \rightarrow \infty$ => $H_{iron} \rightarrow 0$

- Induction magnetic field in the airgap:

$$B_{air} = \mu_0 \frac{ni}{2e}$$

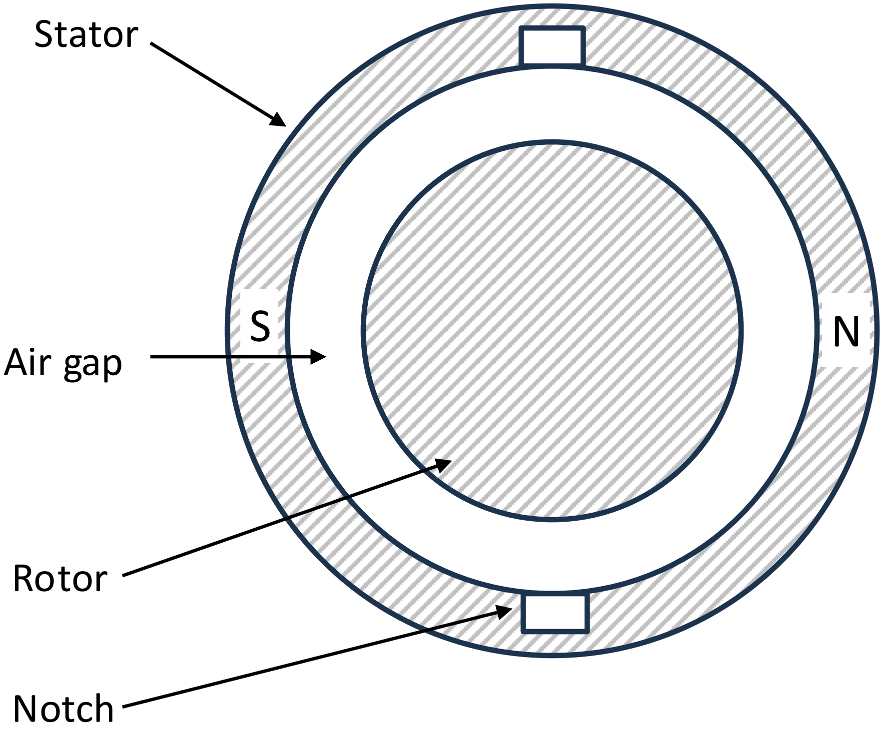
n number of turns, e airgap width

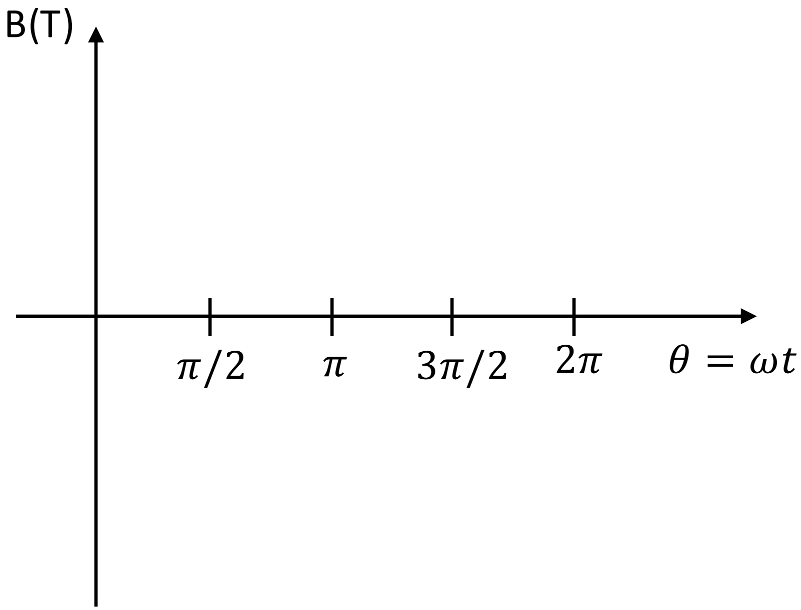


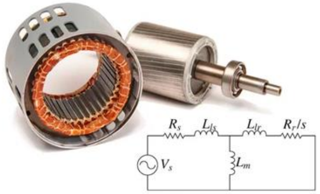
3-phase motors

- Stator: field distribution

- Field created in an air gap by one or several turns: **case of the bipolar stator + 2 notches**





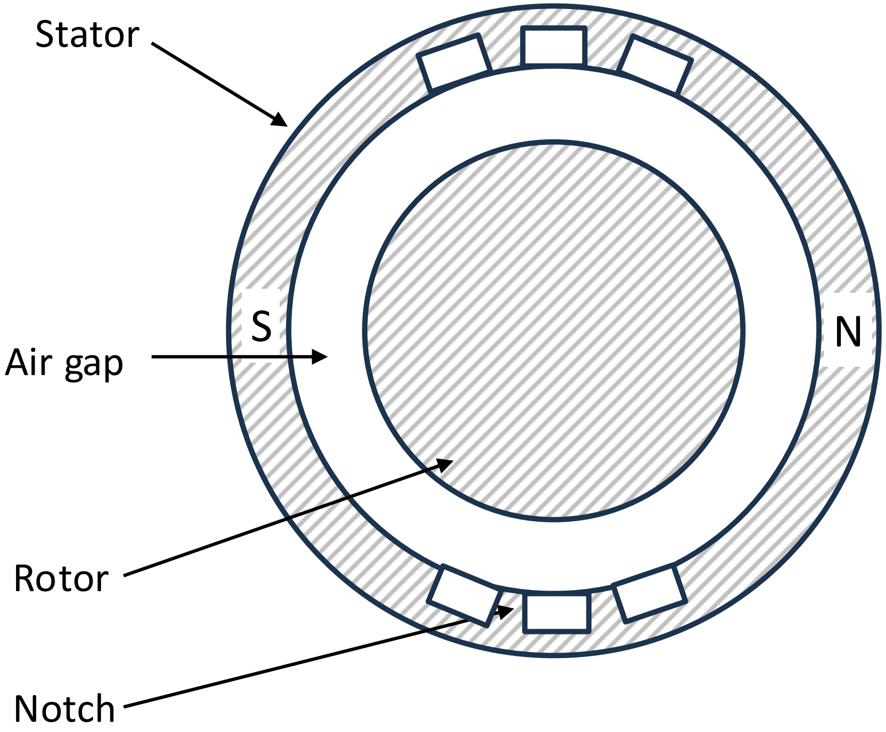


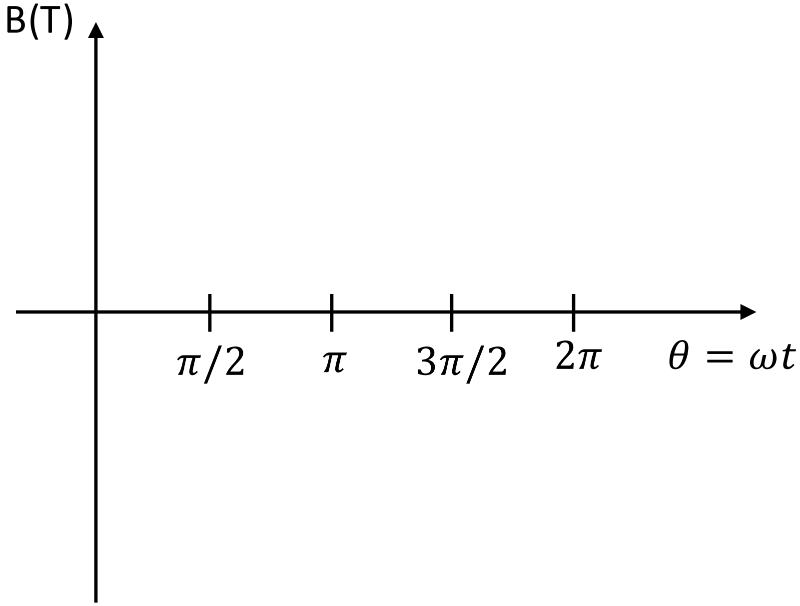
3-phase motors

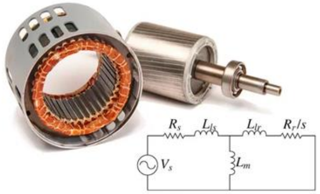
- Stator: field distribution

- Field created in an air gap by one or several turns: **case of the bipolar stator + 3 notches pairs**

$$B_{air\ 1} = \mu_0 \frac{ni}{2e} \qquad B_{air\ 2} = \mu_0 \frac{ni}{6e}$$



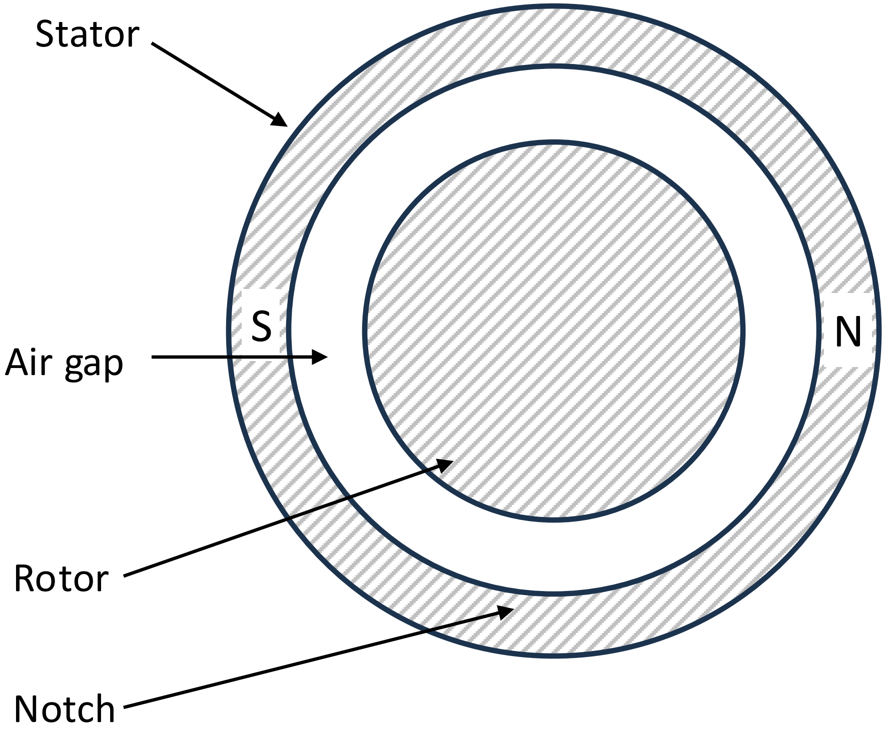


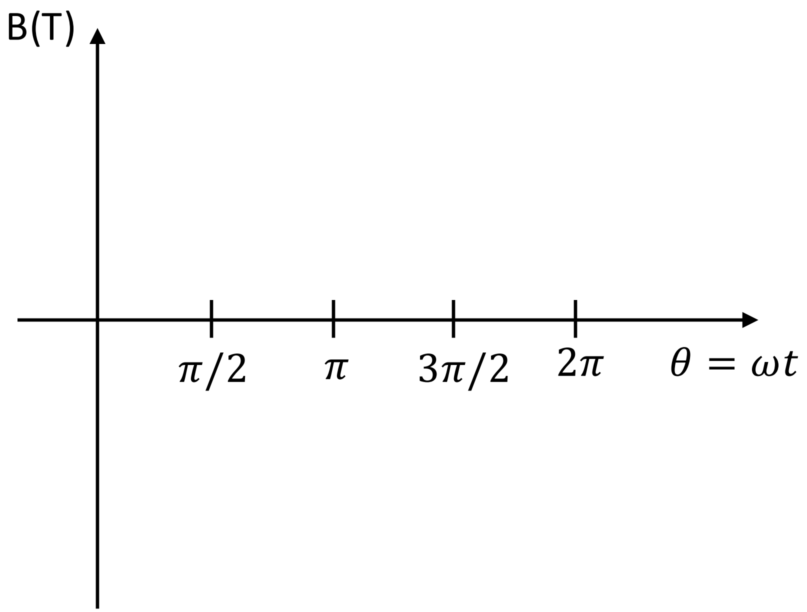


3-phase motors

- Stator: field distribution

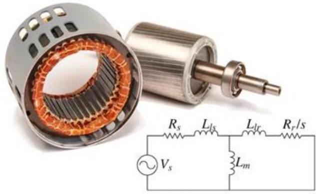
- Field created in an air gap by one or several turns: **bipolar stator** + **notches all around the stator**





$$B(\theta) = \mu_0 \frac{ni}{2e} cos\theta = \hat{B} cos\theta$$

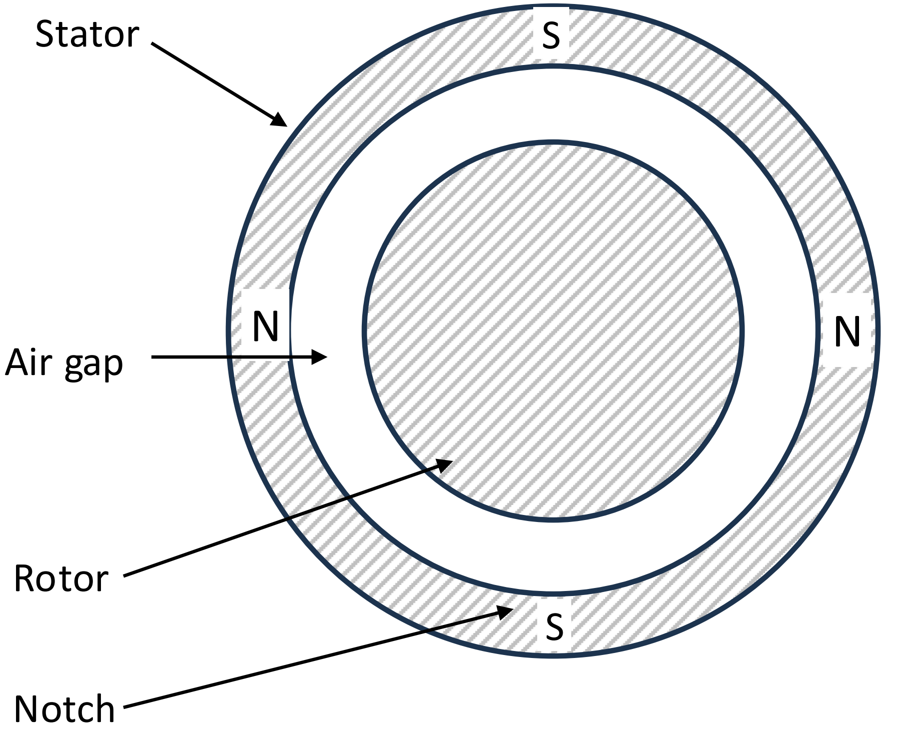
=> B field with sinusoidal spatial distribution

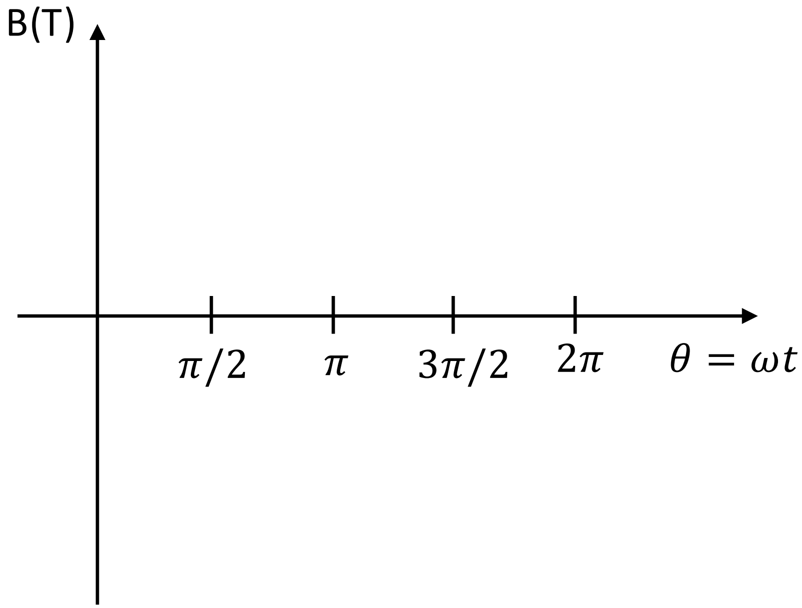


3-phase motors

- Stator: field distribution

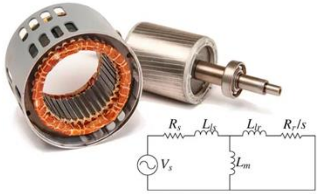
- Field created in an air gap by one or several turns: **multipolar stator (p pole pairs)**





$$B(\theta) = \mu_0 \frac{ni}{2e} cosp\theta = \hat{B} cosp\theta$$

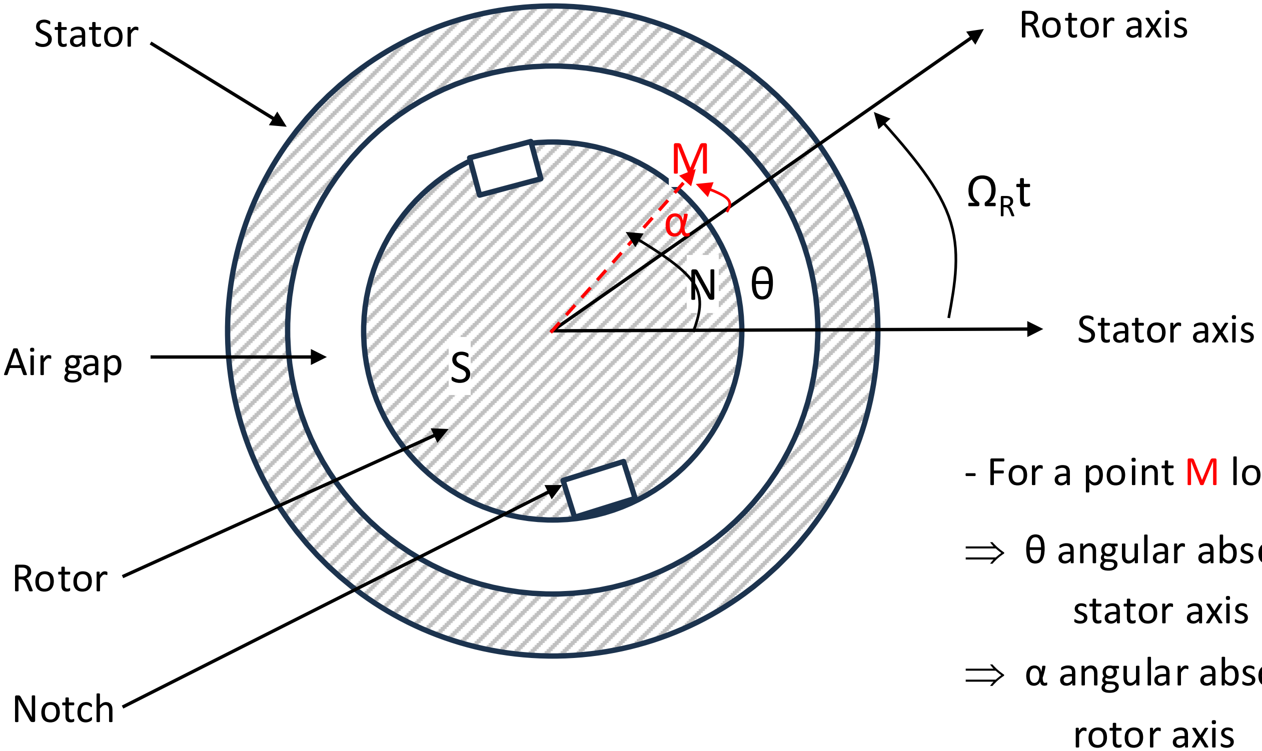
=> Static B field distribution (no rotation)



3-phase motors

- Stator: field distribution

- Rotating B field created by a rotor in which a DC current flows: **bipolar rotor**



$\Omega_R t$ is the rotor rotation speed

- For a point M located in the air gap:

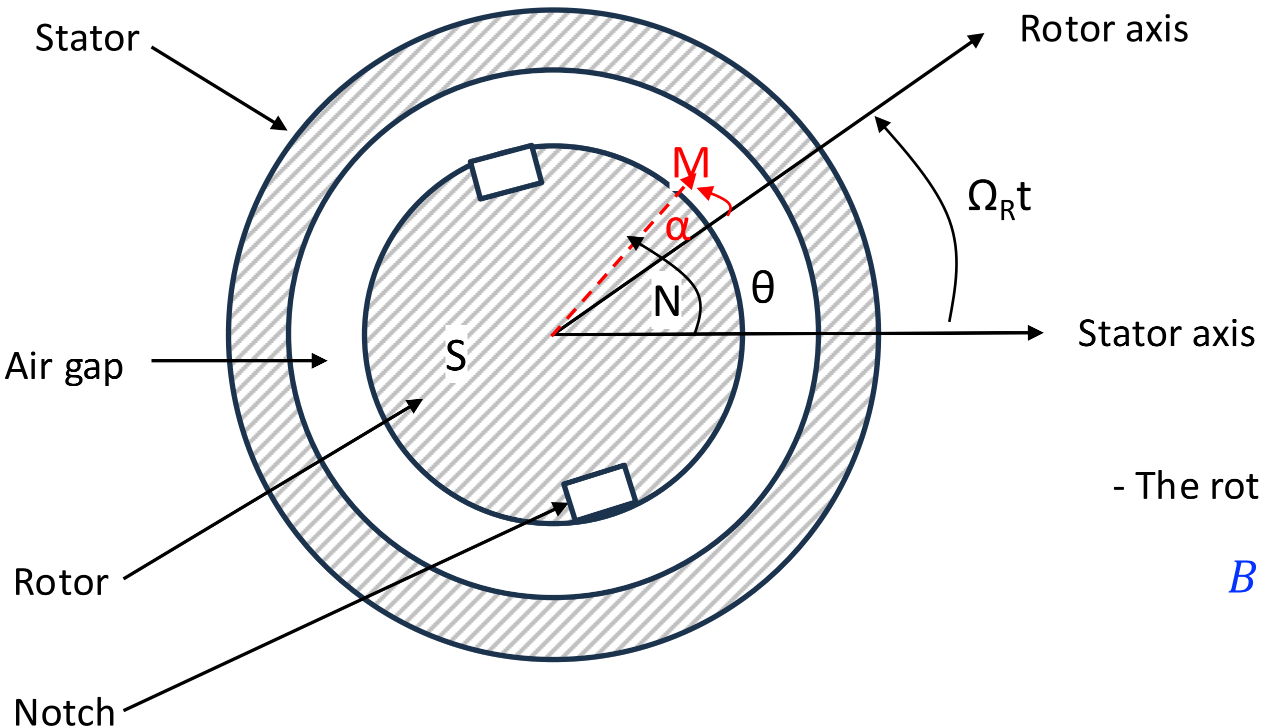
⇒ θ angular abscissa with respect to the fixed stator axis

⇒ α angular abscissa with respect to the rotor axis

3-phase motors

- Stator: field distribution

- Rotating B field created by a rotor in which a DC current flows: **bipolar rotor**



M sees the B field rotating at $\Omega_R=\omega$

$$\theta = \omega t + \alpha$$

- The rotor winding creates at point M:

$$B(\theta, t) = \hat{B}\cos(\theta - \omega t)$$

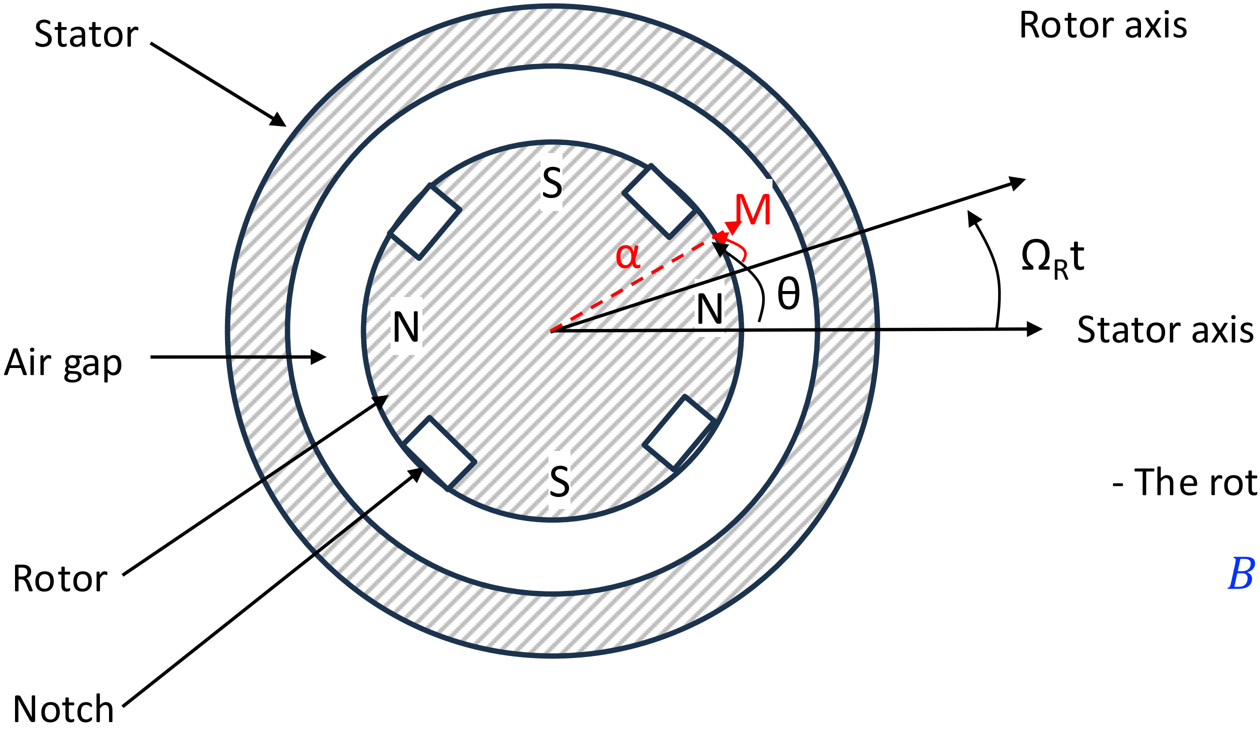
=> At a fixed point, it can be observed a B(t) field varying sinusoidally with time

=> At a fixed point and time t, a static B(θ) distribution is observed => rotating distribution field

3-phase motors

- Stator: field distribution

- Rotating B field created by a rotor in which a DC current flows: **multipolar rotor**



M sees the B field rotating at

$\Omega_R = \omega/p$

$$\theta = \frac{\omega}{p} t + \alpha$$

- The rotor winding creates at point M:

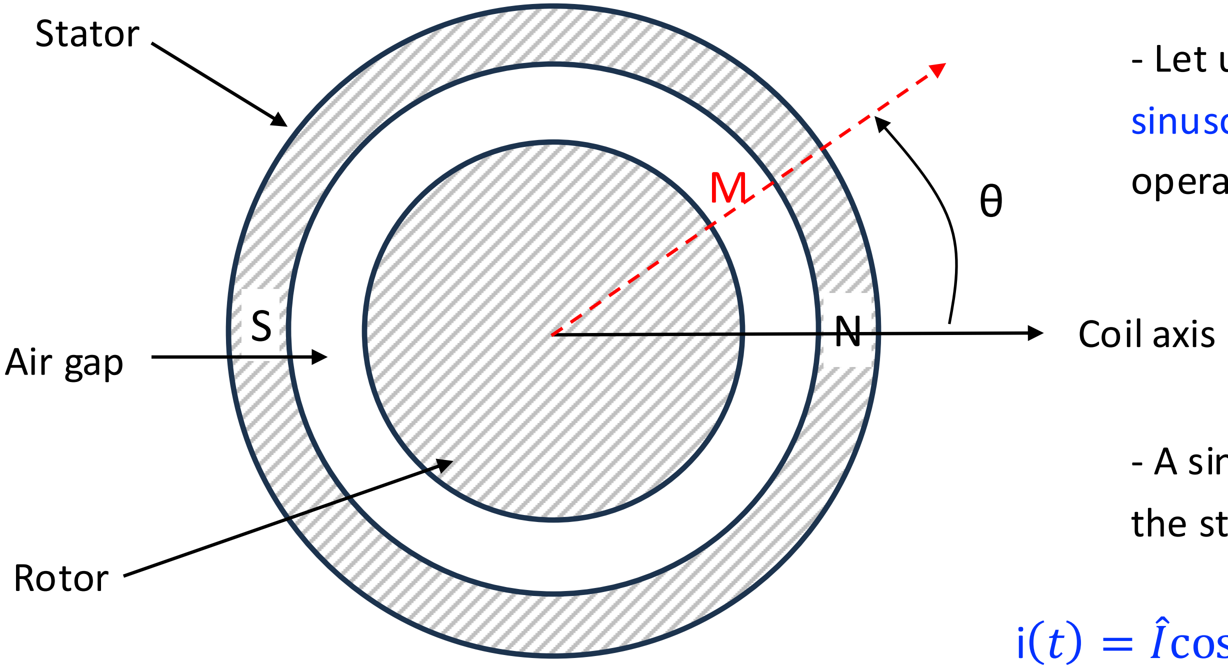
$$B(\theta, t) = \hat{B}\cos(p\theta - \omega t)$$

=> Sinusoidal distribution of rotating B field

3-phase motors

- Stator: field distribution

- Rotating B fields created by a **single-phase stator** in which an AC current flows: **bipolar stator**

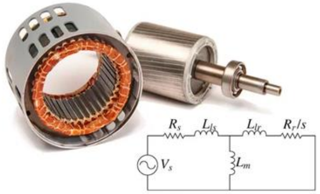


- Let us consider a stator with sinusoidal spatial distribution operating in unsaturated mode regime

- A sinusoidal current flows through the stator winding

$$i(t) = \hat{I}\cos(\omega t)$$

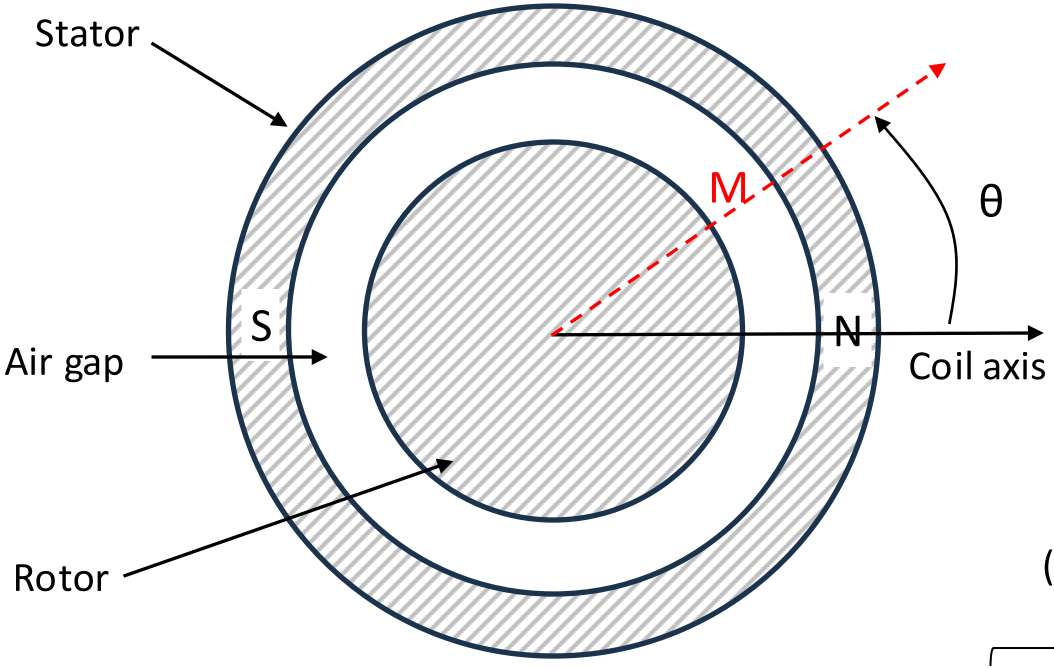
$$B(\theta, t) = k.i(t)\cos(\theta)$$



3-phase motors

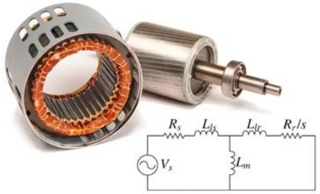
- Stator: field distribution

- Rotating B fields created by a **single-phase stator** in which an AC current flows: **bipolar stator**



- The winding creates 2 rotating fields of same amplitude, with sinusoidal spatial distribution, rotating in opposite directions at the same speed.

$$B(\theta,t)=\overbrace{\frac{1}{2}k\hat{I}\cos(\theta-\omega t)}^{B_d\ \text{(direct field)}}+\overbrace{\frac{1}{2}k\hat{I}\cos(\theta+\omega t)}^{B_i\ \text{(indirect field)}}$$



3-phase motors

- Stator: field distribution

- Rotating B fields created by a **single-phase stator** in which an AC current flows: **multipolar stator**

=> Leblanc's theorem

- A fixed, single-phase, p-polar stator with sinusoidal spatial distribution, through which flows a sinusoidal current of pulsation ω leads to two rotating fields (direct and inverse):

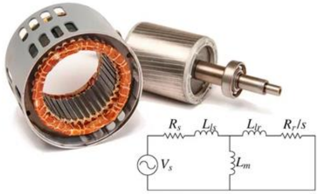
=> Of same amplitude, with sinusoidal spatial distribution

=> Turning in opposite directions to each other at rotation speed $\Omega_S = \dfrac{\omega}{p}$

=> whose axes coincide with the winding axis at maximum current

$$B(\theta,t) = k\hat{I}\cos(\omega t)\cos(\theta)$$

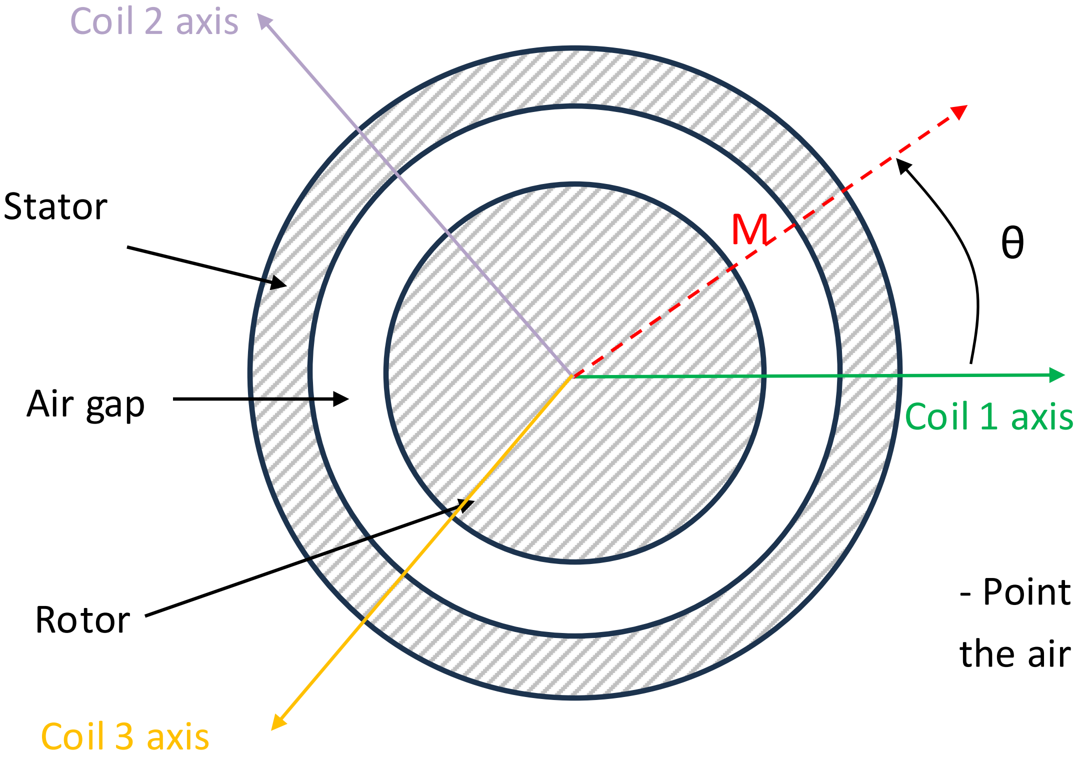
$$B(\theta,t) = \frac{1}{2}k\hat{I}\cos(p\theta - \omega t) + \frac{1}{2}k\hat{I}\cos(p\theta + \omega t)$$



3-phase motors

- Stator: field distribution

- Rotating B fields created by a **3-phase stator** in which AC currents flow: **bipolar stator**

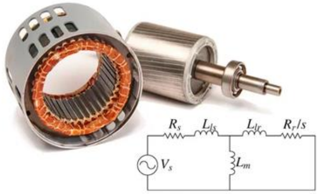


Construction:

- 3-phase winding
- Each coil creates a sinusoidal spatial distribution
- The coil axes are shifted by an angle of 2π/3.

- Point M located in the air gap at angles:

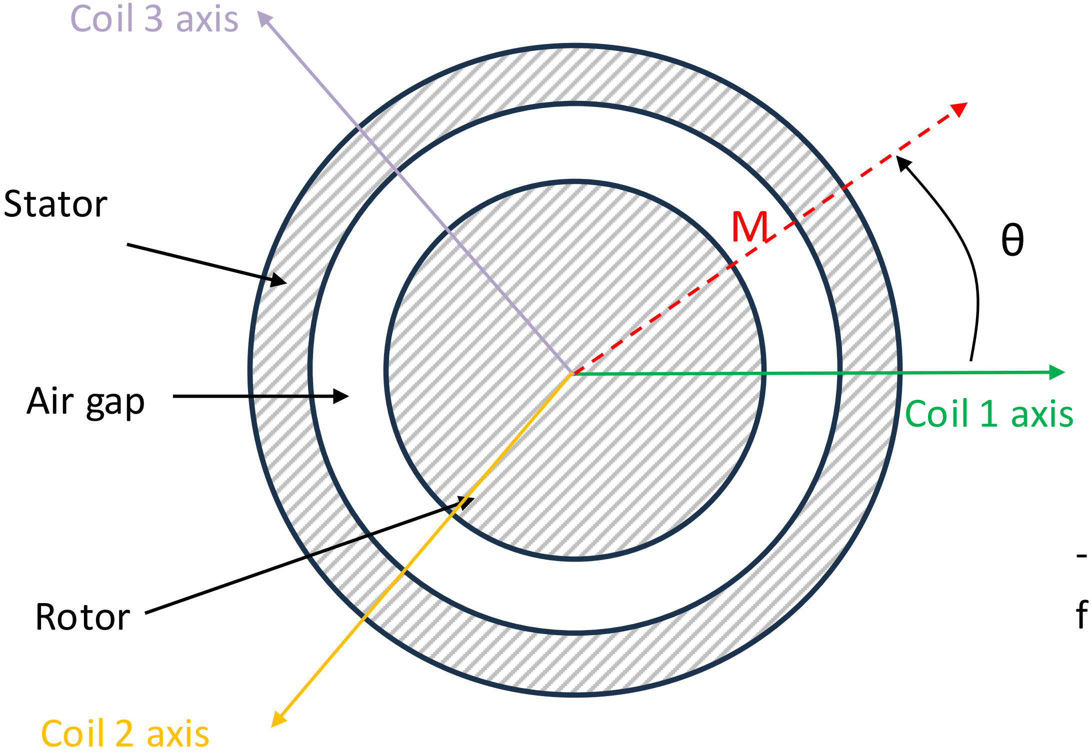
$$\begin{cases} \theta_1 = \theta \\ \theta_2 = \theta + \dfrac{4\pi}{3} \\ \theta_3 = \theta + \dfrac{2\pi}{3} \end{cases}$$



3-phase motors

- Stator: field distribution

- Rotating B fields created by a **3-phase stator** in which AC currents flow: **bipolar stator**



- 3-phase current systems at the windings:

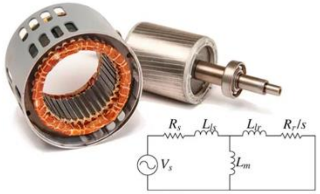
$$i_1(t) = \hat{I}\cos(\omega t)$$

$$i_2(t) = \hat{I}\cos(\omega t - \frac{2\pi}{3})$$

$$i_3(t) = \hat{I}\cos(\omega t - \frac{4\pi}{3})$$

- The resulting B-field is the sum of the fields generated by each winding:

$$B(\theta, t) = \frac{3}{2}k\hat{I}\cos(\omega t - \theta)$$



3-phase motors

- Stator: field distribution

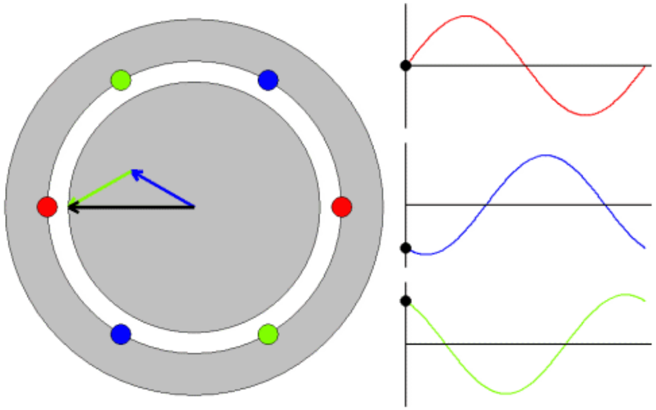
- Rotating B fields created by a **3-phase stator** in which AC currents flow: **multipolar stator**

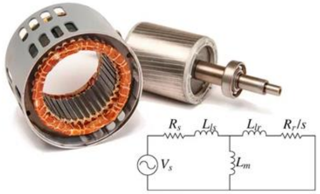
=> Ferraris's theorem

- A fixed, 3-phase, p-polar stator with sinusoidal spatial distribution, through which flows a sinusoidal current of pulsation ω leads to one rotating field (direct and inverse):

=> rotation speed $\Omega_S = \dfrac{\omega}{p}$

$$B(\theta, t) = \frac{3}{2} k \hat{I} \cos(\omega t - p\theta)$$

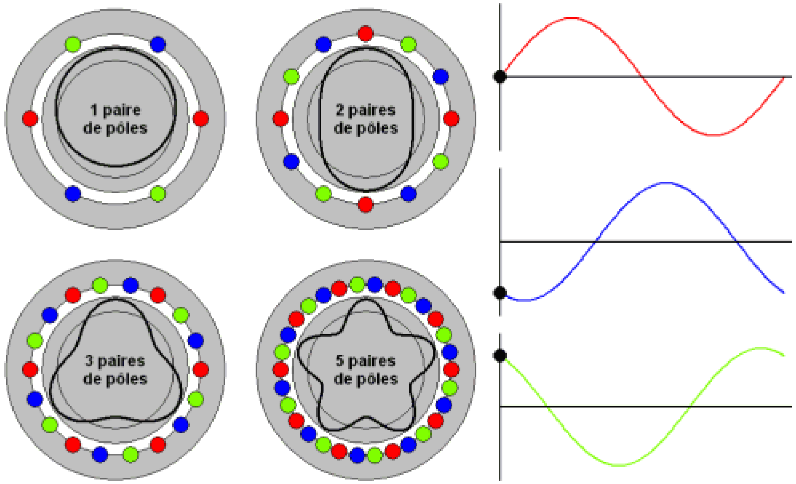




3-phase motors

- Stator: rotating field speed

- Influence of the number of pole pairs (p)



Number of pole pairs	f = 50Hz	f = 60Hz
1	3000 tr/min	3600 tr/min
2	1500 tr/min	1800 tr/min
3	1000 tr/min	1200 tr/min
4	750 tr/min	900 tr/min

NB: Rotation of fields distribution

3-phase motors

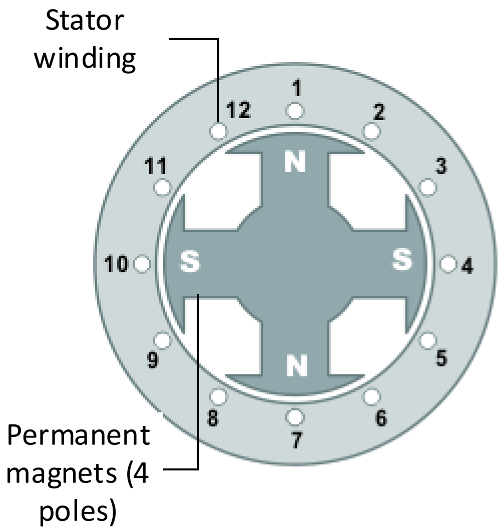
- Synchronous machine: =>limited to the study of the synchronous alternator

- Construction:

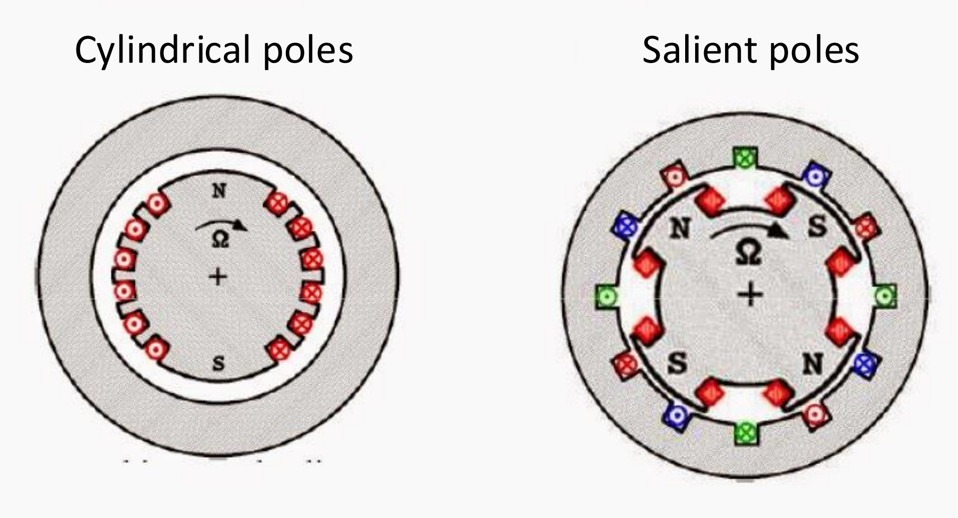
=> 3-phase stator generating a rotating magnetic field (see before)

=> DC rotor : different configurations

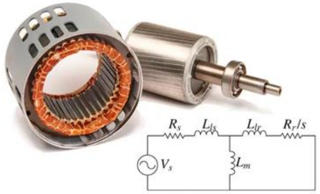
Permanent magnet rotor



Wound rotor (DC current)



Synchronous machine: The rotor rotates at the same speed as that of the rotation magnetic field



3-phase motors

- Synchronous machine:

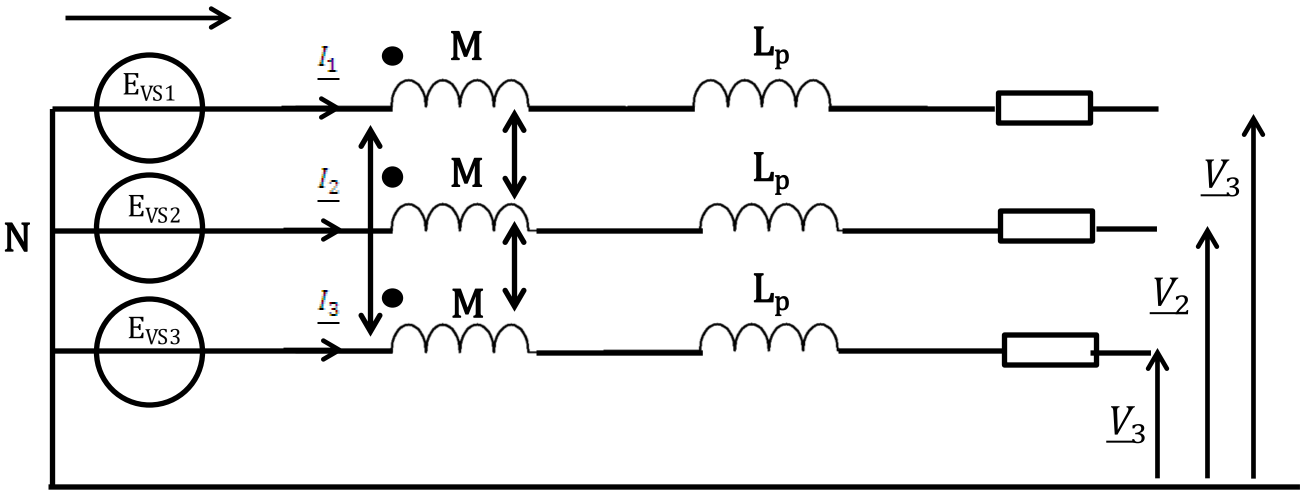
- Equivalent circuit in the linear domain:

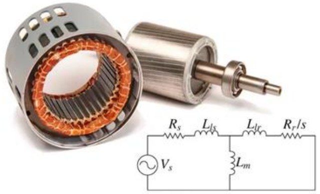
=> The armature circuit (stator for the alternator) can be represented, for each phase, by the series connection of a no-load emf, a resistor (winding), a self-inductance and a mutual inductance with the two other phases.

=> With the no-load electromotive force:

$$E_v = K\Omega_S I_e$$

- Ω_S: rotation speed (rad/s)
- I_e: rotor excitation current

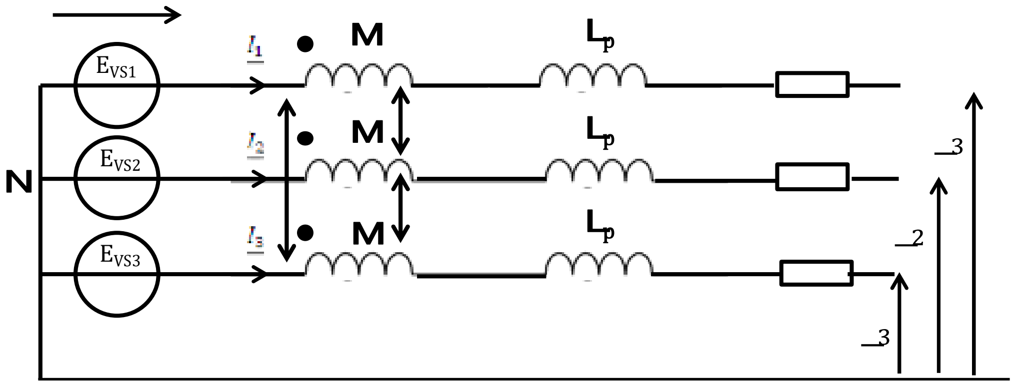




3-phase motors

- Synchronous machine:

- Equivalent circuit in the linear domain:

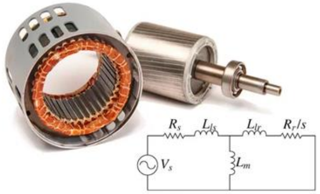


- With:

$$\underline{I_1} + \underline{I_2} + \underline{I_3} = 0$$

$$\begin{cases} \underline{E_{v1}} = j.M.\omega.\underline{I_2} + j.M.\omega.\underline{I_3} + j.L_p.\omega.\underline{I_1} + R.\underline{I_1} + \underline{V_1} \\ \underline{E_{v2}} = j.M.\omega.\underline{I_1} + j.M.\omega.\underline{I_3} + j.L_p.\omega.\underline{I_2} + R.\underline{I_2} + \underline{V_2} \\ \underline{E_{v3}} = j.M.\omega.\underline{I_1} + j.M.\omega.\underline{I_2} + j.L_p.\omega.\underline{I_3} + R.\underline{I_3} + \underline{V_3} \end{cases} \Rightarrow \begin{cases} \underline{E_{v1}} = j.\left(L_p - M\right).\omega.\underline{I_1} + R.\underline{I_1} + \underline{V_1} \\ \underline{E_{v2}} = j.\left(L_p - M\right).\omega.\underline{I_2} + R.\underline{I_2} + \underline{V_2} \\ \underline{E_{v3}} = j.\left(L_p - M\right).\omega.\underline{I_3} + R.\underline{I_3} + \underline{V_3} \end{cases}$$

With $L_s = L_p - M$ the **cyclic inductance**, each phase can be decoupled

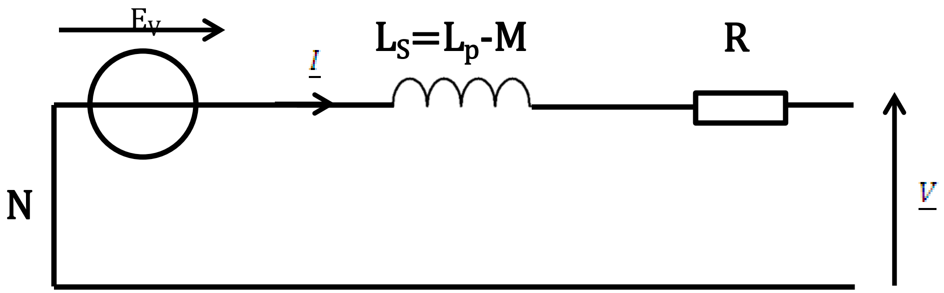


3-phase motors

- Synchronous machine:

- Each phase can be modelled with an equivalent single-phase circuit

=> Behn-Eschenburg's model



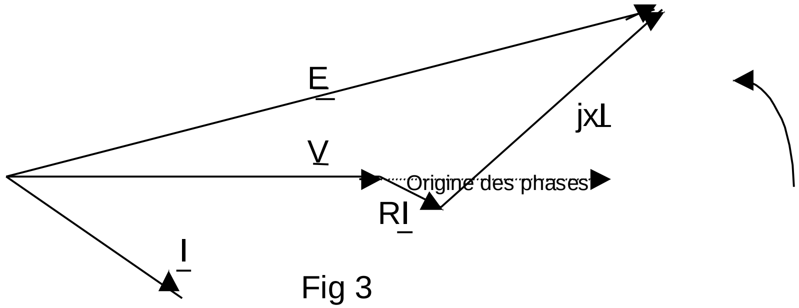
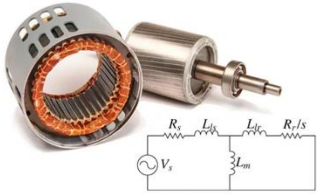


Fig 3

$$\underline{V} = \underline{E_v} - jL_s.\omega.\underline{I} - R.\underline{I}$$

- L_S: synchronous inductance
- X_S: synchronous reactance
- R: winding resistance
- E_v: no-load electromotive force



3-phase motors

- Synchronous machine:

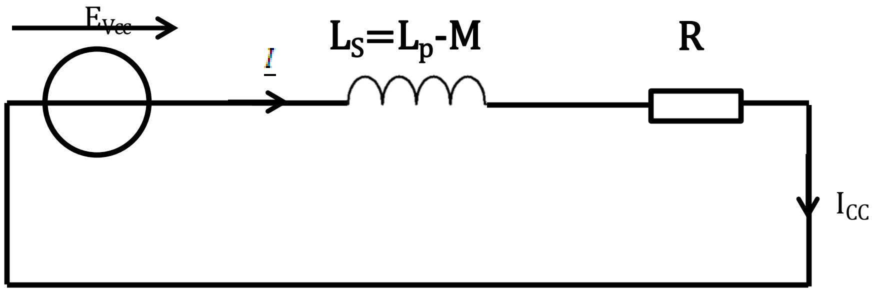
- Determining the elements of the equivalent circuit diagram

=> E_v is measured as a function of the excitation current I_e - $E_v(I_e)$: No-load characteristic of the SM

=> No-load test = the SM is driven at nominal speed by an auxiliary motor (e. g. a DC motor)

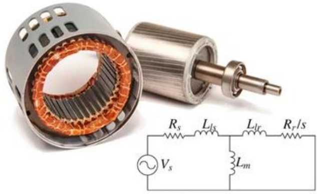
=> R is directly calculated from imposed current and voltage

=> X_s is obtained from a short-circuit test performed at nominal speed with reduced excitation



$$\underline{E_{vcc}} = jL_s.\omega.\underline{I_{cc}} - R.\underline{I_{cc}}$$

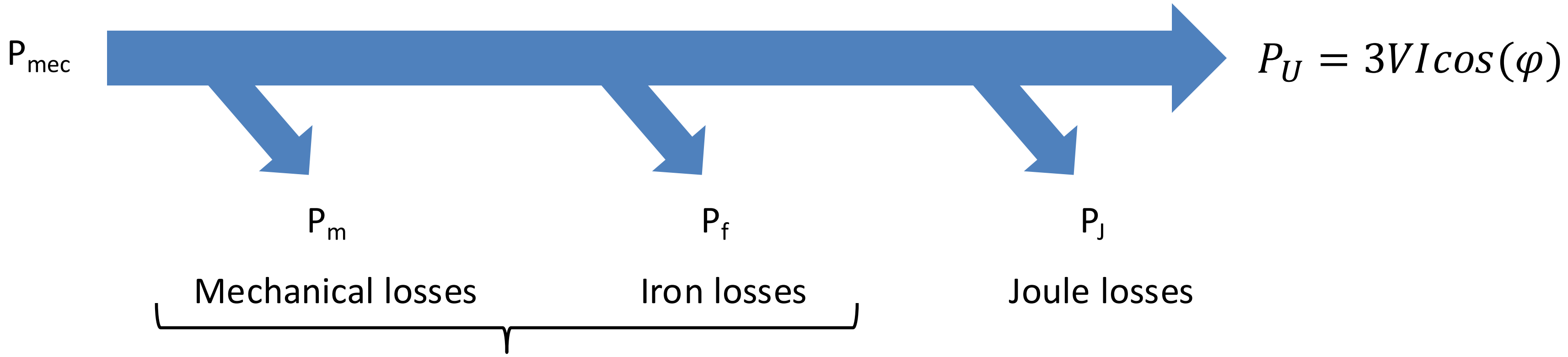
$$X_S = L_S\omega = \sqrt{\left(\frac{\underline{E_{vcc}}}{\underline{I_{cc}}}\right)^2 - R^2}$$



3-phase motors

- Synchronous machine:

- Power balance



P_{mec}

$P_U = 3VIcos(\varphi)$

P_m Mechanical losses

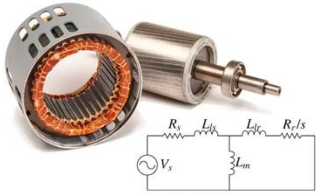
P_f Iron losses

P_J Joule losses

Considered constant since they only depend on the rotation speed et the supply voltage

$P_J = 3RI^2$

- Therefore, the efficiency: $\eta = \dfrac{P_U}{P_U + P_m + P_f + P_J}$



3-phase motors

- Induction motor/ Asynchronous motor:

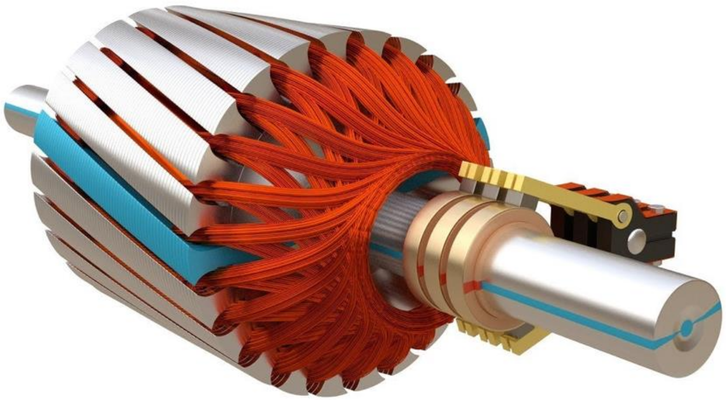
- Construction:

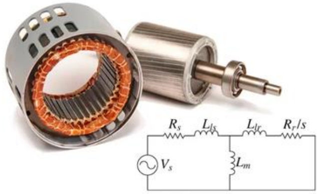
=> 3-phase stator generating a rotating magnetic field (see before)

=> 3-phase rotor : different configurations

Squirrel cage rotor

3-phase wound rotor





3-phase motors

- Induction motor/ Asynchronous motor: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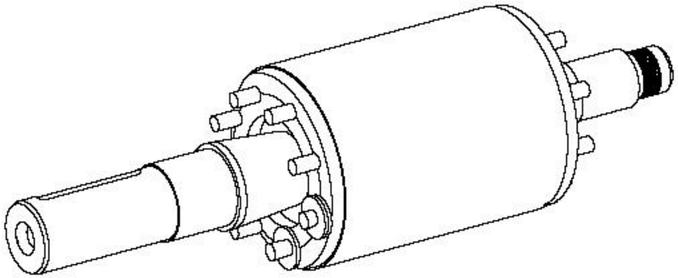
- **Squirrel cage rotor**:

=> Aluminum conductors or bars are placed in the cylinder slots and short-circuited at each end

- **3-phase wound rotor** or **Slip-ring rotor**:

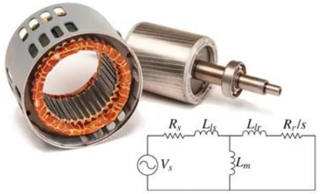


⇒ Windings identical to those of the stator are housed in the slots on the periphery of the rotor

⇒ The rotor is three-phase and wye-coupled (short-circuit)

⇒ The ends of the windings are connected to three copper rings, insulated and fixed to the rotor

Short-cut necessary at the rotor to generate induced currents



3-phase motors

- Induction motor/ Asynchronous motor:

- Working principle:

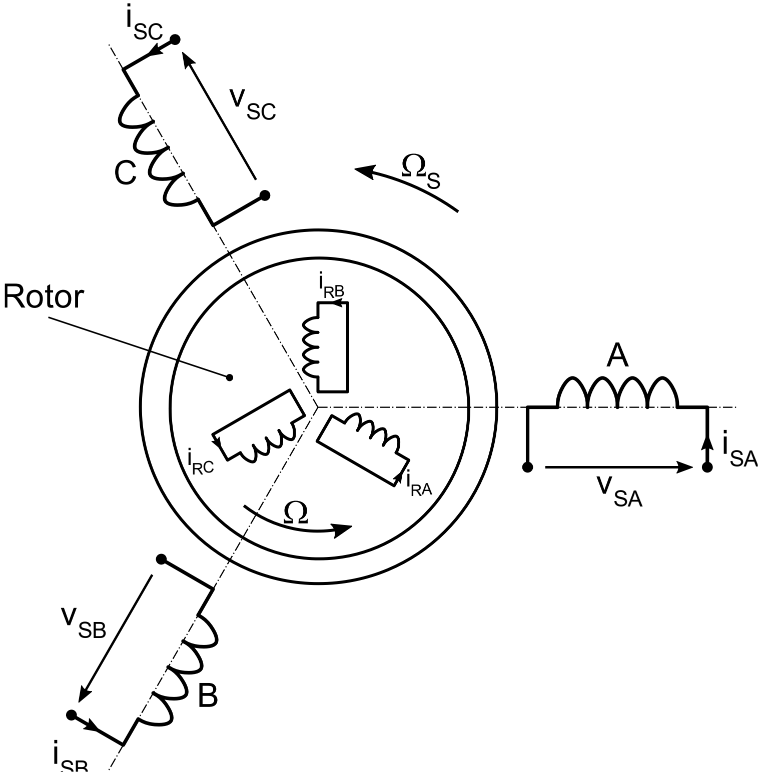
=> The 3-phase stator generates a rotating magnetic field

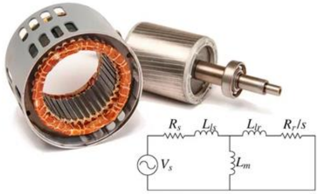
=> The rotating field induces eddy currents in the rotor windings

=> According to Lenz's law, these currents oppose the cause that gave rise to them and generate a magneto-motive force that make the rotor rotate

=> Therefore, the rotor moves with the stator field at speed Ω_R tending towards Ω_S, but never reaches it

$$\Omega_R < \Omega_S$$





3-phase motors

- Induction motor/ Asynchronous motor:

- The slip:

=> The rotor slip with respect to the stator field is defined as the relative deviation of the rotor rotation speed from synchronous speed

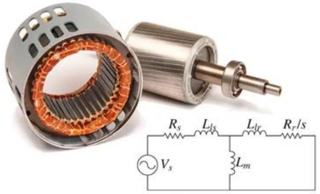
$$g = \frac{\Omega_S - \Omega_R}{\Omega_S}$$

=> At synchronous speed: $\Omega_R = \Omega_S, g = 0$

=> O induced current => deceleration of rotor

=> When stopped, and at starting-up: $\Omega_R = 0, g = 1$

=> Generator operation: $\Omega_R > \Omega_S, g < 0$

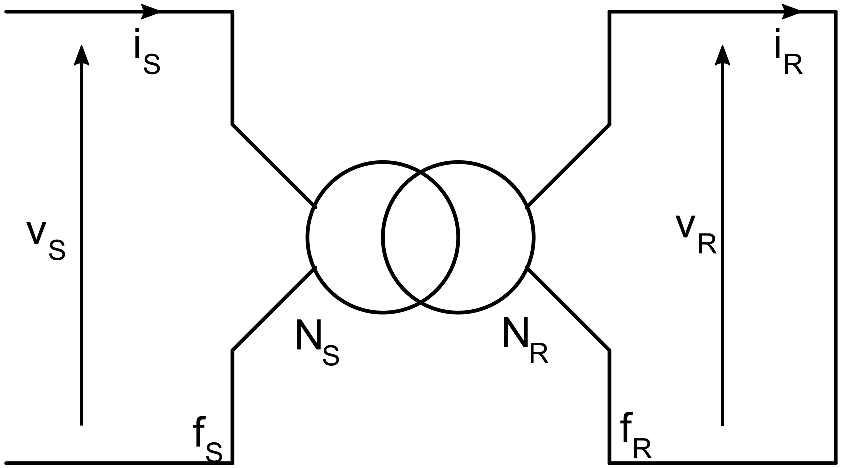


3-phase motors

- Induction motor/ Asynchronous motor:
- Equivalent circuit diagram of the induction motor

=> Ideal model: the perfect Asynchronous motor

=> Very similar to the transformer with short-circuited secondary side



V_S: RMS voltage across the stator winding

I_S: RMS current flowing in the stator winding

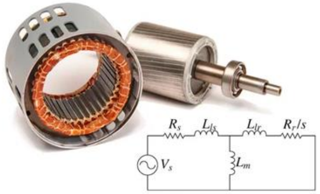
V_R: RMS voltage across the rotor winding

I_R: RMS current flowing in the rotor winding

f_S : frequency of stator or grid

f_R : frequency of rotor currents

m: number-of-turns ratio between rotor and stator: $m=N_R/N_S$



3-phase motors

- Induction motor/ Asynchronous motor:

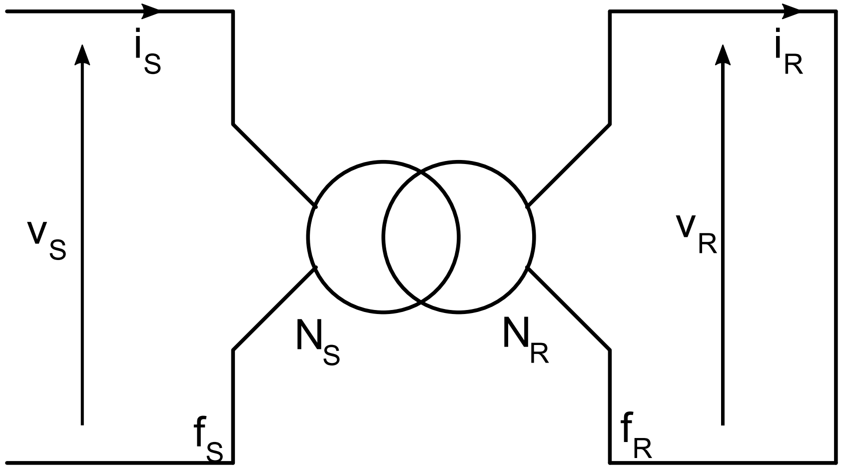
- Equivalent circuit diagram of the induction motor for each phase

=> Ideal model: the perfect Asynchronous motor

- At the stator, frequency of the grid f_S:

$$f_S = \frac{\Omega_S}{2\pi} \; for \; p = 1$$

- Frequency of currents induced at the rotor: f_R



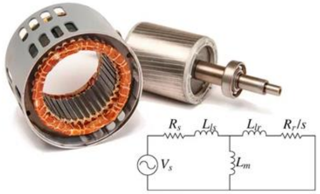
$$g = \frac{\Omega_S - \Omega_R}{\Omega_S}$$

$$g\Omega_S = \Omega_S - \Omega_R$$

$$f_R = \frac{\Omega_S - \Omega_R}{2\pi} = \frac{g\Omega_S}{2\pi}$$

=> Frequency transformation ratio

$$\boldsymbol{f_R = g f_S}$$



3-phase motors

- Induction motor/ Asynchronous motor:
- Equivalent circuit diagram of the induction motor

  => Ideal model: the perfect Asynchronous motor

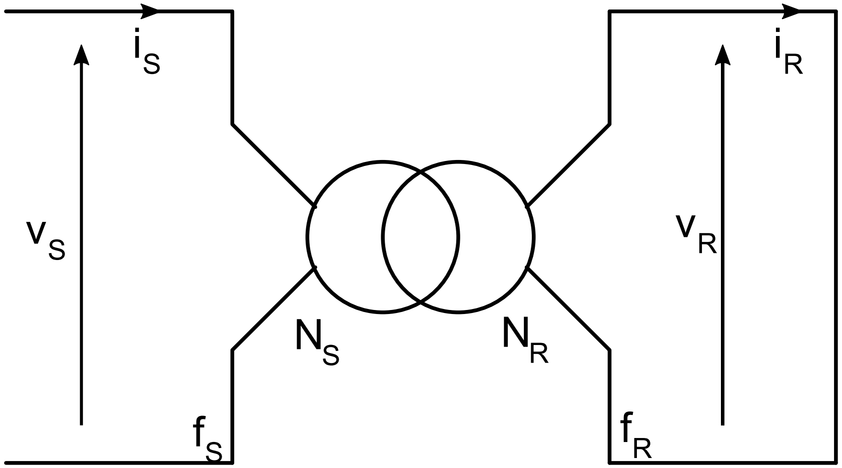
$$V_R = 4{,}44.S.N_R.\hat{B}.f_R$$

$$V_S = 4{,}44.S.N_S.\hat{B}.f_S$$

- Voltage transformation ratio (from Boucherot's formula)

$$\frac{V_R}{V_S} = \frac{N_R}{N_S}.\frac{f_R}{f_S} = m.g$$

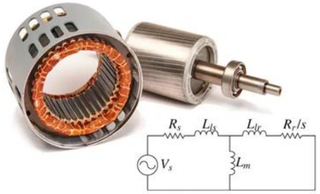
$$\boldsymbol{V_R = m.g.V_S}$$



- Current transformation ratio

$$N_R.I_R - N_S.I_S = 0$$

$$\boldsymbol{\frac{I_R}{I_S} = \frac{1}{m} = \frac{N_S}{N_R}}$$

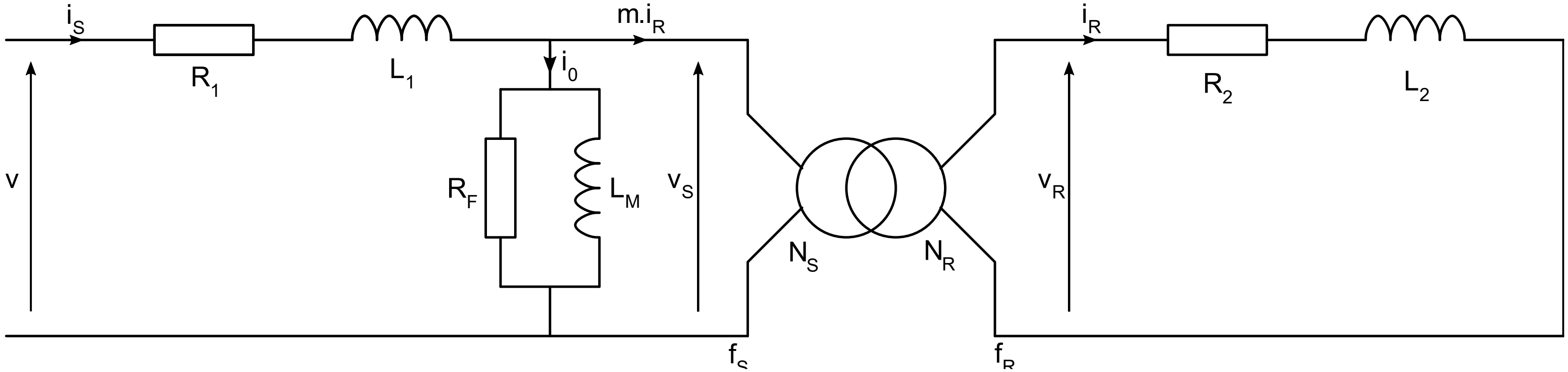


3-phase motors

- Induction motor/ Asynchronous motor:

- Equivalent circuit diagram of the induction motor

=> Intermediate model (from the transformer)



R_1: resistance of stator conductors

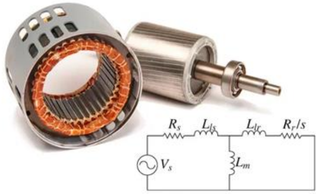
L_1: stator winding leakage inductance

R_2: resistance of rotor conductors

L_2: rotor winding leakage inductance

R_F: resistance modeling iron losses

L_M: magnetizing inductance of the magnetic circuit

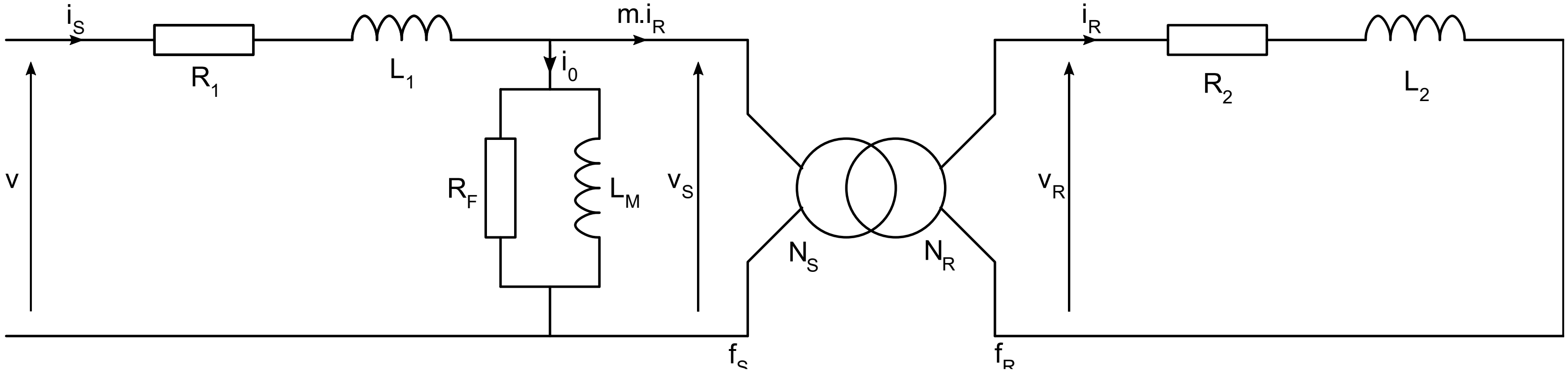


3-phase motors

- Induction motor/ Asynchronous motor:

- Equivalent circuit diagram of the induction motor

=> Intermediate model (from the transformer)



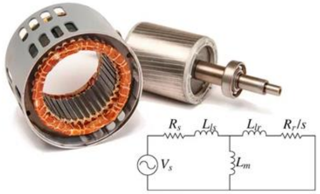
- Voltages at the stator

$$\underline{\boldsymbol{V}} = \boldsymbol{R_1}.\underline{\boldsymbol{I_S}} + \boldsymbol{j}.\boldsymbol{L_1}.\boldsymbol{\omega_S}.\underline{\boldsymbol{I_S}} + \underline{\boldsymbol{V_S}}$$

- Voltages at the rotor

$$\underline{V_R} - R_2.\underline{I_R} - j.L_2.g.\omega_S.\underline{I_R} = 0$$

$$\boldsymbol{m}.\underline{\boldsymbol{V_S}} - \frac{\boldsymbol{R_2}}{\boldsymbol{g}}.\underline{\boldsymbol{I_R}} - \boldsymbol{j}.\boldsymbol{L_2}.\boldsymbol{\omega_S}.\underline{\boldsymbol{I_R}} = \boldsymbol{0}$$

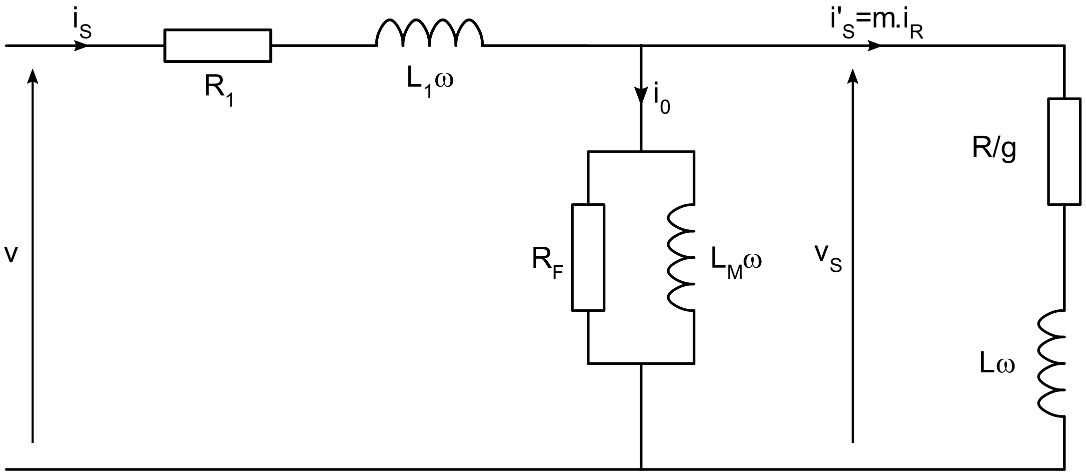


3-phase motors

- Induction motor/ Asynchronous motor:

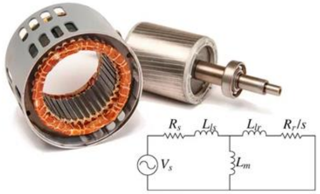
- Equivalent circuit diagram of the induction motor

=> The real model (equivalent primary model)



- Impedance transfer from rotor to stator: $m.\underline{V_S} - \frac{R_2}{g}.\underline{I_R} - j.L_2.\omega_S.\underline{I_R} = 0$

$$\mathrm{L} = \frac{L_2}{m^2} \qquad \frac{R}{g} = \frac{R_2}{g.m^2} \qquad \frac{I_R}{I_S} = \frac{1}{m}$$



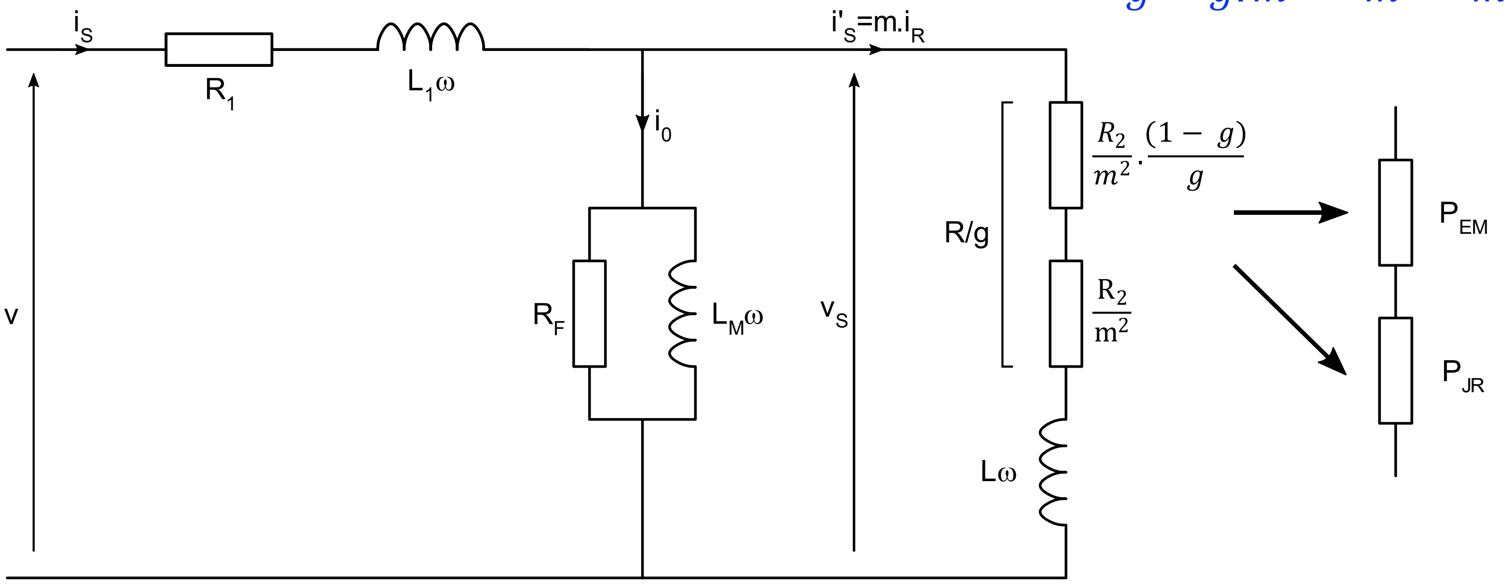
3-phase motors

- Induction motor/ Asynchronous motor:

- Equivalent circuit diagram of the induction motor

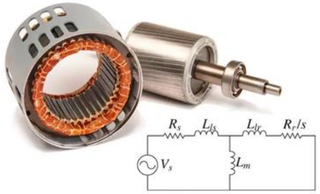
=> The real model (equivalent primary model)

$$\frac{R}{g} = \frac{R_2}{g.m^2} = \frac{R_2}{m^2} + \frac{R_2}{m^2}.\frac{(1-g)}{g}$$



$\frac{R_2}{m^2}$ = related to Joule losses at the rotor

$\frac{R_2}{m^2}.\frac{(1-g)}{g}$ = related to the power transferred to the rotor

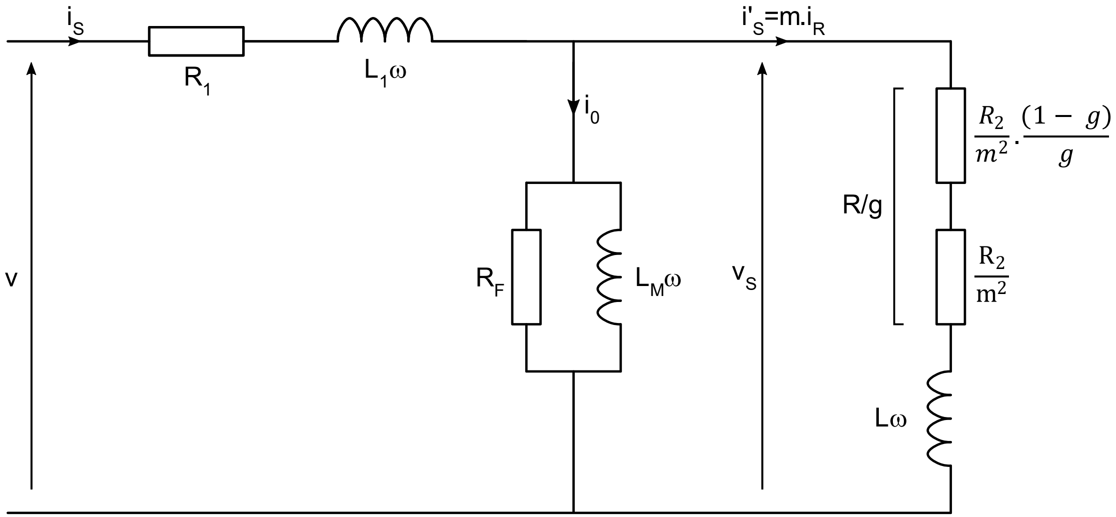


3-phase motors

- Induction motor/ Asynchronous motor:
- Equivalent circuit diagram of the induction motor

=> The real model (equivalent primary model)

Remark : inductances are cyclic inductances



R_F: resistance modeling iron losses

L_M: magnetizing inductance of the magnetic circuit

R_1: resistance of stator conductors/ Joule losses

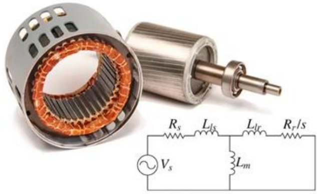
L_1: stator winding leakage inductance

R/g: resistance of rotor conductors/motional resistance

R_2/m^2: resistance of rotor conductors/ Joule losses

R_2/m^2 ((1-g)/g) resistance modelling the electromagnetic power transferred to the rotor

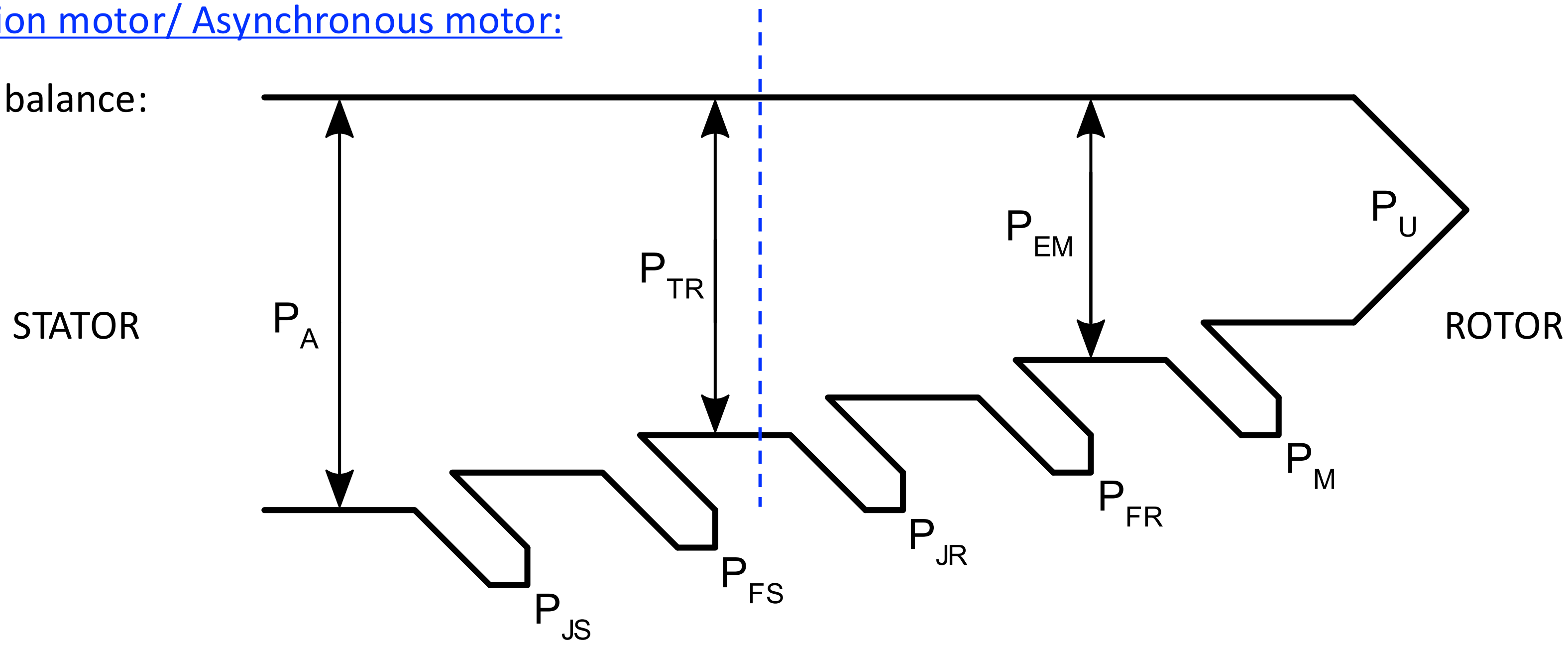
L: stator winding leakage inductance



3-phase motors

- Induction motor/ Asynchronous motor:

- Power balance:



P_a: Active power absorbed at the stator

$$P_a = 3V_S I_S cos\varphi_S$$

P_{JS}: Joule losses at the stator

$$P_{JS} = 3R_S I_S^2$$

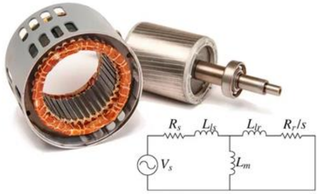
P_{FS}: Iron losses at the stator

$$P_{FS} = 3.\frac{{V_S}^2}{R_F}$$

P_{TR}: Power transmitted to the rotor

P_U: Useful power (mechanical)

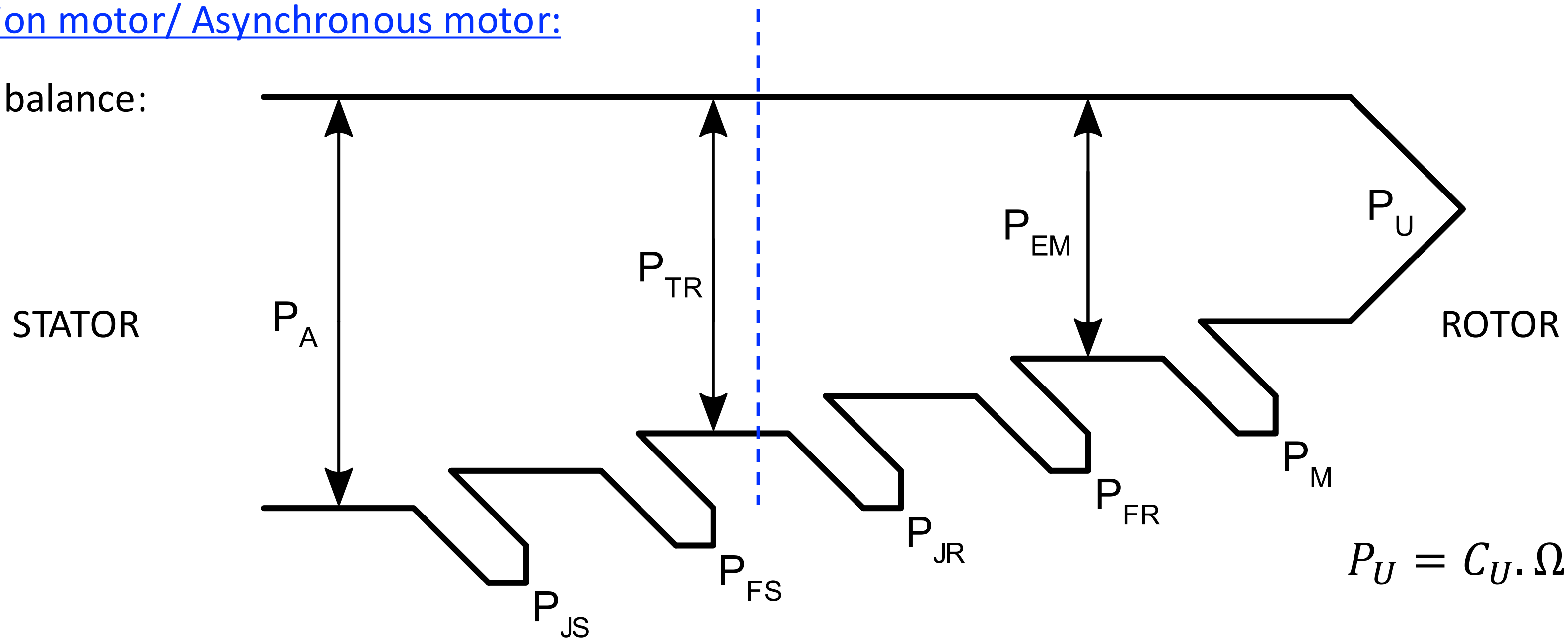
$$P_{TR} = C_{EM}.\Omega_S$$



3-phase motors

- Induction motor/ Asynchronous motor:

- Power balance:



$$P_U = C_U.\Omega$$

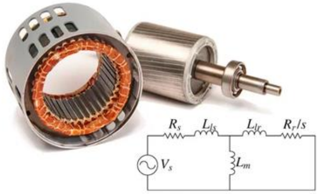
P_{JR}: Joule losses at the rotor $P_{TR} = 3.R_2.I_R{}^2$

P_{FR}: Iron losses at the stator (almost 0 usually, low f_R)

$$\eta = \frac{P_U}{P_U + P_{JS} + P_{FS} + P_{JR} + P_{FR} + p_M}$$

P_{EM}: Electromagnetic power $P_{EM} = C_{EM}.\Omega$

p_M: Mechanical losses (bearing losses, aerodynamic friction of the fan…)



3-phase motors

- Induction motor/ Asynchronous motor:

- Determining the elements of the equivalent circuit diagram: **Test at g = 0**

=> Test at nominal voltage

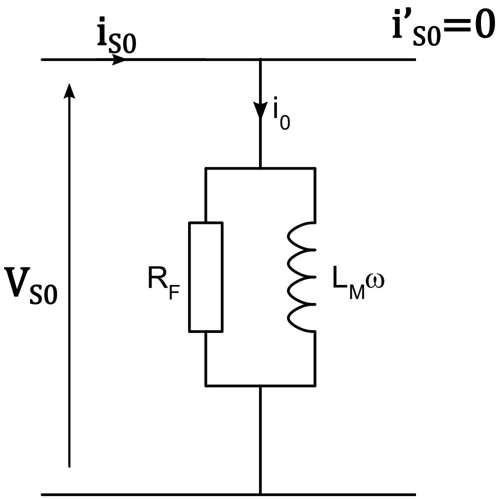
=> The synchronous machine is driven at synchronous speed (Ω_S) by an auxiliary motor (g=0)

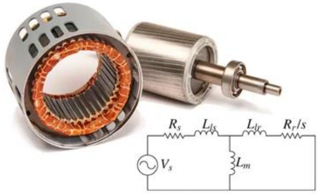
=> Under such conditions, $P_{EM} = 0$

- Measured quantities: V_{S0}, I_{S0}, P_{S0}, Q_{S0}

$$P_{S0} = P_{JS0} + P_{FS0} \cong P_{FS}$$

$$R_F = \frac{3V_{S0}^2}{P_{S0}} \qquad L_m\omega = \frac{3V_{S0}^2}{Q_{S0}}$$





3-phase motors

- Induction motor/ Asynchronous motor:

- Determining the elements of the equivalent circuit diagram: **Test at g = 1**

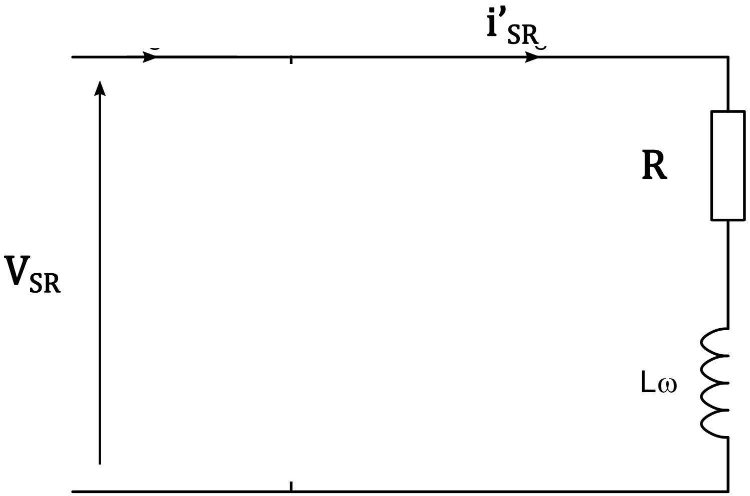
=> Test at reduced voltage, close to the nominal current

=> A brake blocks the rotor (g=1, (Ω_R=0)

=> Iron losses are assumed to be negligible – Powers are all consumed at the rotor

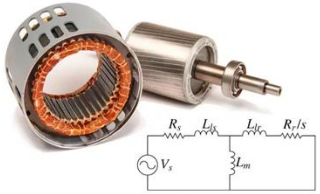
- Measured quantities: V_{SR}, I_{SR}, P_{SR}, Q_{SR}

$$R = \frac{P_{SR}}{3I_{SR}^2} - R_1 \qquad L\omega = \frac{Q_{SR}}{3I_{SR}^2}$$



- R_1 is measured by volt-amperemetric at the stator

=> Hot, continuous and nominal voltage and current

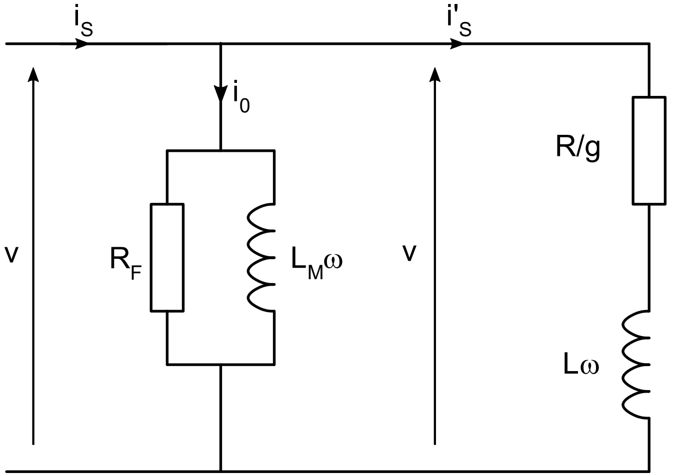


3-phase motors

- Induction motor/ Asynchronous motor:

- Expression of the electromagnetic torque

- R_1 and L_1 are neglected at $V = V_S$

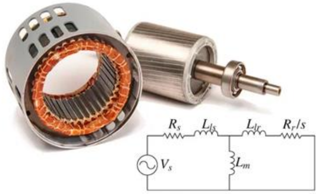


$$I'_S = \frac{V}{\sqrt{\left(\frac{R}{g}\right)^2 + (L.\omega)^2}}$$

$$P_{TR} = 3.\frac{R}{g}.I'^2_S$$

$$P_{TR} = C_{EM}.\Omega_S$$

$$C_{EM} = 3.\frac{\boldsymbol{p}}{\boldsymbol{\omega}}.\frac{\boldsymbol{V^2.R}}{\frac{\boldsymbol{R^2}}{\boldsymbol{g}} + \boldsymbol{g}.(\boldsymbol{L.\omega})^2}$$



3-phase motors

- Induction motor/ Asynchronous motor:
- Electromagnetic torque

$$C_{EM} = 3.\frac{p}{\omega}.\frac{V^2.R}{\frac{R^2}{g} + g.(L.\omega)^2}$$

$$C_{MAX} = k.\frac{V^2}{f^2} \qquad g_{max} = \frac{R}{L.\omega}$$

$$C_{DEM} = 3.\frac{p}{\omega}.\frac{V^2.R}{R^2 + (L.\omega)^2}$$