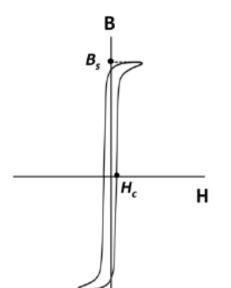


## **Reminders: magnetic materials**

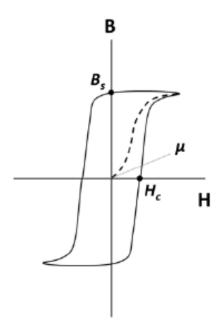
- The shape of the hysteresis loop varies according to the magnetic material

#### Soft magnetic material

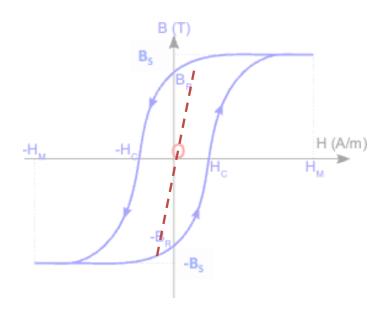


$$B_S = 1.5 - 2T$$
  
 $B_R < 1T$   
 $H_C = 1 - 10 \text{ A/m}$ 

#### Hard magnetic material



$$B_R - 1T$$
  
 $H_C = 10^3 \text{ A/m}$ 

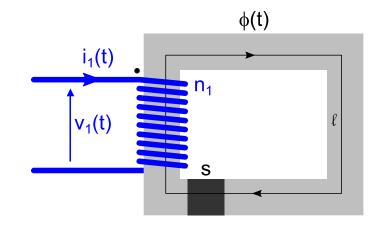


Linear approximation outside the saturation region:  $B=\mu H$ 





- Linear homogeneous magnetic circuits:
- Materials and geometry are chosen to concentrate flux density as much as possible, thus creating the strongest possible induction => limitation of both mass and size/volume
- Homogeneous = a single magnetic material
- Homogeneous = constant cross section
- Linear = outside the saturation regime
- ℓ (schematic) = mean field line



$$\oint \overrightarrow{H}.\overrightarrow{d\ell} = n.I$$

$$H.\ell = n.I$$

- The nI quantity is also called magnetomotive force



#### **Magnetic circuits**

- Hopkinson's relation:
- In a linear homogeneous magnetic circuit, the material exhibits a constant permeability

$$\mu = \mu_0 \mu_r$$

- Therefore:  $\mathbf{B} = \mu H$
- We have shown previously that  $\Phi = B.S$
- By considering the Ampere's theorem as well, it can be shown that:

$$NI = \Re \Phi$$
 Hopkinson's relation, with the reluctance  $\Re = \frac{\ell}{\mu S}$  (in H<sup>-1</sup>)

- The reluctance is the opposition that a ferromagnetic material produces to the establishment of a magnetic field





# Magnetic circuits: analogy with electrical circuits

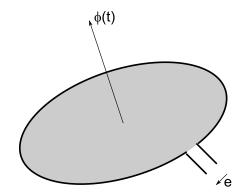
Electrical circuit	Magnetic circuit
R	$\phi(t)$ $v_1(t)$ $s$
Electromotive force in V : V	Magnetomotive force in A.tr : ε = n.l
Electrical current in A : I	Magnetic flux in Wb : Φ
Electrical resistance in $\Omega$ : R	Magnetic reluctance: R
$R = \rho \times \frac{\ell}{S}$ • Electrical resistivity in $\Omega$ .m : $\rho$ • Conductor length in m : $\ell$ • Conductor cross section in m² : s	$\mathcal{R} = \frac{1}{\mu_0 \times \mu_R} \times \frac{\ell}{S}$ • Magnetic permeability in H/m : $\mu$ • Circuit length in m : $\ell$ • Circuit cross section in m² : s
Ohm's law: $V = R.I$	Hopkinson's law: $n.I=\mathcal{R}.\Phi$
Electrical field in V/m : E	Excitation magnetic field in A/m : H
Current density in A.m <sup>-2</sup> : $J = \frac{I}{s} = \sigma$ . $E$	Induction magnetic field in T: $B=\frac{\Phi}{s}=\mu$ . $H$



- Lenz's law and Faraday's law:
- Lenz's law (qualitative law): Induced currents and fields oppose the causes that gave rise to them
  - => The induced field and current oppose the change in flux through the circuit
- Faraday's law (quantitative law): Any flux variation produces an induced electromotive force across a circuit
- => For a coil, an electromotive force is produced across each turn of the winding

$$e(t) = -\frac{d\phi(t)}{dt}$$
 (receptor convention)

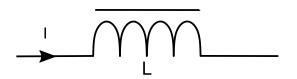
For n turns: 
$$e(t) = -n \times \frac{d\phi(t)}{dt}$$

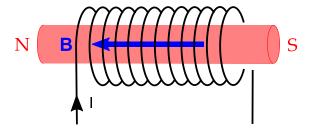






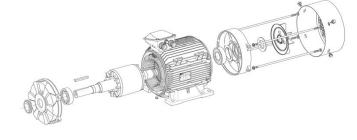
- The iron-core coil:
  - A winding of copper wire wrapped on a magnetic circuit of ferromagnetic material forms an iron-core coil
    - => transformers, electromagnets, motors









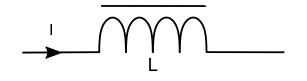




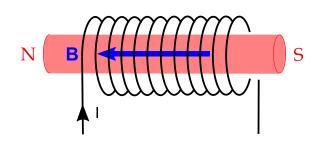


- The iron-core coil:
- Consider an iron-core coil with N turns
- => Each turn is crossed by the flux  $\phi$  created in the material => Total flux  $\phi_T = N\phi$
- By considering both Hopkinson's relation and Faraday's law:

$$Ni = \Re \Phi$$
  $e(t) = -\frac{d\phi_T(t)}{dt} = -N\frac{d\phi(t)}{dt}$ 

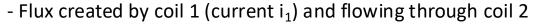


- We introduce the inductance L such that:  $\phi_T = Li$
- With L (in H):  $L = \frac{N^2}{\Re} = \frac{N^2 \mu S}{\ell}$
- We retrieve the expression:  $e(t) = -L \frac{di(t)}{dt}$

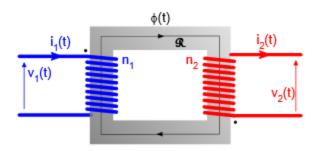




- Mutual inductance: (case of the homogeneous linear magnetic circuit)
- Mutual inductance occurs when the magnetic circuit has at least two windings
- => Each current has an influence on the flux flowing in the circuit.



$$\phi_{1\to 2} = \frac{N_1 i_1}{\Re} \qquad \longrightarrow \qquad \phi_{T2} = N_2 \phi_{1\to 2}$$



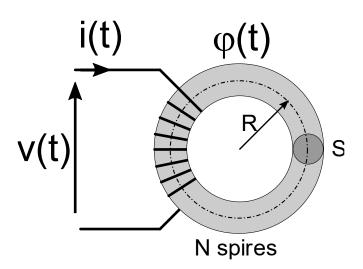
- By analogy with the definition of the inductance:  $\phi_{T2}=N_2\phi_{1 o 2}=M_{12}i_1$
- With M (in H):  $M_{12} = \frac{N_1 N_2}{\Re}$   $e_2(t) = -M_{12} \frac{di_1(t)}{dt}$
- Voltage across winding k among n other windings: mutual inductances + self-inductance

$$-e_{k}(t) = M_{1k}\frac{di_{1}(t)}{dt} + M_{2k}\frac{di_{2}(t)}{dt} + \dots + M_{nk}\frac{di_{n}(t)}{dt} + L\frac{di_{k}(t)}{dt}$$





- Boucherot's formula: Ideal coil (no losses)
- The winding is subjected to a sinusoidal voltage
  - => Assumption of forced flux



$$v(t) = V\sqrt{2}.\cos(\omega t)$$

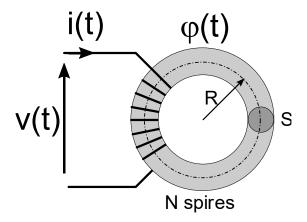
$$\frac{d\varphi(t)}{dt} = \frac{V\sqrt{2}}{N}.\cos(\omega t)$$

$$\varphi(t) = \frac{V\sqrt{2}}{N} \cdot \int_0^t \cos(\omega t) \cdot dt$$

$$\varphi(t) = \frac{V\sqrt{2}}{N\omega} \cdot \sin(\omega t) + \varphi(0)$$



- Boucherot's formula: Ideal coil (no losses)
- $\phi(0)$ =0: no permanent magnet, no remanent flux, no second DC winding).



- The flux lags the voltage
- The relation Nφ=Li (Hopkinson's law) gives the current

$$\varphi(t) = \frac{V\sqrt{2}}{N\omega} \cdot \sin(\omega t) + \varphi(0)$$

$$\varphi(t) = \frac{V\sqrt{2}}{N\omega} \cdot \sin(\omega t)$$

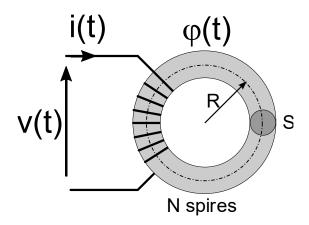
$$\varphi(t) = \frac{V\sqrt{2}}{N\omega} \cdot \cos(\omega t - \pi/2)$$

$$i(t) = \frac{V\sqrt{2}}{L\omega} \cdot \cos(\omega t - \pi/2)$$





- Boucherot's formula: Ideal coil (no losses)



$$\begin{cases} \varphi(t) = \frac{V\sqrt{2}}{N\omega} \cdot \cos(\omega t - \frac{\pi}{2}) \\ \varphi(t) = \Phi_M \cdot \cos(\omega t - \frac{\pi}{2}) \end{cases}$$

$$\frac{V\sqrt{2}}{N\omega} = \Phi_M$$

$$V = \frac{N \cdot \omega \cdot \Phi_M}{\sqrt{2}}$$

$$V = \frac{N \cdot (2 \cdot \pi \cdot f) \cdot S \cdot B_M}{\sqrt{2}}$$

$$V = \frac{2 \cdot \pi}{\sqrt{2}} \cdot S \cdot N \cdot B_M \cdot f$$

$$V = 4, 44. S. N. B_M. f$$

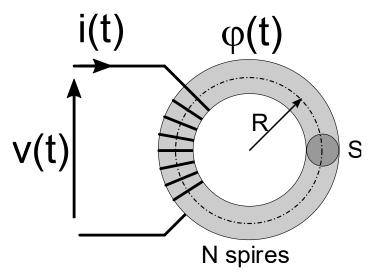




- Boucherot's formula: Ideal coil (no losses)

$$V = 4, 44. S. N. B_M. f$$

If the flux  $\phi$  of a coil is due to the current **i** flowing through it (Hopkinson's law), its maximum value  $\phi_M$  depends only on the RMS value of the voltage **V** (at constant frequency).



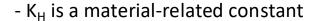
The voltage imposes a flux and the winding draws the current accordingly



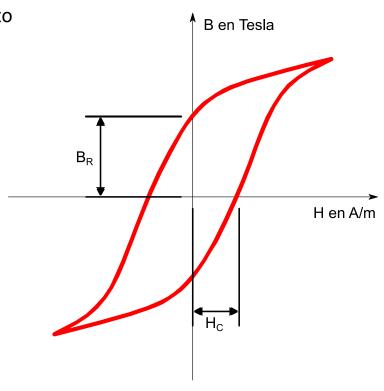


- <u>Defects of the iron-core coil</u>: hysteresis losses
- Hysteresis losses correspond to the power required to magnetize and demagnetize the material over its hysteresis loop.
- Empirical formula:

$$P_H = K_H l S f B_{max}^n$$



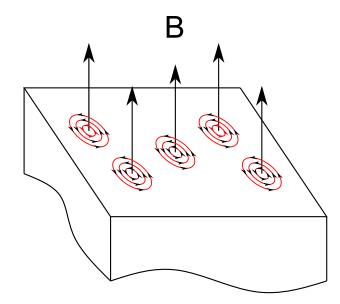
- n is the Steinmetz coefficient (around 1.8)
- I is the mean field line, f the frequency,  $\mathbf{B}_{\text{max}}$  the maximum induction field





- Defects of the iron-core coil: Eddy currents
- Currents induced in the magnetic material in which they flow freely
- These currents cause losses in the form of power dissipated by Joule effect





- K<sub>F</sub> is a material-related constant
- d is the thickness of the foil in the case of a laminated material
- I is the mean field line, f the frequency,  $B_{\text{max}}$  the maximum induction field
- Defects of the iron-core coil: IRON LOSSES  $P_F$   $P_F = P_H + P_E$





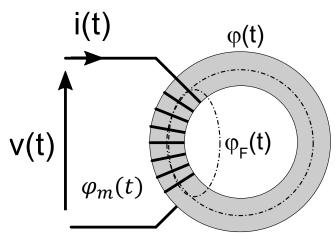
- Defects of the iron-core coil: Leak inductance
- Magnetic materials are never perfect, and never channel all the field lines
  - => Part of the magnetic flux propagates in the air via less reluctant paths
  - => This corresponds to magnetic leaks (flux outside the magnetic circuit)
- The magnetic field channeled in the magnetic circuit is called

"magnetizing flux"

$$\phi = \phi_m + \phi_f$$

$$v(t) = -N\frac{d\phi}{dt} = -(L_m\frac{di}{dt} + L_f\frac{di}{dt})$$

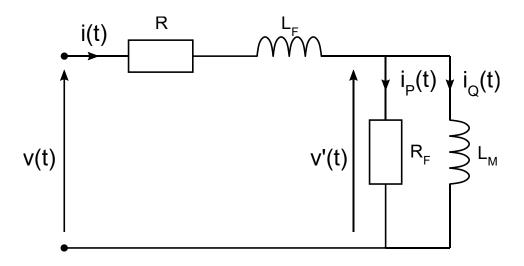
- L<sub>m</sub> is the magnetizing inductance, L<sub>f</sub> is the leak inductance







- Linear model of the iron-core coil:



- R is the resistance of the coil N turns
- L<sub>F</sub> is the leak inductance, L<sub>M</sub> is the magnetizing inductance
- R<sub>F</sub> is the resistor modelling the iron losses



## **Single-phase transformer**

- The transformer enables a change in the RMS value of an AC voltage to be achieved with high efficiency => step-up transformer or step-down transformer
- A single-phase transformer consists of two windings wound on the same magnetic circuit

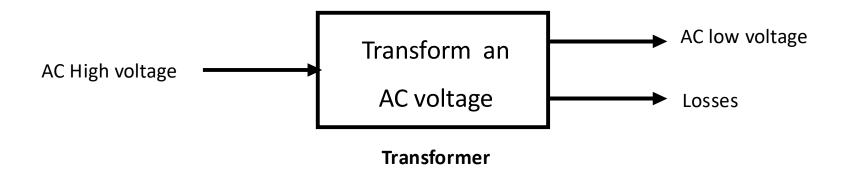
=> Usually, the two windings have different numbers of turns

# 



## **Single-phase transformer**

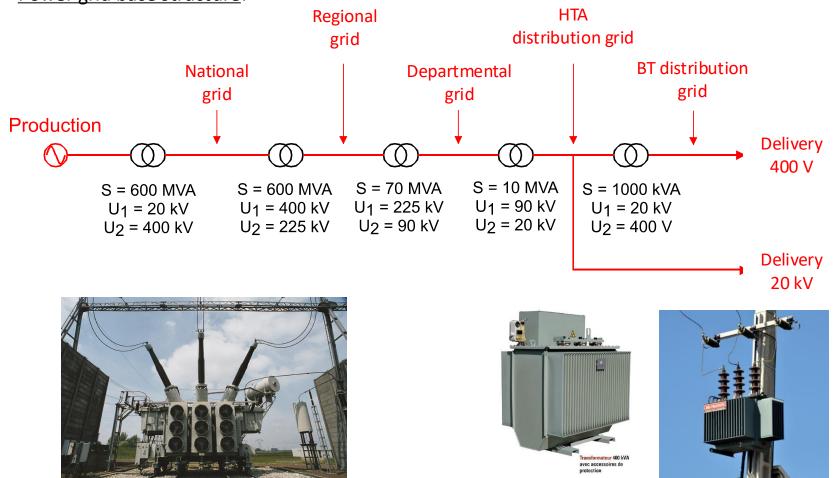
- <u>Functional approach</u>: More precisely, the transformer is a static machine that allows sinusoidal quantities (voltages, currents) to be modified without changing their frequency.
  - => Voltage adaptation: step-down or step-up transformer
  - => Galvanic insulation: insulation transformer





## **Single-phase transformer**

- Power grid base structure:

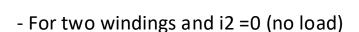






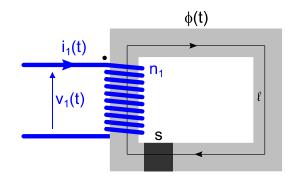
- <u>Principle</u>:
- Hopkinson's relation in the case of a single winding at the primary side:

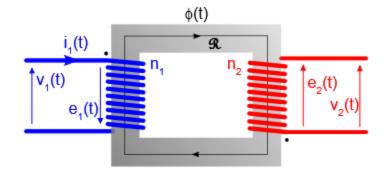
$$n_1$$
.  $i_1 = \mathcal{R}$ .  $\Phi$ 



$$e_1 = -v_1 = -n_1 rac{darphi}{dt}$$
  $v_1 = n_1 rac{darphi}{dt}$ 

$$e_2 = -n_2 \frac{d\varphi}{dt} = v_2$$









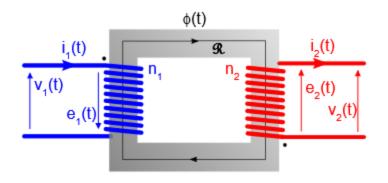
- <u>Principle</u>:

- For two windings and i2 ≠ 0 (presence of a load)

$$n_1 \cdot i_1 + n_2 \cdot i_2 = \mathcal{R} \boldsymbol{\phi}$$

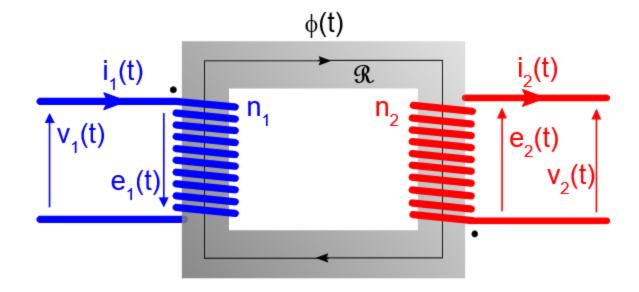
$$e_1 = -v_1 = -n_1 \frac{d\varphi}{dt}$$
$$v_1 = n_1 \frac{d\varphi}{dt}$$

$$e_2 = -n_2 \frac{d\varphi}{dt} = v_2$$





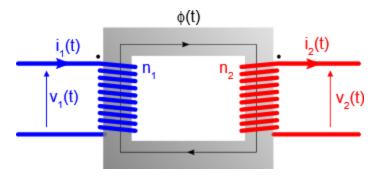
- <u>Principle</u>: receptor convention/generator convention
- Seen from the primary grid, the transformer is a receptor
  - => Receptor convention at the primary side
- Seen from the load, the transformer is a generator
  - => Generator convention at the secondary side

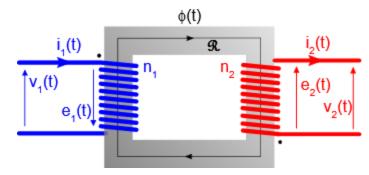






- <u>Principle</u>: homonymous terminals
- The winding direction is marked with dots (●). Terminals marked in this way are called "homonymous terminals". They correspond to points of the same instantaneous polarity.
- Sign conventions for magnetic and electrical quantities:
- => A positive current entering through a homonymous terminal creates a positive flux in the magnetic circuit
- => According to Hopkinson's law, the magnetomotive force is preceded by the sign + if the current orientation arrow enters through a homologous terminal, and by the sign otherwise.



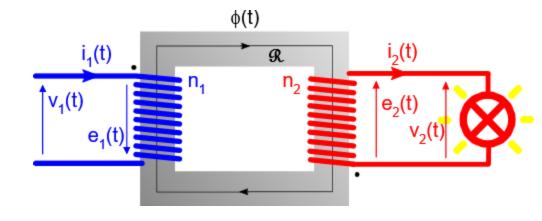






- <u>Principle</u>: the ideal transformer voltages
- Ideal windings = no voltage drop, no losses
- Ideal magnetic circuit = zero reluctance

$$v_1 = -e_1 = n_1 \frac{d\varphi}{dt}$$
$$v_2 = e_2 = -n_2 \frac{d\varphi}{dt}$$



- The **transformation ratio** is given by:

$$m=\frac{n_2}{n_1}=-\frac{v_2}{v_1}$$

m > 1: Step-up transformer

m < 1: Step-down transformer

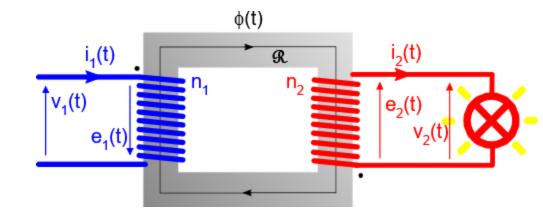


- Principle: the ideal transformer currents
- Ideal windings = no voltage drop, no losses
- Ideal magnetic circuit = zero reluctance

- Hopkinson's relation:

$$n_1. i_1 + n_2. i_2 = \mathcal{R}\phi$$

$$\downarrow$$
 $n_1. i_1 + n_2. i_2 = \mathcal{R}\phi = \mathbf{0}$ 



$$rac{oldsymbol{i}_2}{oldsymbol{i}_1} = -rac{oldsymbol{n}_1}{oldsymbol{n}_2}$$

$$\frac{i_2}{i_1}=-\frac{1}{m}$$

$$i_1 = -m.i_2$$

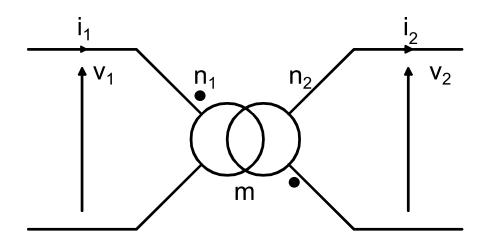




- <u>Principle</u>: the ideal transformer - symbol

$$m = \frac{n_2}{n_1} = -\frac{V_2}{V_1} = -\frac{I_1}{I_2}$$

$$\eta = \frac{P_2}{P_1} = \frac{V_2 \cdot I_2}{V_1 \cdot I_1} = 1$$



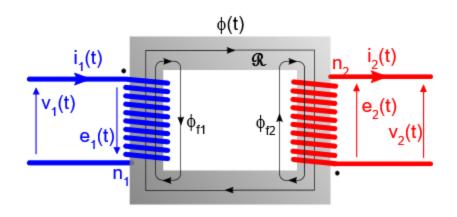
- Conservation of power, for the ideal transformer only





- Principle: winding imperfection
- Joule losses (or copper losses)

$$P_{J}=r_{1}.\,I_{1}^{2}+r_{2}.\,I_{2}^{2}$$
 
$$R=\rho.\frac{\ell}{s}\quad \rho_{\Theta}=\rho_{0}(1+a\Theta)$$
 Copper (Cu):  $\rho_{0}$  = 1,6.10-8  $\Omega$ .m, a = 0,39 Aluminum (Al):  $\rho_{0}$  = 2,42.10-8  $\Omega$ .m, a = 0,43 
$$v_{1}=-e_{1}^{\prime}+r_{1}.\,i_{1}$$
 
$$v_{2}=e_{2}^{\prime}-r_{2}.\,i_{2}$$



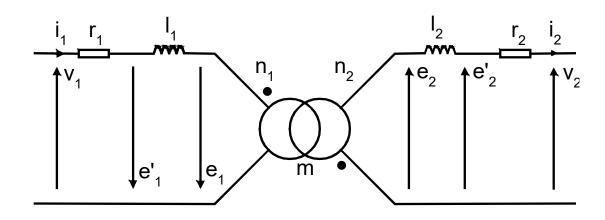
- Magnetic leaks

$$\phi_{T} = \phi_{f1} + \phi_{f2} + \phi$$

$$e'_{1} = n_{1} \cdot \frac{d\phi}{dt} + n_{1} \cdot \frac{d\phi_{f1}}{dt}$$

$$e'_{1} = e_{1} + l_{1} \cdot \frac{di_{1}}{dt}$$

$$e'_{2} = e_{2} + l_{2} \cdot \frac{di_{2}}{dt}$$



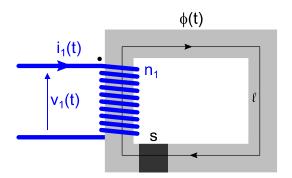


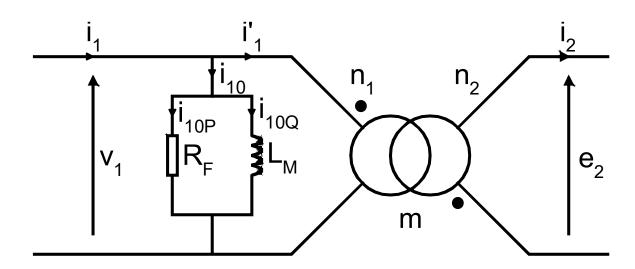


- <u>Principle</u>: magnetic circuit imperfection
- Finite permeability and non zero reluctance of the magnetic circuit

=> A primary current is consumed at no load

$$\mathcal{R}\Phi = n_1 \cdot i_{10}$$

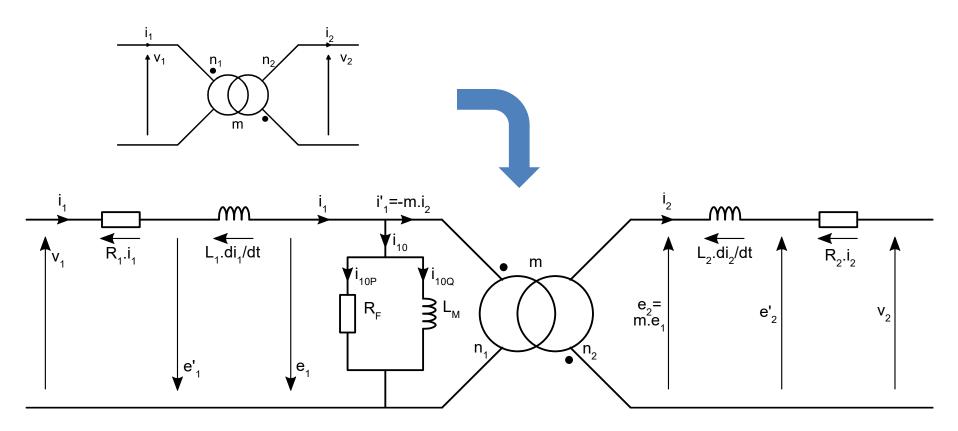








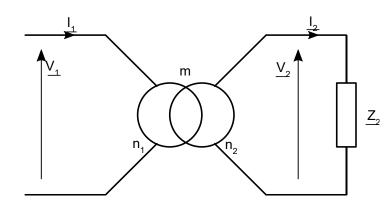
- Equivalent model of the real transformer

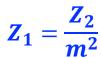


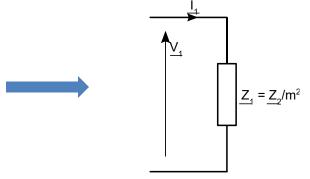




- Equivalent model of the real transformer: Impedance transfer
- A transformer, in which an impedance  $Z_2$  is connected to the secondary side is equivalent to an impedance  $Z_1$  connected to the primary side with:







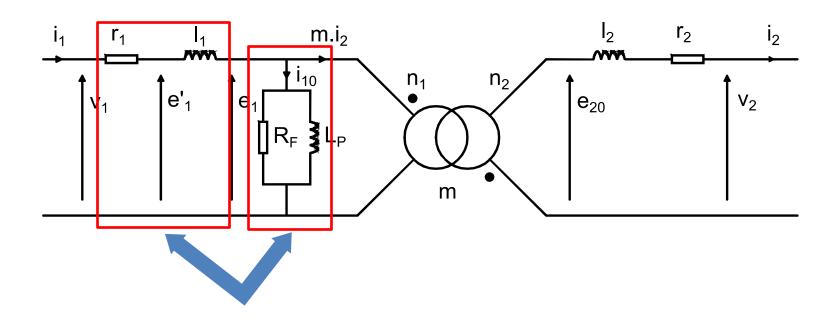
- We remind:  $I_1 = m$ .  $I_2$  et  $V_1 = \frac{V_2}{m}$
- We write:  $Z_2 = \frac{V_2}{I_2}$  et  $Z_1 = \frac{V_1}{I_1}$   $Z_1 = \frac{V_1}{I_1} = \frac{V_2/m}{m.\,I_2} = \frac{1}{m^2}.\frac{V_2}{I_2} = \frac{Z_2}{m^2}$

$$Z_1 = \frac{Z_2}{m^2}$$



- Equivalent model of the real transformer: Kapp's assumption
- The voltage drop across  $R_1$  and  $L_1$  is small compared to the voltages  $e_1$  and  $V_1$

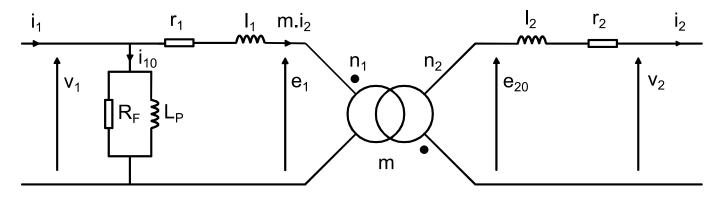
 $=> R_1/L_1$  and  $R_F/L_p$  can be swapped





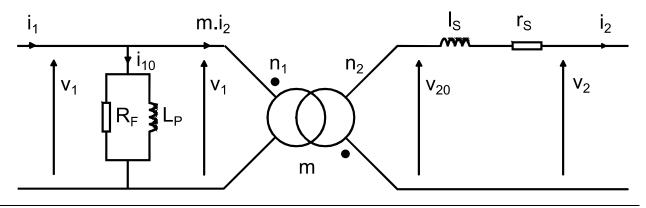


- Equivalent model of the real transformer: Kapp's assumption



- Kapp's equivalent secondary model (R<sub>1</sub> and L<sub>1</sub> are transferred to the secondary side)

$$r_S = r_2 + m^2 \cdot r_1$$
  
 $l_S = l_2 + m^2 \cdot l_1$   
 $m = \frac{n_2}{n_1} = \frac{V_{20}}{V_1} \cong \frac{I_1}{I_2}$ 





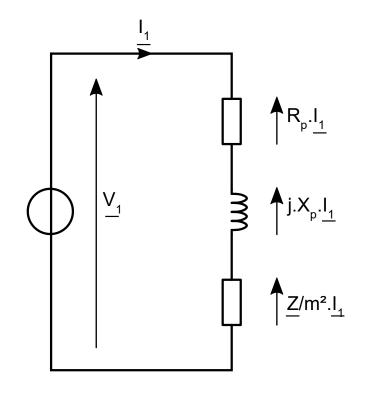


- Equivalent model of the real transformer: Kapp's assumption
- Kapp's equivalent primary model (R<sub>2</sub> and L<sub>2</sub> are transferred to the primary side)

$$R_{\mathrm{P}}=R_{1}+rac{R_{2}}{m^{2}}$$
: total winding resistance transferred to the primary side  $X_{\mathrm{P}}=X_{1}+rac{X_{2}}{m^{2}}$ : total winding reactance transferred to the primary side  $Z_{\mathrm{p}}=R_{\mathrm{p}}+\mathrm{j}X_{p}$ : total winding impedance transferred to the primary side  $rac{Z}{m^{2}}$ : secondary side load transferred to the primary side.

$$\underline{V_1} = \frac{\underline{Z}}{m^2} \cdot \underline{I_1} + \underline{Z_P} \cdot \underline{I_1}$$

(this model will be useful for the induction motor)

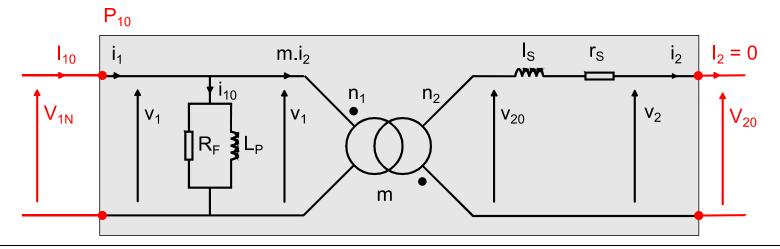






- <u>Determining the elements of the Kapp secondary equivalent model</u>:
- Test at no load:
- conditions: Transformer supplied at <u>rated</u> voltage  $(V_{1N})$  + open secondary side  $(I_2 = 0)$
- Measured quantities:  $V_{1N}$ ,  $I_{10}$ ,  $V_{20}$ ,  $P_{10}$  (iron losses) and  $Q_{10}$

$$R_F = \frac{V_{1N}^2}{P_{10}}$$
  $X_P = L_P$ .  $\omega = \frac{V_{1N}^2}{Q_{10}}$  with  $Q_{10} = \sqrt{(V_{1N} \cdot I_{10})^2 - P_{10}^2}$   $m = \frac{V_{20}}{V_1}$ 

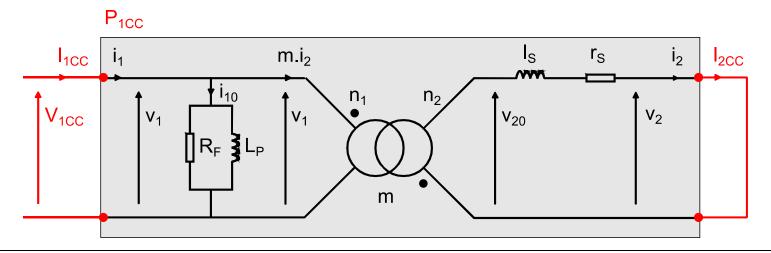






- Determining the elements of the Kapp secondary equivalent model:
- Short-circuit test: (short circuit at the secondary side)
- conditions: Transformer supplied at <u>reduced</u> voltage  $(V_{1CC})$  + <u>rated</u> secondary side current  $(I_{2CC} = I_{2N})$
- Measured quantities:  $V_{1CC}$ ,  $I_{1CC}$ ,  $I_{2CC}$ ,  $P_{1CC}$  (Joule losses) and  $Q_{1CC}$

$$R_S = \frac{P_{1CC}}{I_{2CC}^2}$$
  $X_S = L_S. \omega = \frac{Q_{1CC}}{I_{2CC}^2}$  with  $Q_{1CC} = \sqrt{(V_{1CC}.I_{1CC})^2 - P_{1CC}^2}$ 





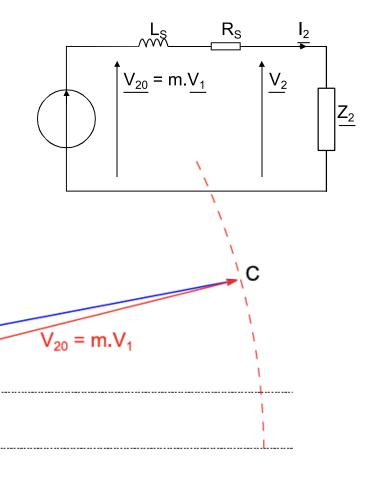


- Voltage drop at the secondary side of the transformer:
- Kapp's triangle: case of the inductive load ( $\phi_2 > 0$ )

 $X_S.I_2$ 

$$\underline{V_{20}} = m.\underline{V_1} = (R_S + jX_S).\underline{I_2} + \underline{V_2}$$

$$\Delta V_2 \approx R_S \cdot I_2 \cos \varphi_2 + X_S \cdot I_2 \cdot \sin \varphi_2$$



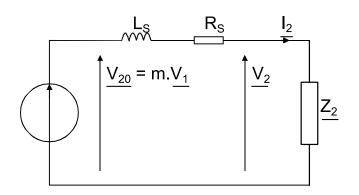


 $R_s.I_2$ 

## **Single-phase transformer**

- Efficiency:

$$\eta = \frac{V_2. I_2. \cos \varphi_2}{V_2. I_2. \cos \varphi_2 + R_S. I_2^2 + P_F}$$



- P<sub>F</sub> is obtained from the test at no load (P<sub>10</sub>)

- Optimum current obtained from

$$\eta = \frac{V_2 \cdot \cos \varphi_2}{V_2 \cdot \cos \varphi_2 + R_S \cdot I_2 + \frac{P_{Fer}}{I_2}}$$

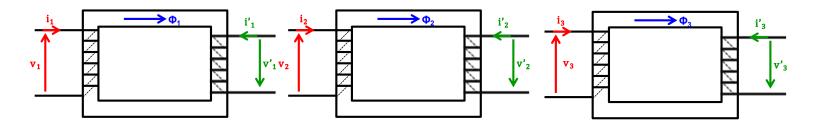
- The maximum efficiency is given for

$$R_S.\,I_{2Opt}=rac{P_{Fer}}{I_{2Opt}}$$
 and  $I_{2Opt}=\sqrt{rac{P_{Fer}}{R_S}}$ 

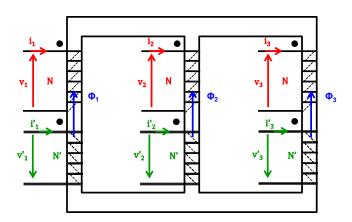




- Construction and connection:
- At first glance, the three-phase transformer can be considered as the combination of 3 single-phase transformers



 Or it can be integrated on a single magnetic circuit comprising 3 columns, each carrying the primary winding and the secondary winding





## **Three-phase transformer**

- Construction and connection:
- Both the primary and the secondary sides of the transformer need to be connected (delta or Y)
- The nature of these couplings is designated by letters, using upper case letters for the high voltage side and lower case letters for the secondary side

1st letter (upper case): connection on H.T. side

- Y: « star »

- **D** or  $\Delta$ : delta

2nd letter (lower case): coupling on B.T. side

- **y**: star

- d: delta

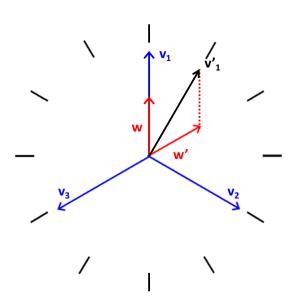
Add letter "N ou n" if neutral is out.



#### **Three-phase transformer**

- Hourly index: Phase shift angle between primary and secondary voltages expressed in hours

- Marked from 0 to 11, each hour angle is always a multiple of 30°:
- => 0 for 0
- => 1 for 30
- => 2 for 60°, and
- => 6 for 180
- => ...



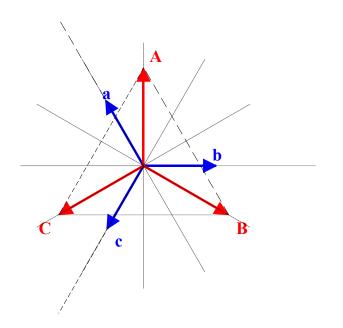
- That's why a specific representation has been chosen:
- => On the same Fresnel diagram, we plot two vectors representing two homologous voltages, one on the primary side, the other on the secondary side.
- => The voltage on the primary side is shown vertically, pointing upwards.

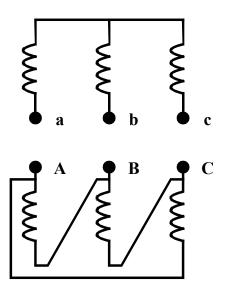
If we consider these two vectors as the two hands of a watch, the time indicated by the watch is by definition the transformer hourly index

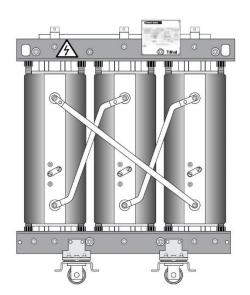




#### - Example of Dyn 11 connection:







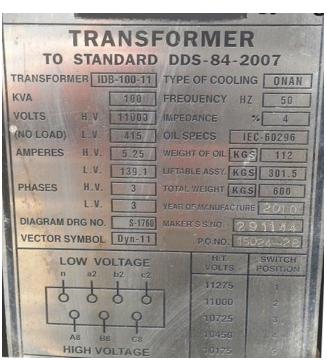
- High-voltage side => delta coupling.
- Low-voltage side => Y connection with neutral out.
- 330° phase shift (11x30) between primary and secondary.

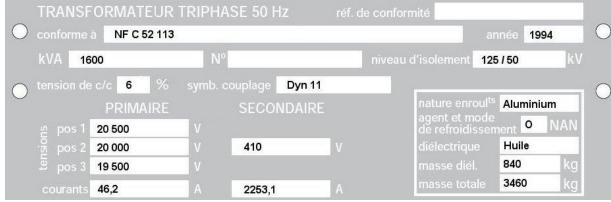




## **Three-phase transformer**

- <u>Transformer name plate</u>: gives all the rating







#### **Three-phase transformer**

- Equivalent model of the real transformer:
- Each transformer column can be modeled separately by a single-phase equivalent diagram at the secondary (see before)
- Each element of the secondary model can be determined by the same tests as those of the singlephase transformer
- Test at no load:

$$R_F = \frac{3V_N^2}{P_0}$$
  $X_P = L_P$ .  $\omega = \frac{3V_N^2}{Q_0}$  with  $Q_0 = \sqrt{(V_N \cdot I_0)^2 - P_0^2}$   $m = \frac{V_N'}{V_N}$ 

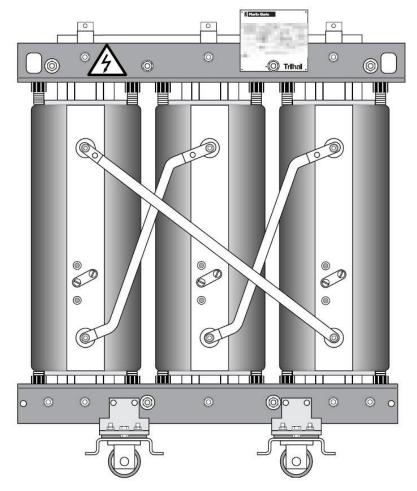
- Short-circuit test: (short circuit at the secondary side)

$$R_S = \frac{P_{CC}}{3I_{CC}^{'2}}$$
  $X_S = L_S. \omega = \frac{Q_{CC}}{3I_{CC}^{'2}}$  with  $Q_{CC} = \sqrt{(V_{CC}.I_{CC})^2 - P_{CC}^2}$ 





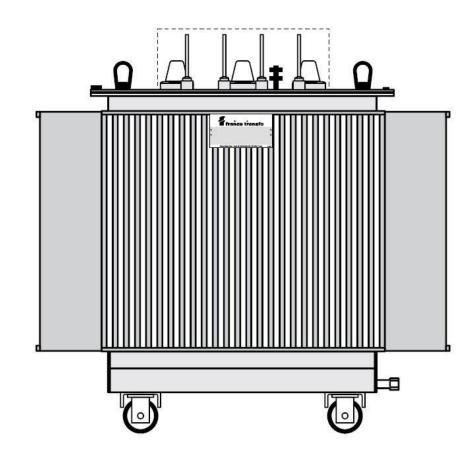
- HTA/BT distribution transformers: Dry transformer
- Active parts coated in protective resins (often epoxies) and mounted on a support frame in the open air.
- Good ventilation of the device and the room is required
- Dust removal from ambient air recommended







- HTA/BT distribution transformers: Immersed transformer
- Sealed with Total/Integral filling
- Hermetically sealed transformer
- Flexible tank
- Flexible tank
- Accordion-folded walls to absorb changes in dielectric volume as it heats up







- <u>HTA/BT distribution transformers</u>: Dry and Immersed transformers





