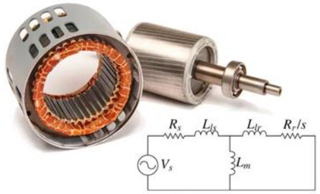
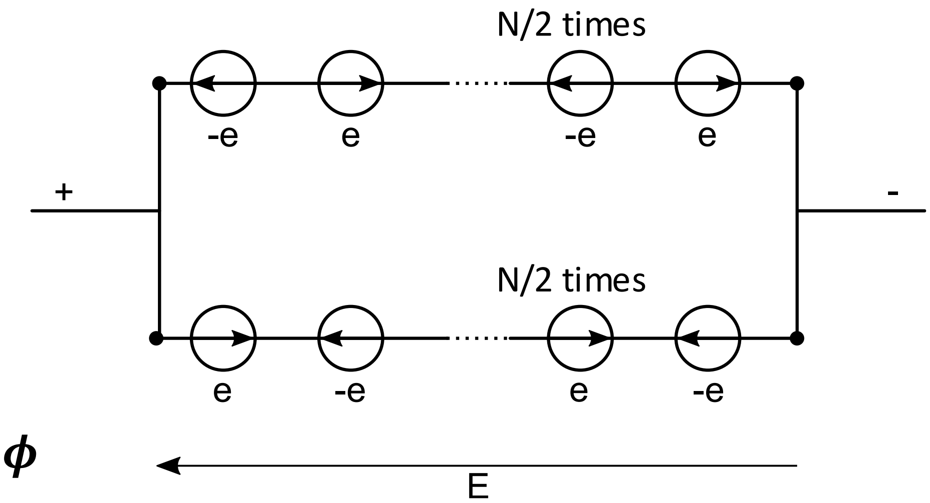


The DC motor

- Electromechanic relations: counter electromotive force

- The armature has "N" active conductors and "a" winding paths

- The armature's counter-electromotive force is written:

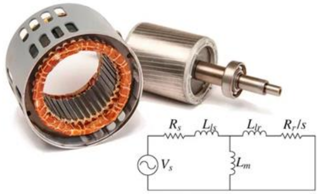


$$E = \frac{N}{2a}.e \qquad E = \frac{N}{2a}.2p.n.\phi \qquad \boldsymbol{E = \frac{p}{a}.N.n.\phi}$$

- Usually written: $\boldsymbol{E = k.n.\phi}$ with k=(p/a)N a constant related to the construction of the machine

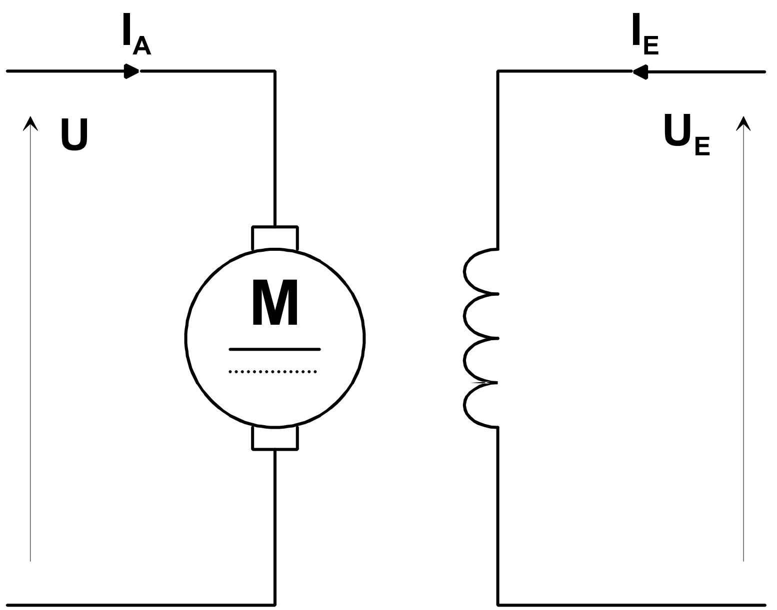
- Or also: $\boldsymbol{E = k_n.n}$ if Φ remains constant, permanent magnet motor or i_E =Cste

k_n: speed constant, given by the manufacturer, in $V.tr^{-1}.s$ or $V.tr^{-1}.min$

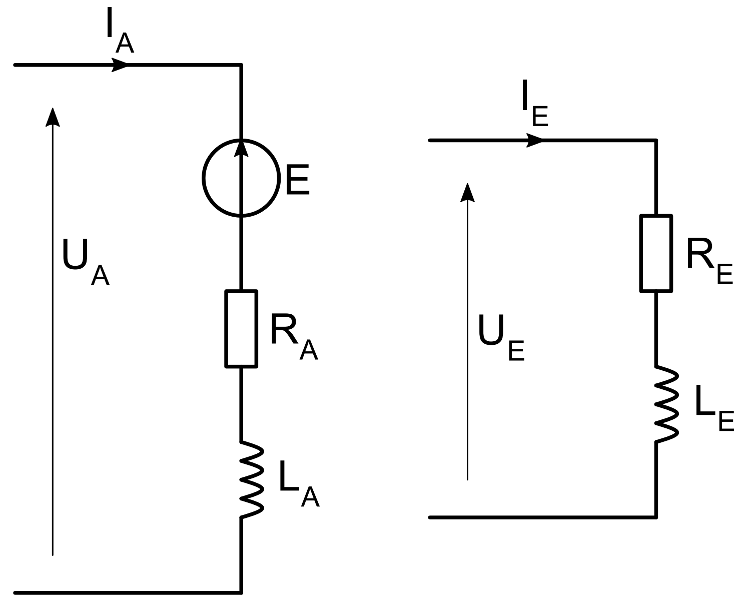


The DC motor

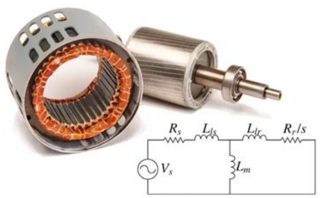
- Equivalent circuit of the DC motor:



Armature   Inductor



Armature   Inductor



The DC motor

- Equivalent circuit of the DC motor:

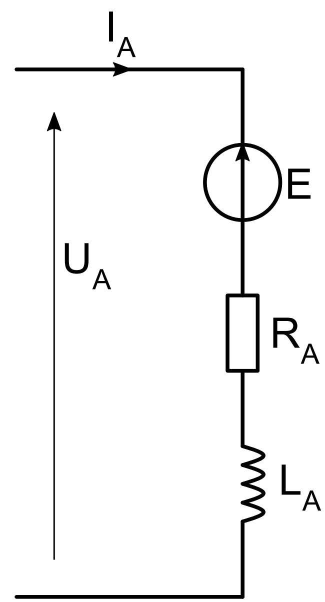
- Armature: Each active conductor is made of copper, has a length and a cross-section, and therefore has a resistance to current flow

=> The total armature resistance is called R_A

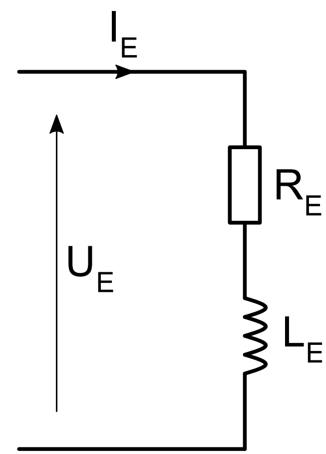
- The equivalent models of both armature and inductor windings actually consist of a resistor R and an inductance L with no steady-state effect

$\Rightarrow$ I_A = Cste and $L.dI_A/dt = 0$

$\Rightarrow$ L usually discarded for this reason



Armature

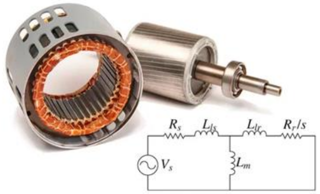


Inductor

- Expression of the voltage at the terminals of the motor armature:

$$U_A = E + R_A . I_A$$

- U_A: supply voltage in Volts
- E :.c.e.m.f in Volts
- R_A: armature resistance in Ohms
- I_A : armature current in Amperes



The DC motor

- Characteristic equations:

- Expression of rotation speed: $$n = \frac{E}{k.\Phi} = \frac{U_A - R.I_A}{k.\Phi} \cong \frac{U}{k.\Phi}$$

=> If the flux disappears, the motor goes into overdrive

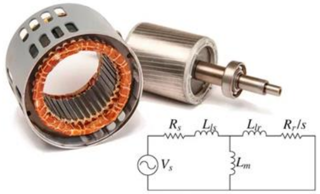
=> The speed is proportional to voltage if the flux is constant

$$n \cong \frac{U}{k_n.}$$

- Power absorbed by the armature: $$\mathrm{P_A = U_A.I_A}$$

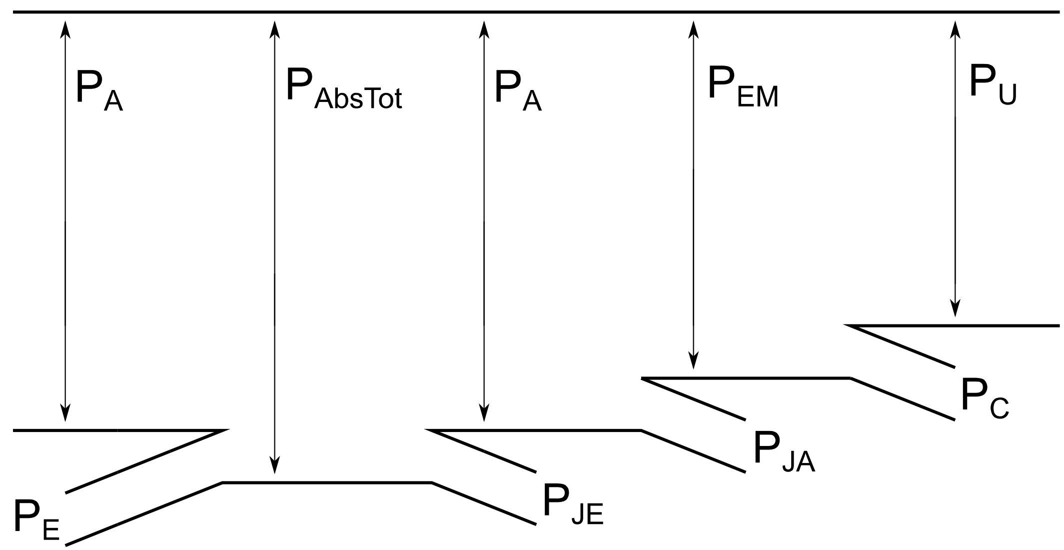
- Power absorbed by the inductor: $$\mathrm{P_E = U_E.I_E}$$

- Total electrical power supplied to the motor: $$\mathrm{P_{AbsTot} = U_A.I_A + U_E.I_E}$$



The DC motor

- Characteristic equations:



- Inductor Joule losses:

$$\mathbf{P_{JE}} = \mathbf{R}_E.\mathbf{I}_E{}^2$$

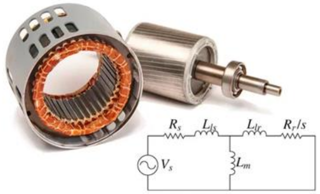
- Armature Joule losses:

$$\mathbf{P_{JA}} = \mathbf{R_A}.\mathbf{I_A}{}^2$$

- Useful electrical power / electromagnetic power: $\mathbf{P_{EM}} = \mathbf{E}.\mathbf{I_A} = \mathbf{U_A}.\mathbf{I_A} - \mathbf{P_{JA}}$

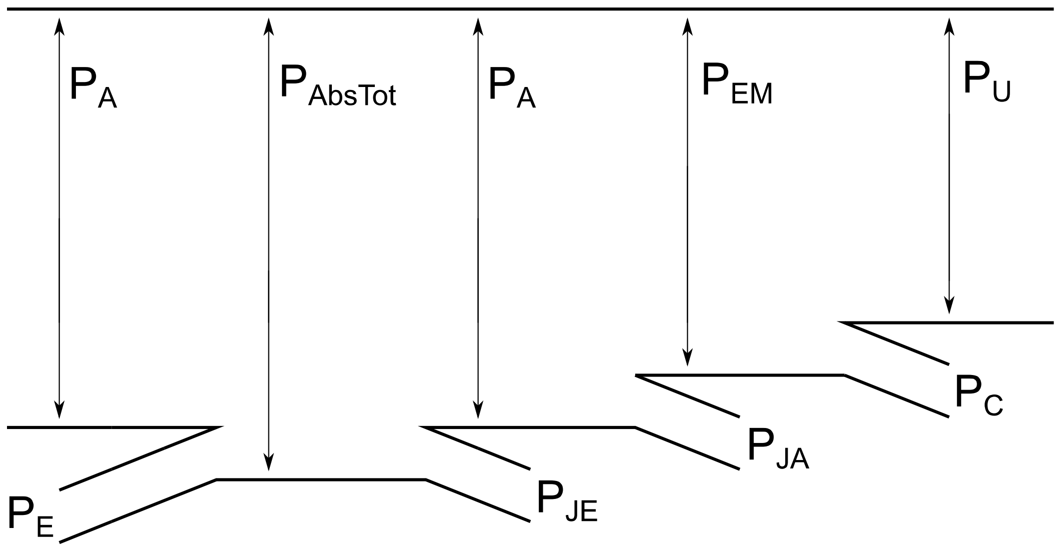
- Constant losses: iron losses + mechanical losses: $\mathbf{P_C} = \mathbf{P_f} + \mathbf{P_m} = \mathbf{P}_0 - \mathbf{R_A}.\mathbf{I_A}{}^2 = \mathbf{C_P}.\boldsymbol{\Omega}$

=> Can be represented by a loss torque (Cp)



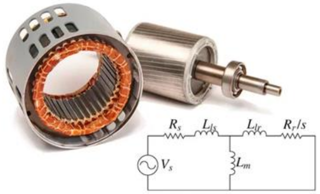
The DC motor

- Characteristic equations:



- Useful power: $\mathbf{P_U = U_A . I_A - P_{JA} - P_C = C_U . \Omega}$

- Efficiency: $\boldsymbol{\eta} = \dfrac{\mathbf{P_U}}{\mathbf{P_{AbsTot}}}$



The DC motor

- Expressions of torques:

- Electromagnetic torque:

$$C_{EM} = \frac{E.I_A}{\Omega} = \frac{E.I_A}{2.\pi.n}$$

$$C_{EM} = \frac{1}{2.\pi}.\frac{p}{a}.N.\Phi.I_A = k.\Phi.I_A$$

$$C_{EM} = k_C.I_A$$

=> If the flux is constant, C_{EM} is proportional to I_A (k_C: torque constant, in $N.m.A^{-1}$ (manufacturer's data))

- Loss torque:

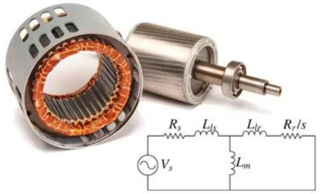
$$C_P = \frac{P_0 - R_A.I_{A0}^{\ 2}}{2.\pi.n}$$

=> Due to iron losses in the rotor (hysteresis + eddy currents) and to mechanical losses (bearing friction, brush friction on the collector). It can be estimated from a no-load test

- Useful torque:

$$C_U = C_{EM} - C_P = \frac{P_U}{2.\pi.n}$$

=> Also called driving torque



The DC motor:summary

- Mechanical equations:

- Fundamental principle of dynamics applied to a rotating solid: $$J\frac{d\Omega}{dt} = C_M - C_R$$

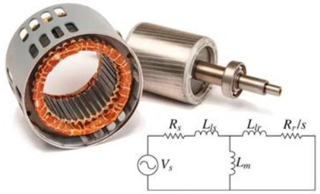
=> C_M is the driving torque, C_R the resistant torque and J the moment of inertia

- With the electromagnetic torque: $$\boldsymbol{C_{EM} = k.\Phi.I_A}$$

- Electrical equations:

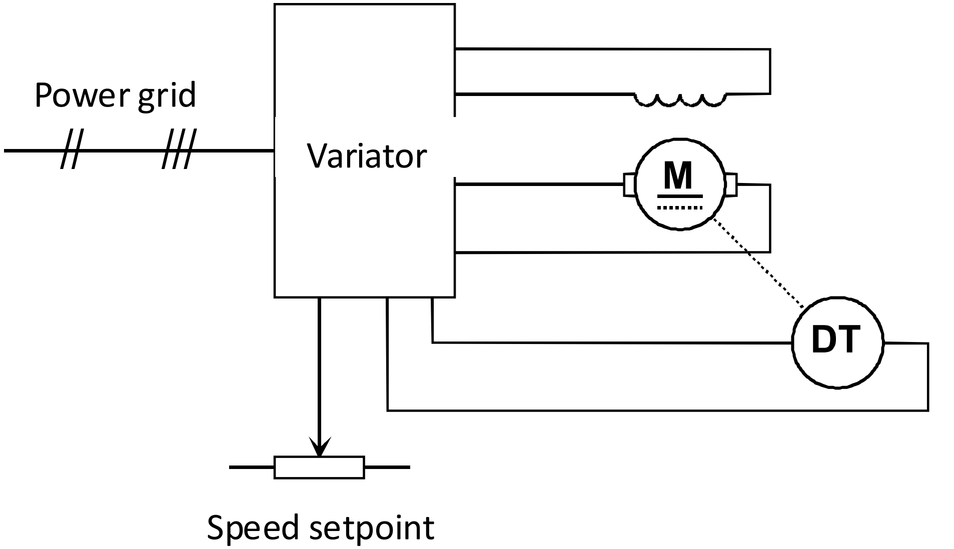
- Counter electromotive force: $$\boldsymbol{E = k.\Phi.\Omega}$$

- Voltage at the terminals of the armature: $$\boldsymbol{U - E = R.i_A + L.\frac{di_A}{dt}}$$ $$U = E + R.I_A$$



The DC motor

- Speed variation:



- Industrial variable speed drive consist of an all-thyristor, single-phase or three-phase, head-to-tail double bridge supplying the armature, enabling operation in all four quadrants, and a mixed bridge for the inductor.

=> Speed varied by playing on the armature voltage

- Small permanent-magnet motor use a switch-mode power supply.