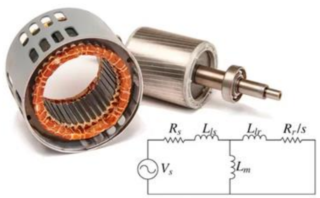


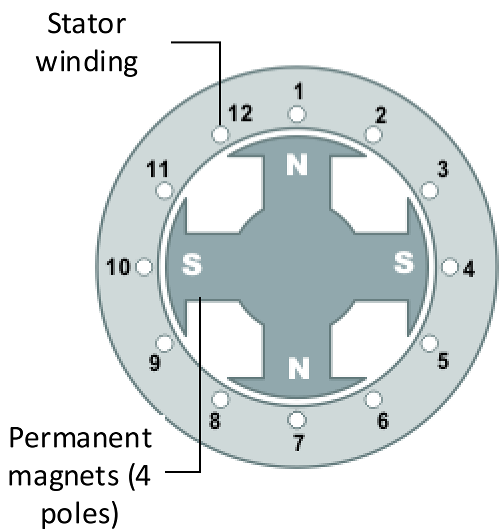
3-phase motors

- Synchronous machine: => limited to the study of the synchronous alternator
- Construction:

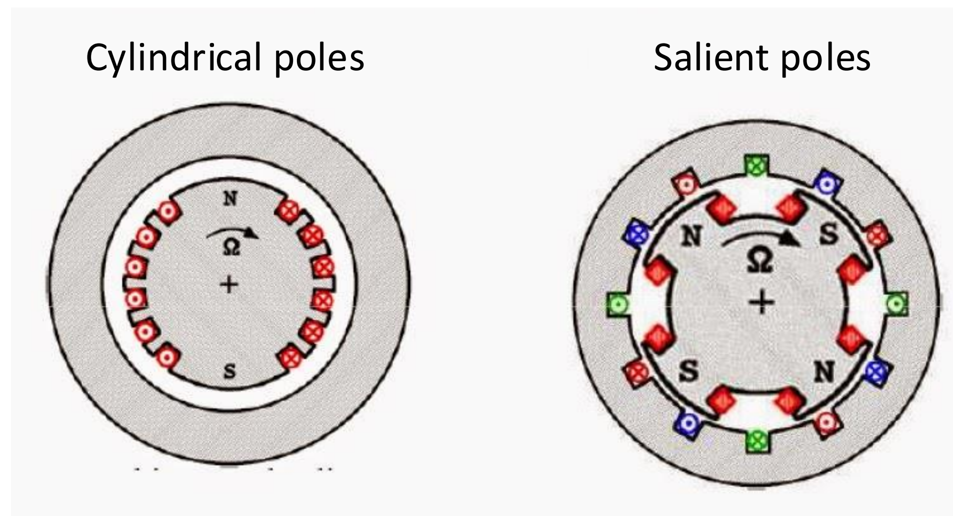
=> 3-phase stator generating a rotating magnetic field (see before)

=> DC rotor : different configurations

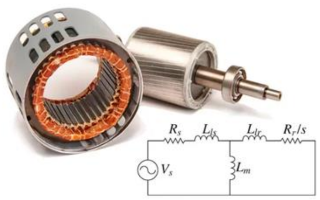
Permanent magnet rotor



Wound rotor (DC current)



Synchronous machine: The rotor rotates at the same speed as that of the rotation magnetic field



3-phase motors

- Synchronous machine:

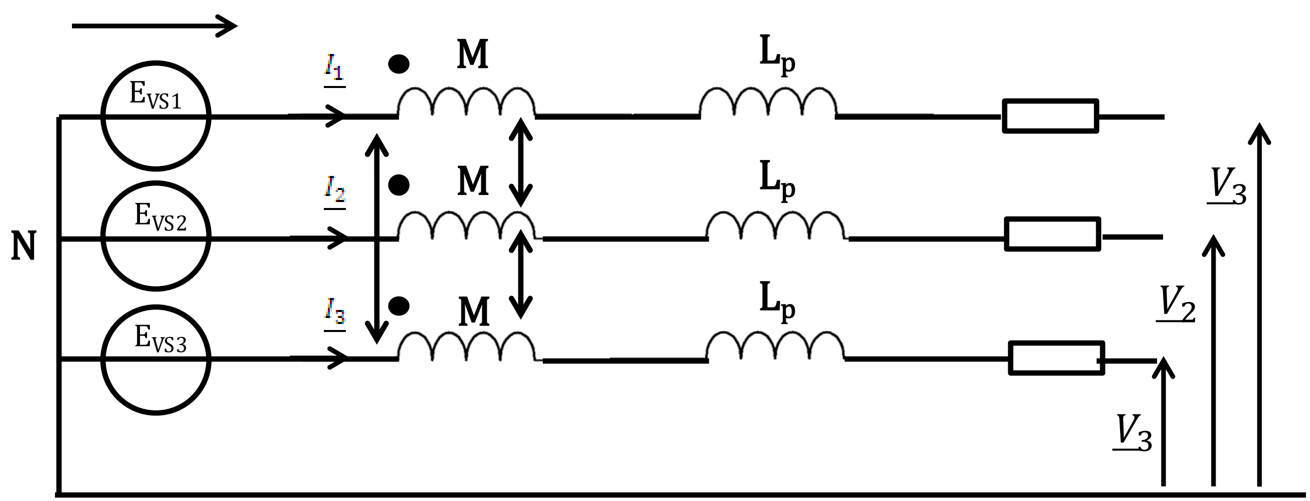
- Equivalent circuit in the linear domain:

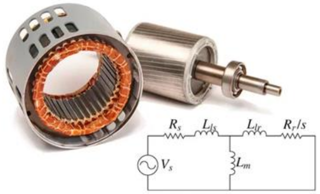
=> The armature circuit (stator for the alternator) can be represented, for each phase, by the series connection of a no-load emf, a resistor (winding), a self-inductance and a mutual inductance with the two other phases.

=> With the no-load electromotive force:

$$E_v = K\Omega_S I_e$$

- Ω_S: rotation speed (rad/s)
- I_e: rotor excitation current

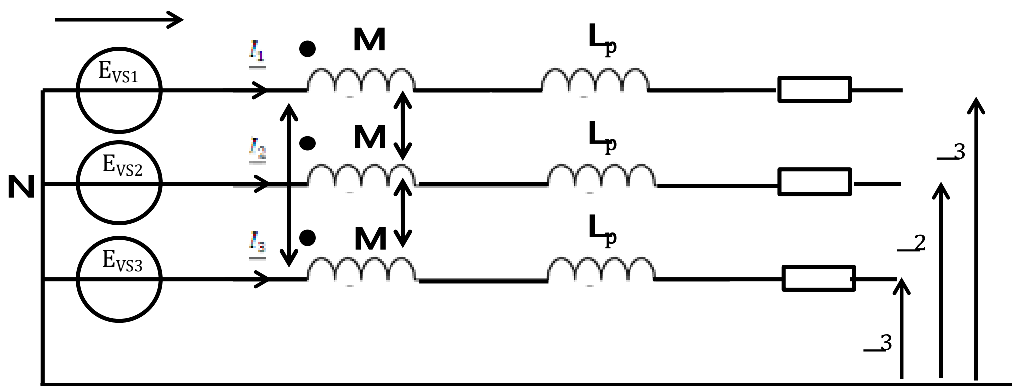




3-phase motors

- Synchronous machine:

- Equivalent circuit in the linear domain:

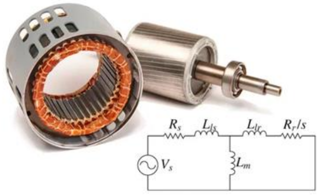


- With:

$$\underline{I_1} + \underline{I_2} + \underline{I_3} = 0$$

$$\begin{cases} \underline{E_{v1}} = j.M.\omega.\underline{I_2} + j.M.\omega.\underline{I_3} + j.L_p.\omega.\underline{I_1} + R.\underline{I_1} + \underline{V_1} \\ \underline{E_{v2}} = j.M.\omega.\underline{I_1} + j.M.\omega.\underline{I_3} + j.L_p.\omega.\underline{I_2} + R.\underline{I_2} + \underline{V_2} \\ \underline{E_{v3}} = j.M.\omega.\underline{I_1} + j.M.\omega.\underline{I_2} + j.L_p.\omega.\underline{I_3} + R.\underline{I_3} + \underline{V_3} \end{cases} \Rightarrow \begin{cases} \underline{E_{v1}} = j.(L_p - M).\omega.\underline{I_1} + R.\underline{I_1} + \underline{V_1} \\ \underline{E_{v2}} = j.(L_p - M).\omega.\underline{I_2} + R.\underline{I_2} + \underline{V_2} \\ \underline{E_{v3}} = j.(L_p - M).\omega.\underline{I_3} + R.\underline{I_3} + \underline{V_3} \end{cases}$$

With $L_s = L_p$-M the **cyclic inductance**, each phase can be decoupled

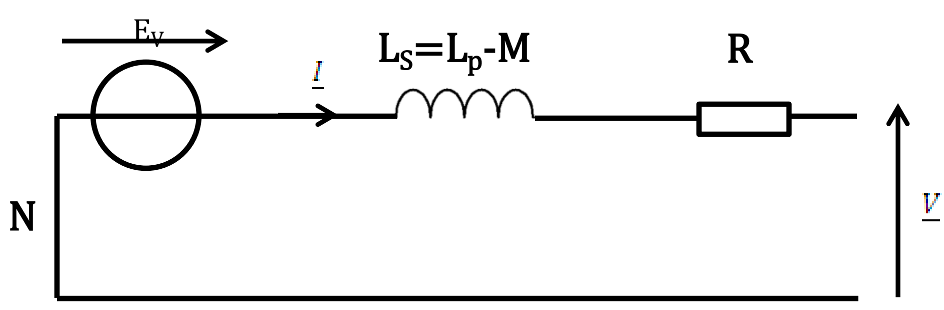


3-phase motors

- Synchronous machine:

- Each phase can be modelled with an equivalent single-phase circuit

=> Behn-Eschenburg's model



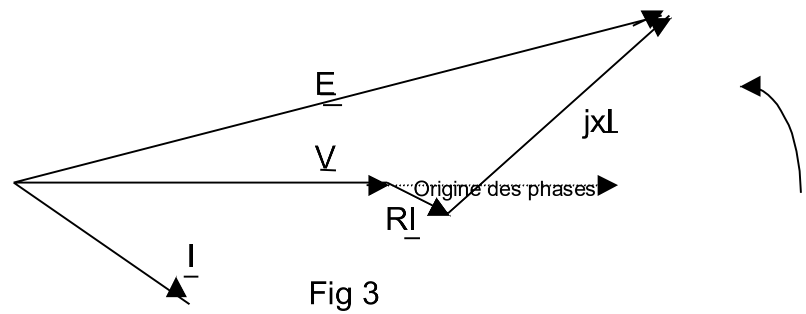
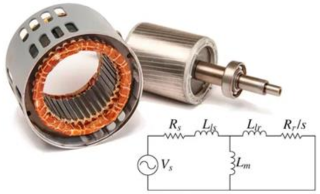


Fig 3

$$\underline{V} = \underline{E_v} - jL_S.\omega.\underline{I} - R.\underline{I}$$

- L_S: synchronous inductance
- X_S: synchronous reactance
- R: winding resistance
- E_v: no-load electromotive force



3-phase motors

- Synchronous machine:

- Determining the elements of the equivalent circuit diagram

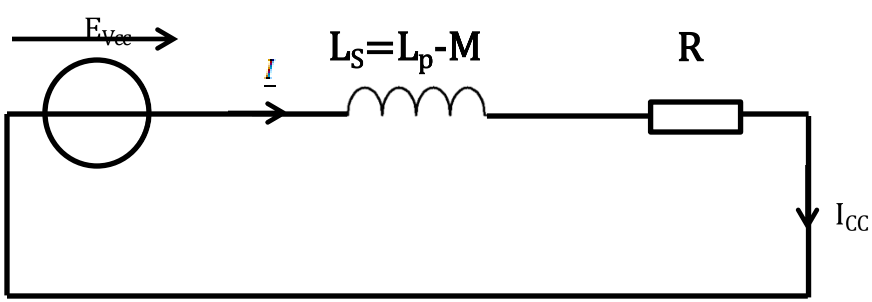
=> E_v is measured as a function of the excitation current I_e - $E_v(I_e)$: No-load characteristic of the SM

=> No-load test = the SM is driven at nominal speed by an auxiliary motor (e. g. a DC motor)

=> R is directly calculated from imposed current and voltage (voltamperemetric method)

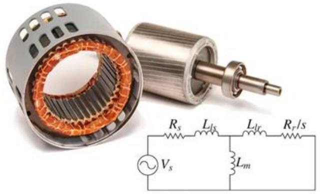
=> X_s is obtained from a short-circuit test performed at nominal speed with reduced excitation (I_e)

=> Plot of I_{cc} as a function of I_e



$$\underline{E_{vcc}} = jL_S.\omega.\underline{I_{cc}} - R.\underline{I_{cc}}$$

$$X_S = L_S\omega = \sqrt{\left(\frac{E_{vcc}}{I_{cc}}\right)^2 - R^2}$$

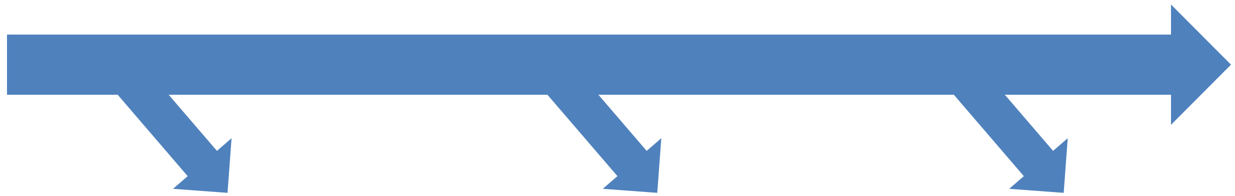


3-phase motors

- Synchronous machine:

- Power balance

P_{mec}



$P_U = 3VIcos(\varphi)$

P_m — Mechanical losses

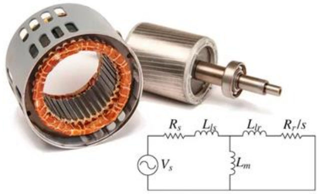
P_f — Iron losses

P_J — Joule losses

(Mechanical losses and iron losses:) Considered constant since they only depend on the rotation speed and the supply voltage

$$P_J = 3RI^2$$

- Therefore, the efficiency: $\eta = \dfrac{P_U}{P_U + P_m + P_f + P_J}$



3-phase motors

- Induction motor/ Asynchronous motor:

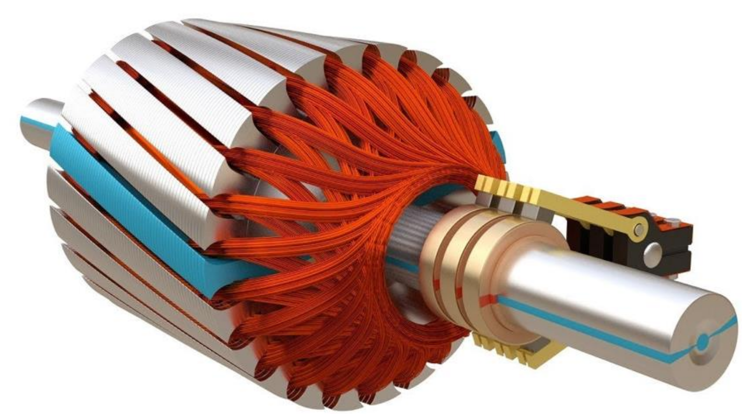
- Construction:

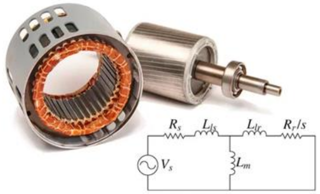
=> 3-phase stator generating a rotating magnetic field (see before)

=> 3-phase rotor : different configurations

Squirrel cage rotor

3-phase wound rotor



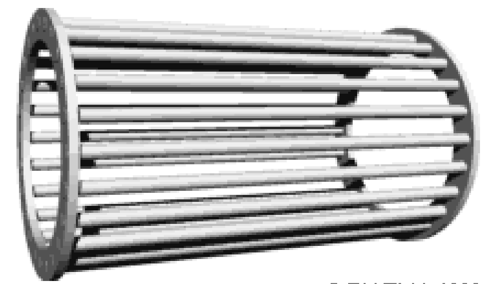


3-phase motors

- Induction motor/ Asynchronous motor:

- **Squirrel cage rotor**:

=> Aluminum conductors or bars are placed in the cylinder slots and short-circuited at each end

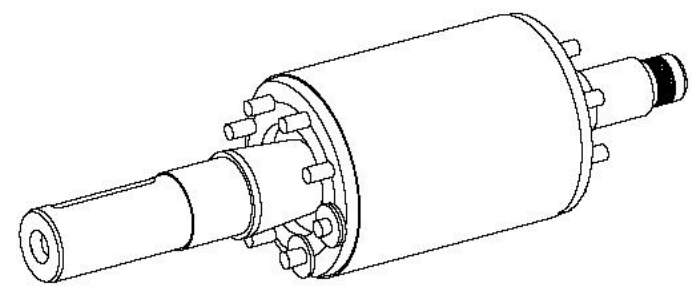


- **3-phase wound rotor** or **Slip-ring rotor**:

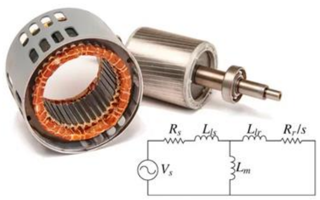
⇒ Windings identical to those of the stator are housed in the slots on the periphery of the rotor

⇒ The rotor is three-phase and wye-coupled (short-circuit)

⇒ The ends of the windings are connected to three copper rings, insulated and fixed to the rotor



Short-cut necessary at the rotor to generate induced currents



3-phase motors

- Induction motor/ Asynchronous motor:

- Working principle:

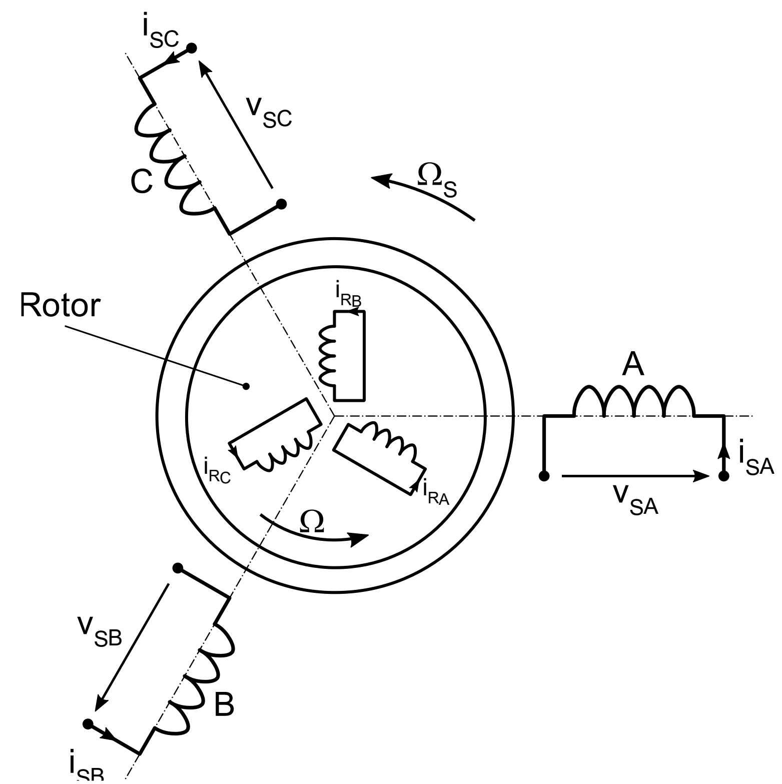
=> The 3-phase stator generates a rotating magnetic field

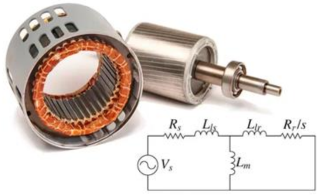
=> The rotating field induces eddy currents in the rotor windings

=> According to Lenz's law, these currents oppose the cause that gave rise to them and generate a magneto-motive force that makes the rotor rotate

=> Therefore, the rotor moves with the stator field at speed Ω_R tending towards Ω_S, but never reaches it

$$\Omega_R < \Omega_S$$





3-phase motors

- Induction motor/ Asynchronous motor:

- The slip g:

=> The rotor slip with respect to the stator field is defined as the relative deviation of the rotor rotation speed from synchronous speed

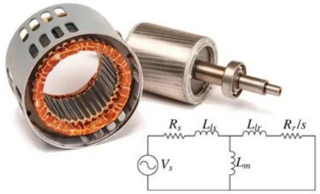
$$g = \frac{\Omega_S - \Omega_R}{\Omega_S}$$

=> At synchronous speed: $\Omega_R = \Omega_S, g = 0$

=> O induced current => deceleration of rotor

=> When stopped, and at starting-up: $\Omega_R = 0, g = 1$

=> Generator operation: $\Omega_R > \Omega_S, g < 0$

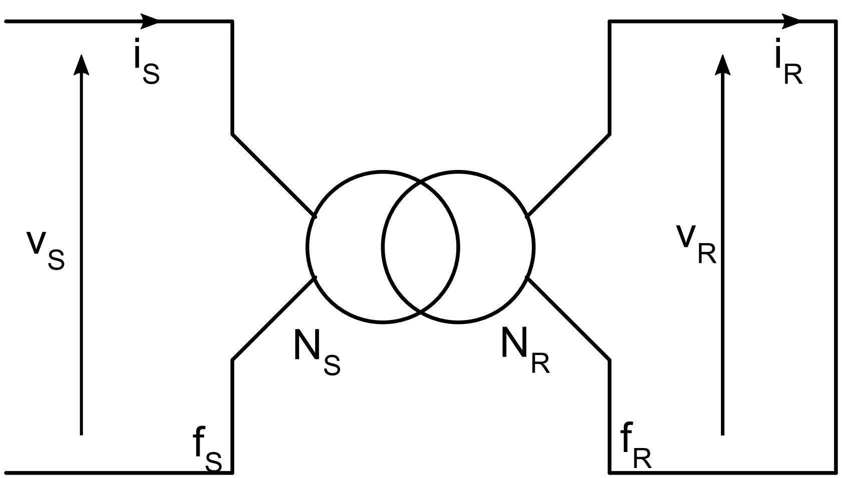


3-phase motors

- Induction motor/ Asynchronous motor:
- Equivalent circuit diagram of the induction motor

=> Ideal model: the perfect Asynchronous motor

=> Very similar to the transformer with short-circuited secondary side



V_S: RMS voltage across the stator winding

I_S: RMS current flowing in the stator winding

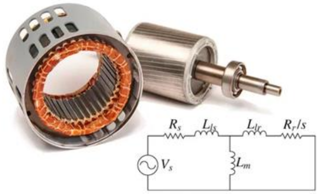
V_R: RMS voltage across the rotor winding

I_R: RMS current flowing in the rotor winding

f_S : frequency of stator or grid

f_R : frequency of rotor currents

m: number-of-turns ratio between rotor and stator: $m=N_R/N_S$



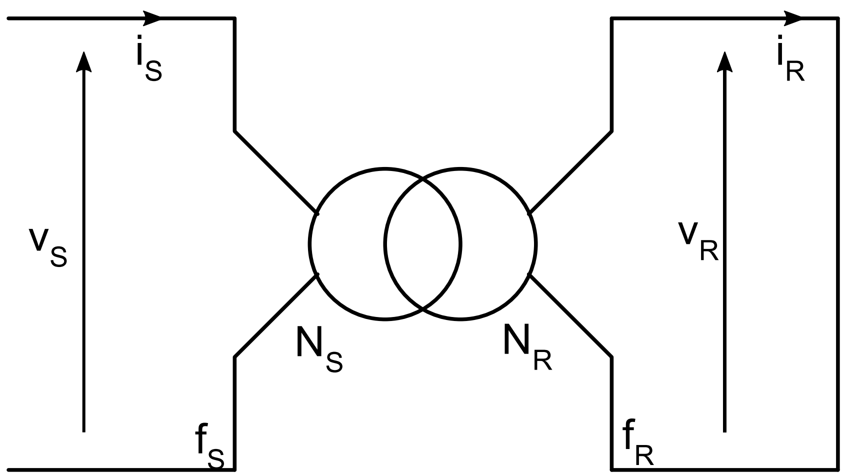
3-phase motors

- Induction motor/ Asynchronous motor:

- Equivalent circuit diagram of the induction motor for each phase

=> Ideal model: the perfect Asynchronous motor

- Frequency of currents induced at the rotor: f_R



=> Frequency transformation ratio

- At the stator, frequency of the grid f_S:

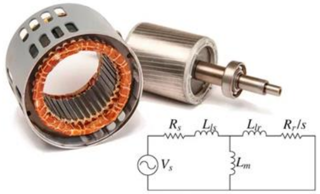
$$f_S = \frac{\Omega_S}{2\pi} \; for \; p = 1$$

$$g = \frac{\Omega_S - \Omega_R}{\Omega_S}$$

$$g\Omega_S = \Omega_S - \Omega_R$$

$$f_R = \frac{\Omega_S - \Omega_R}{2\pi} = \frac{g\Omega_S}{2\pi}$$

$$\boldsymbol{f_R = g f_S}$$



3-phase motors

- Induction motor/ Asynchronous motor:

- Equivalent circuit diagram of the induction motor

=> Ideal model: the perfect Asynchronous motor

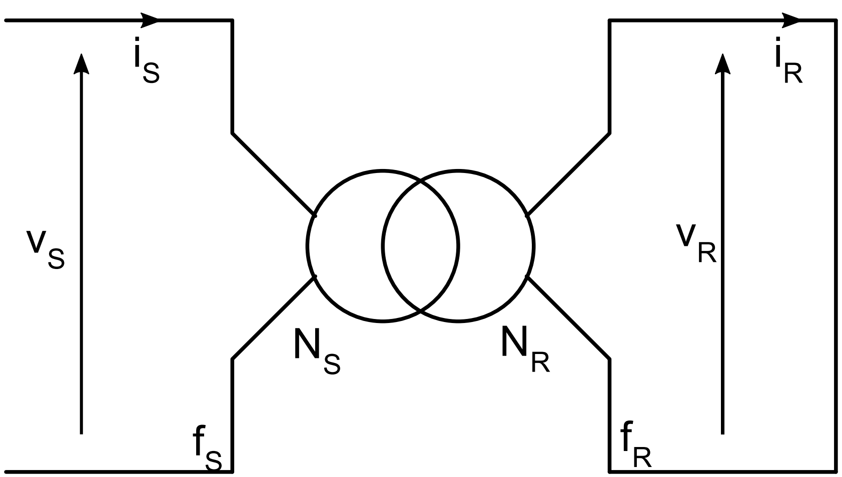
$$V_R = 4{,}44.S.N_R.\hat{B}.f_R$$

$$V_S = 4{,}44.S.N_S.\hat{B}.f_S$$

- Voltage transformation ratio (from Boucherot's formula)

$$\frac{V_R}{V_S} = \frac{N_R}{N_S}.\frac{f_R}{f_S} = m.g$$

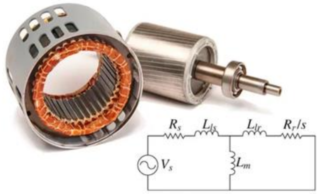
$$\boldsymbol{V_R = m.g.V_S}$$



- Current transformation ratio

$$N_R.I_R - N_S.I_S = 0$$

$$\boldsymbol{\frac{I_R}{I_S} = \frac{1}{m} = \frac{N_S}{N_R}}$$

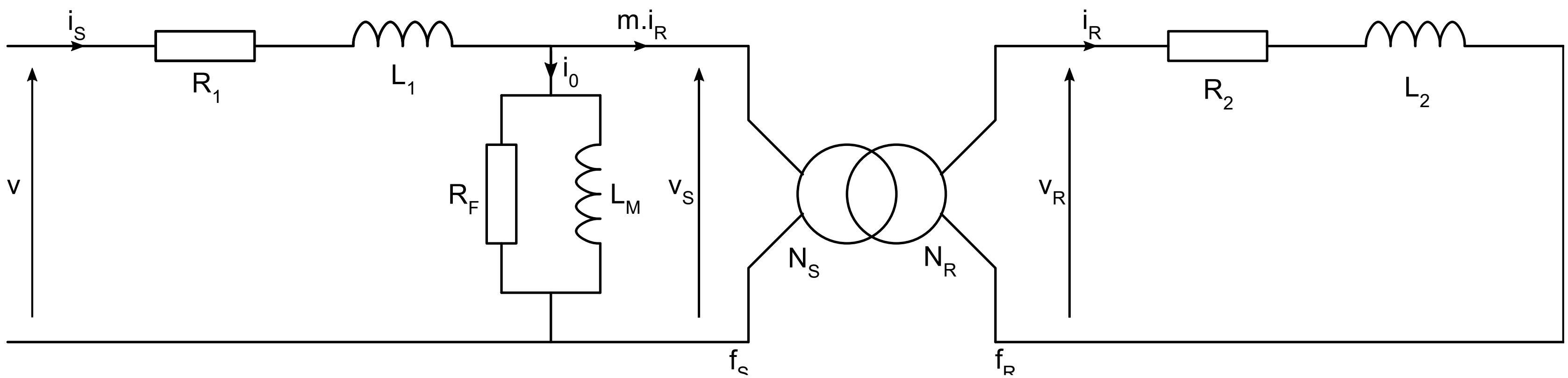


3-phase motors

- Induction motor/ Asynchronous motor:

- Equivalent circuit diagram of the induction motor

=> Intermediate model (from the transformer)



R_1: resistance of stator conductors

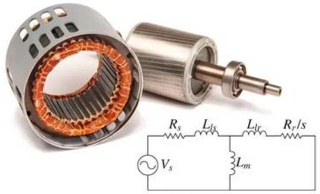
L_1: stator winding leakage inductance

R_2: resistance of rotor conductors

L_2: rotor winding leakage inductance

R_F: resistance modelling iron losses

L_M: magnetizing inductance of the magnetic circuit

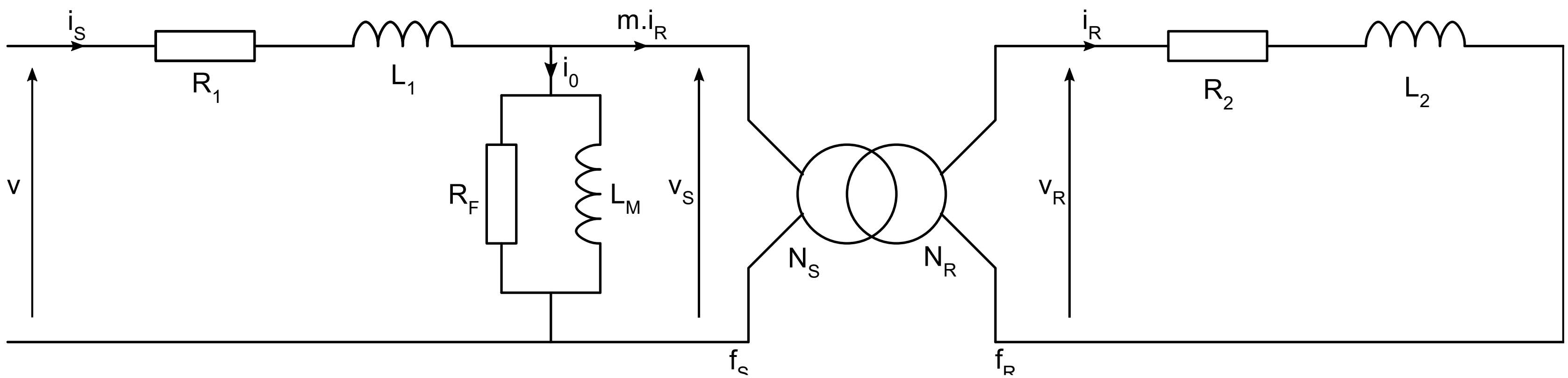


3-phase motors

- Induction motor/ Asynchronous motor:

- Equivalent circuit diagram of the induction motor

  => Intermediate model (from the transformer)



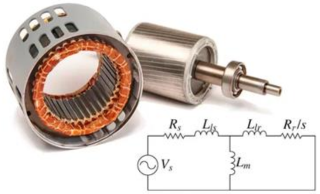
- Voltages at the stator

$$\underline{V} = R_1.\underline{I_S} + j.L_1.\omega_S.\underline{I_S} + \underline{V_S}$$

- Voltages at the rotor

$$\underline{V_R} - R_2.\underline{I_R} - j.L_2.g.\omega_S.\underline{I_R} = 0$$

$$m.\underline{V_S} - \frac{R_2}{g}.\underline{I_R} - j.L_2.\omega_S.\underline{I_R} = 0$$

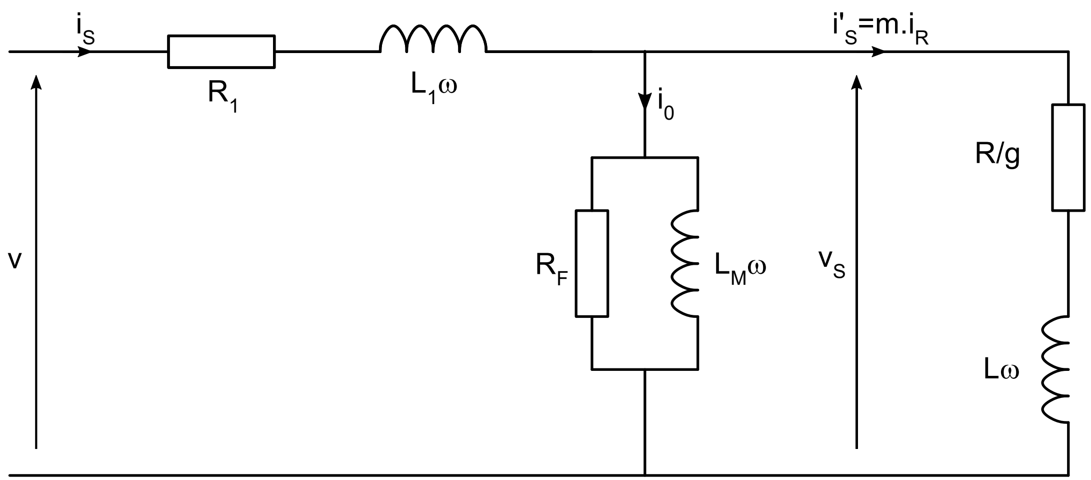


3-phase motors

- Induction motor/ Asynchronous motor:

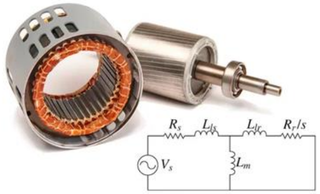
- Equivalent circuit diagram of the induction motor

=> The real model (equivalent primary model)



- Impedance transfer from rotor to stator: $$m.\underline{V_S} - \frac{R_2}{g}.\underline{I_R} - j.L_2.\omega_S.\underline{I_R} = 0$$

$$L = \frac{L_2}{m^2} \qquad \frac{R}{g} = \frac{R_2}{g.m^2} \qquad \frac{I_R}{I_S} = \frac{1}{m}$$



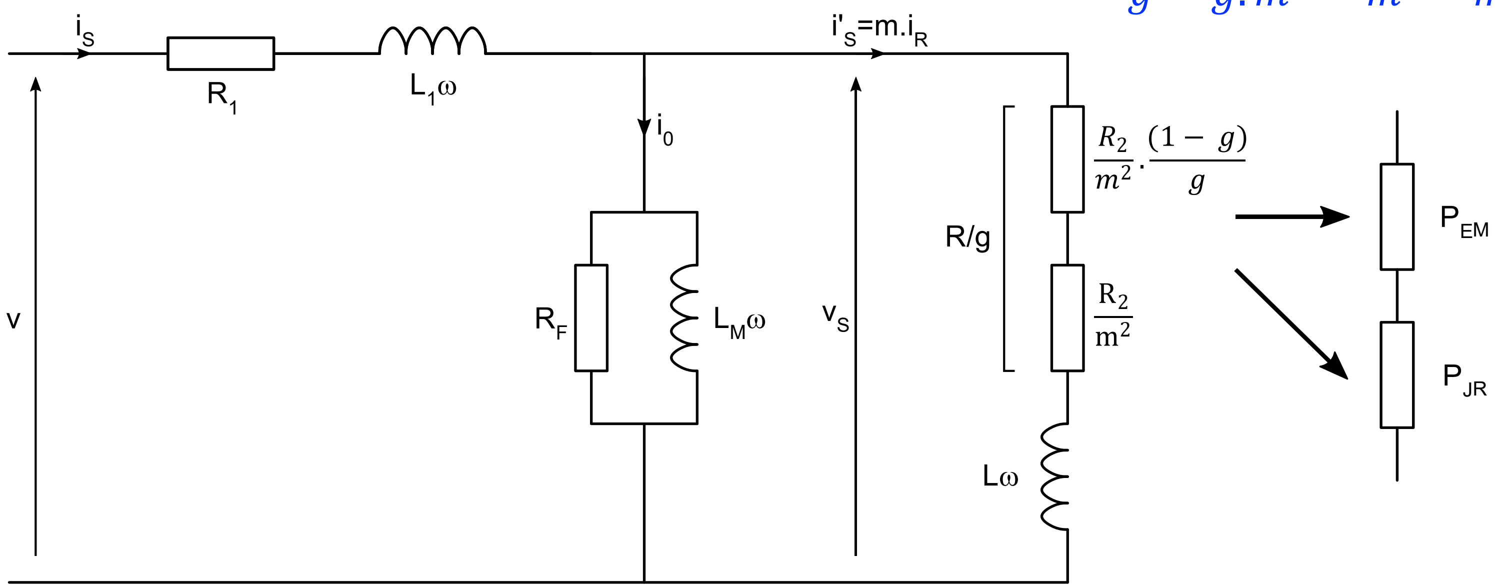
3-phase motors

- Induction motor/ Asynchronous motor:

- Equivalent circuit diagram of the induction motor

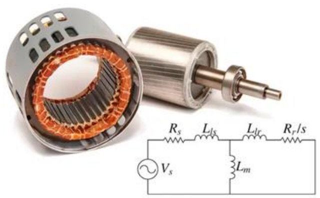
=> The real model (equivalent primary model)

$$\frac{R}{g} = \frac{R_2}{g.m^2} = \frac{R_2}{m^2} + \frac{R_2}{m^2}.\frac{(1-g)}{g}$$



$\frac{R_2}{m^2}$ = related to Joule losses at the rotor

$\frac{R_2}{m^2}.\frac{(1-g)}{g}$= related to the power transferred to the rotor

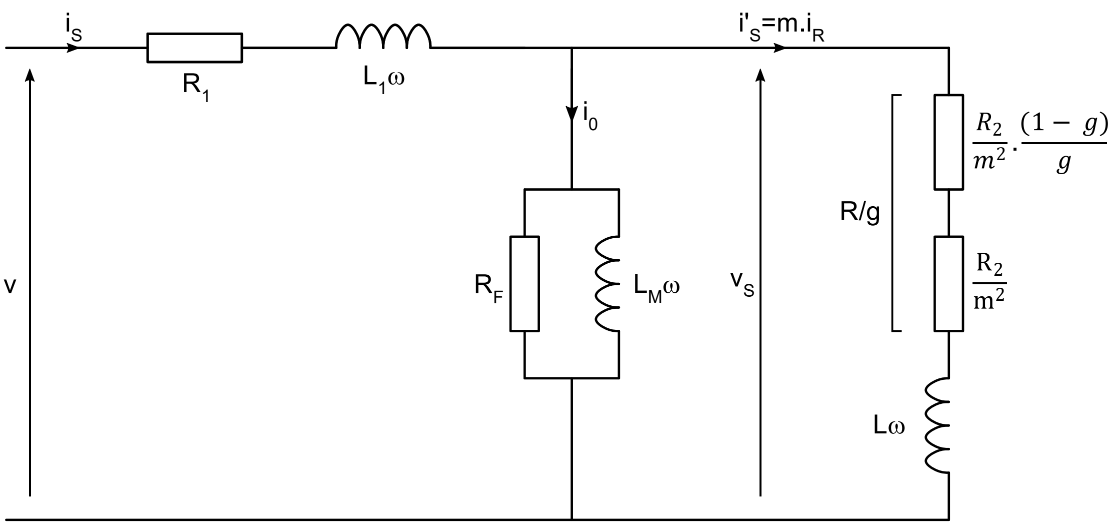


3-phase motors

- Induction motor/ Asynchronous motor:

- Equivalent circuit diagram of the induction motor

=> The real model (equivalent primary model)



R_F: resistance modeling iron losses

L_M: magnetizing inductance of the magnetic circuit

Remark : inductances are cyclic inductances

R_1: resistance of stator conductors/ Joule losses

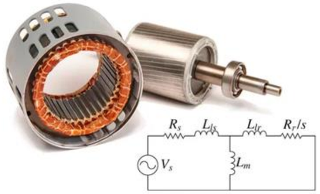
L_1: stator winding leakage inductance

R/g: resistance of rotor conductors/motional resistance

R_2/m^2: resistance of rotor conductors/ Joule losses

R_2/m^2 ((1-g)/g) resistance modelling the electromagnetic power transferred to the rotor

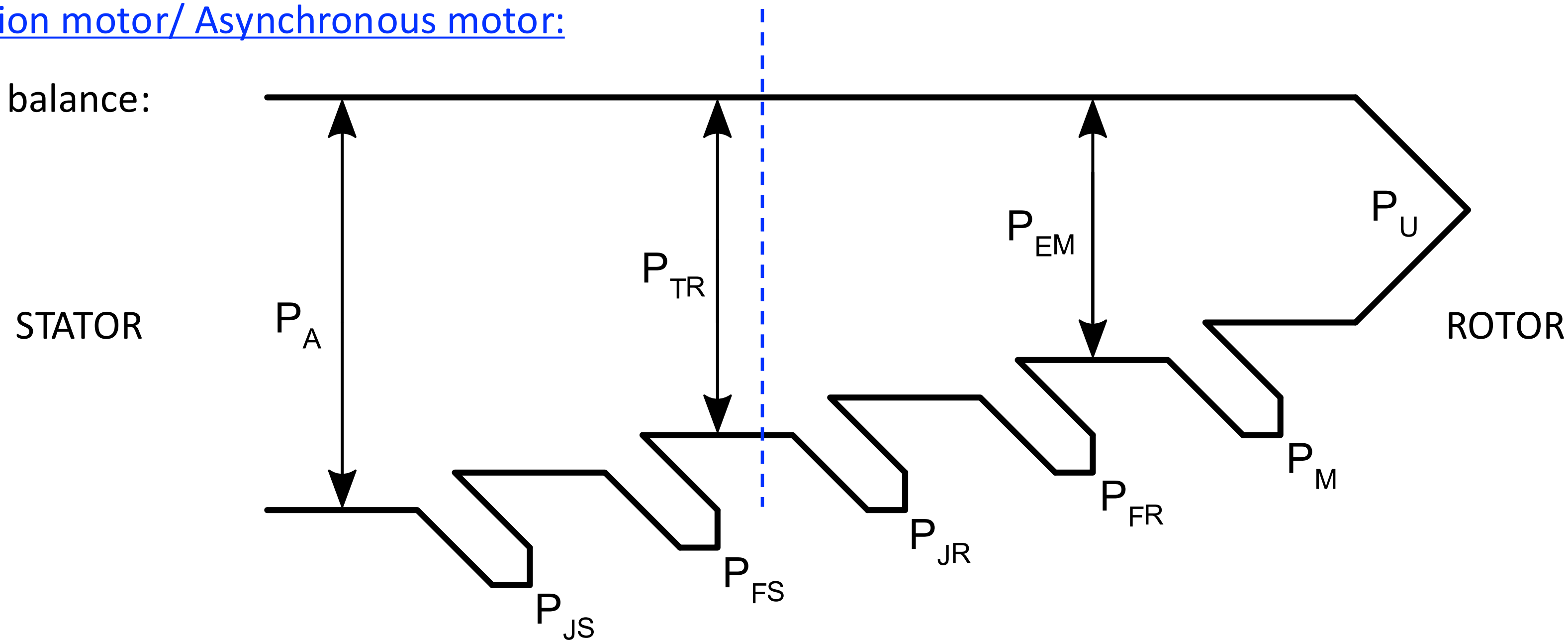
L: rotor winding leakage inductance



3-phase motors

- Induction motor/ Asynchronous motor:

- Power balance:



P$_a$: Active power absorbed at the stator

$$P_a = 3V_S I_S cos\varphi_S$$

P$_U$: Useful power (mechanical)

P$_{JS}$: Joule losses at the stator

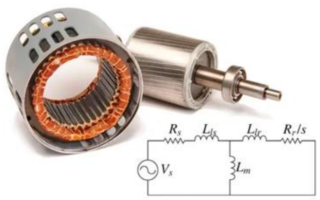
$$P_{JS} = 3R_1 I_S^2$$

P$_{FS}$: Iron losses at the stator

$$P_{FS} = 3.\frac{{V_S}^2}{R_F}$$

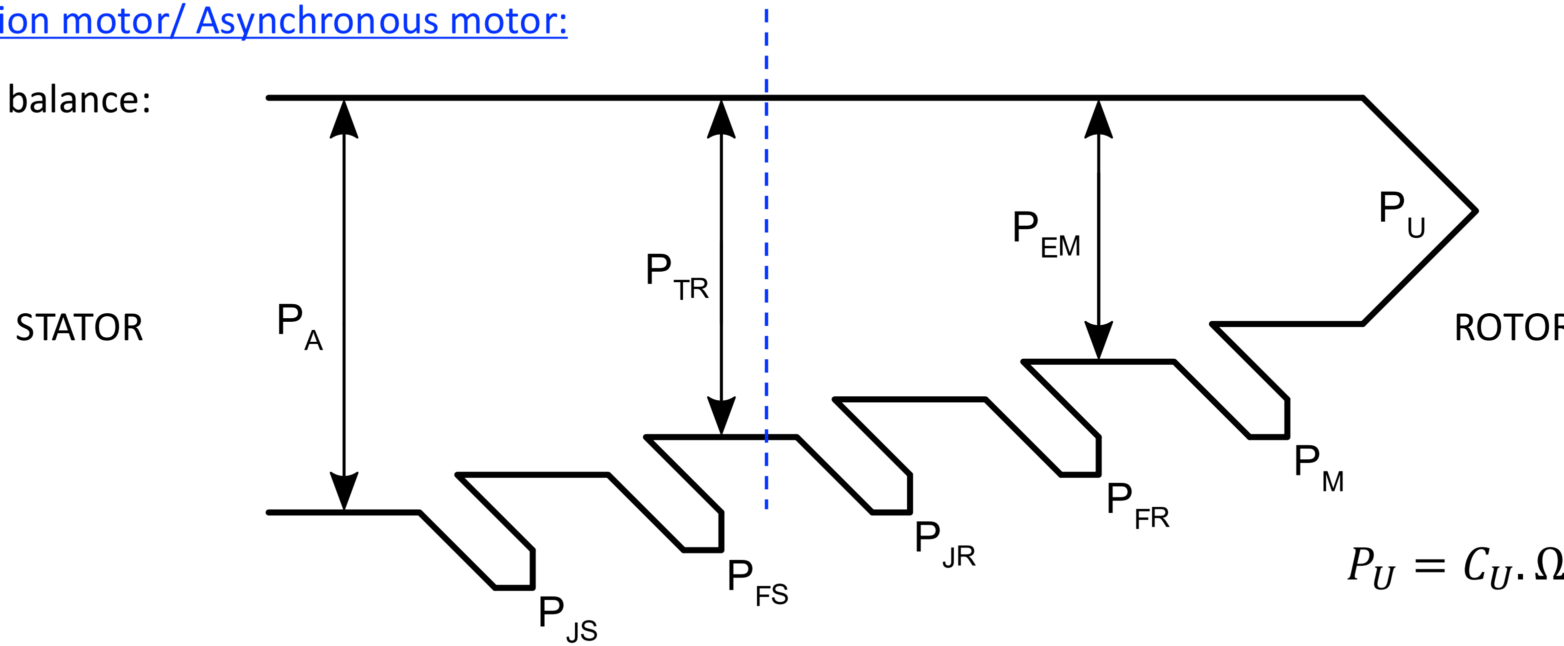
P$_{TR}$: Power transmitted to the rotor

$$P_{TR} = C_{EM}.\Omega_S$$



3-phase motors

- Induction motor/ Asynchronous motor:
- Power balance:



$$P_U = C_U.\Omega_R$$

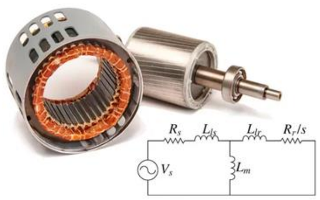
P_{JR}: Joule losses at the rotor $P_{JR} = 3.R_2.{I_R}^2$

P_{FR}: Iron losses at the stator (almost 0 usually, low f_R)

$$\eta = \frac{P_U}{P_U + P_{JS} + P_{FS} + P_{JR} + P_{FR} + p_M}$$

P_{EM}: Electromagnetic power $P_{EM} = C_{EM}.\Omega_R$

p_M: Mechanical losses (bearing losses, aerodynamic friction of the fan…)



3-phase motors

- Induction motor/ Asynchronous motor:

- Determining the elements of the equivalent circuit diagram: **Test at g = 0**

=> Test at nominal voltage

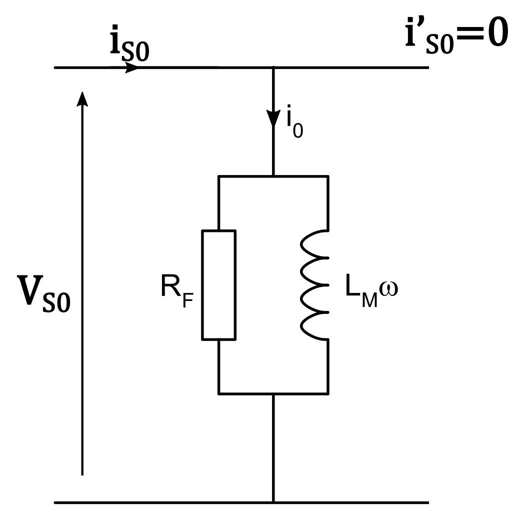
=> The synchronous machine is driven at synchronous speed (Ω_S) by an auxiliary motor (g=0)

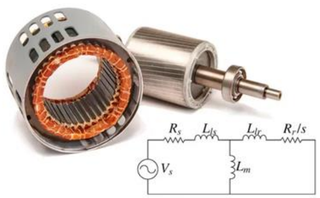
=> Under such conditions, $P_{EM} = 0$

- Measured quantities: V_{S0}, I_{S0}, P_{S0}, Q_{S0}

$$P_{S0} = P_{JS0} + P_{FS0} \cong P_{FS}$$

$$R_F = \frac{3V_{S0}^2}{P_{S0}} \qquad L_m\omega = \frac{3V_{S0}^2}{Q_{S0}}$$





3-phase motors

- Induction motor/ Asynchronous motor:

- Determining the elements of the equivalent circuit diagram: **Test at g = 1**

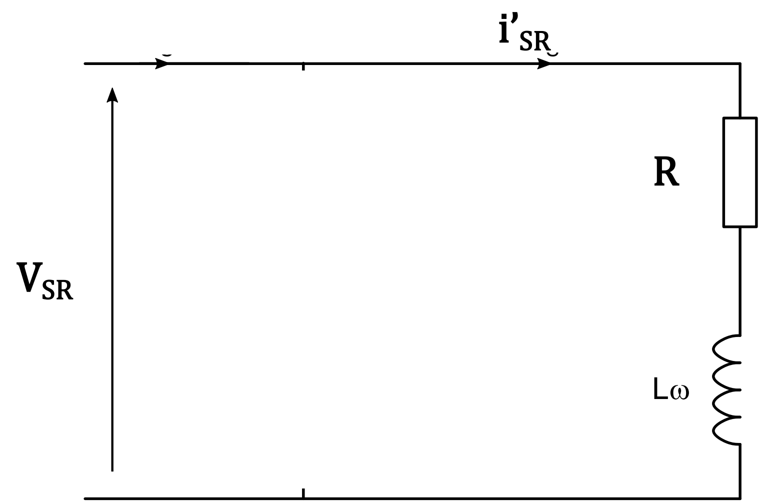
=> Test at reduced voltage, close to the nominal current

=> A brake blocks the rotor (g=1, (Ω_R=0)

=> Iron losses are assumed to be negligible – Powers are all consumed at the rotor

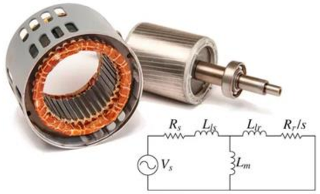
- Measured quantities: V_{SR}, I_{SR}, P_{SR}, Q_{SR}

$$R = \frac{P_{SR}}{3I_{SR}^2} - R_1 \qquad L\omega = \frac{Q_{SR}}{3I_{SR}^2}$$



- R_1 is measured by volt-amperemetric at the stator

=> Hot, continuous and nominal voltage and current

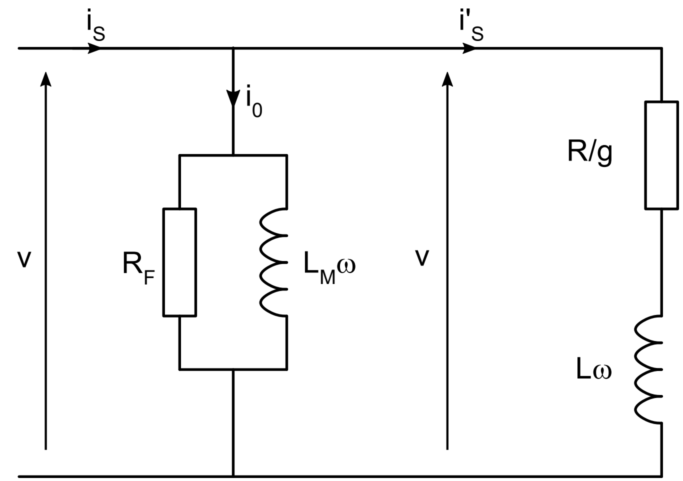


3-phase motors

- Induction motor/ Asynchronous motor:

- Expression of the electromagnetic torque

- R_1 and L_1 are neglected at $V = V_S$

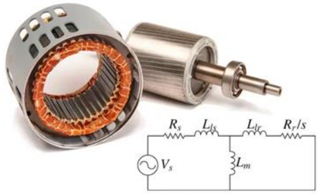


$$I'_S = \frac{V}{\sqrt{\left(\frac{R}{g}\right)^2 + (L.\omega)^2}}$$

$$P_{TR} = 3.\frac{R}{g}.I'^2_S$$

$$P_{TR} = C_{EM}.\Omega_S$$

$$C_{EM} = 3.\frac{\boldsymbol{p}}{\boldsymbol{\omega}}.\frac{\boldsymbol{V^2.R}}{\frac{\boldsymbol{R^2}}{\boldsymbol{g}} + \boldsymbol{g}.(\boldsymbol{L}.\boldsymbol{\omega})^2}$$



3-phase motors

- Induction motor/ Asynchronous motor:

- Electromagnetic torque

$$C_{EM} = 3.\frac{p}{\omega}.\frac{V^2.R}{\frac{R^2}{g} + g.(L.\omega)^2}$$

$$C_{MAX} = k.\frac{V^2}{f^2} \qquad g_{max} = \frac{R}{L.\omega}$$

$$C_{DEM} = 3.\frac{p}{\omega}.\frac{V^2.R}{R^2 + (L.\omega)^2}$$