How Math Can Save Democracy

INSTITUTE FOR MATHEMATICS AND DEMOCRACY

1. Consider the following voting methods suggested by the Institute of Mathematics for Democracy and match the definition with the relevant example.

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WHAT IS *Plurality?*

How it works: The candidate who gets the most votes wins. This method is used in most US elections.

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WHAT IS *Ranked Choice Voting?*

How it works: Voters rank all candidates in order of preference.

If there is a candidate with the majority of first-place votes, they are the winner. Otherwise the candidate with the fewest first-place votes is eliminated. The votes of those who had the eliminated candidate as their first choice are transferred to the next candidate in their rankings, and all the votes are counted again. This process is repeated until there is a majority winner.

Ranked choice voting is currently used in Maine and many cities/counties across the United States.

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WHAT IS *Approval Voting?*

How it works: Voters check any number of candidates they would be okay with being elected. The candidate with the most cumulative votes wins.

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WHAT IS *the Condorcet Method?*

How it works: This is a voting method where each voter ranks all the candidates.

To calculate a winner, we look at one-on-one contests of each candidate: if more people ranked one candidate above the other, that candidate wins that pairwise contest.

The candidate who beats all the other candidates wins.

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WHAT IS *Borda Count?*

How it works: This is a voting method where each voter ranks all the candidates. In an election with n candidates, a candidate gets n-1 points for each first-place vote, n-2 points for each second-place vote, and so on. Last place votes get 0 points. When all the points are added, the candidate with the most points wins.

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WHAT IS *Strategic Voting?*

Strategic voting occurs when a voter's ballot does not match their true preference, often because they do not think their first choice candidate will win.

Examples :

For example, say a family is choosing a flavor of ice cream using plurality. John and Sally love mint ice cream, but they hate vanilla. If they vote honestly, the votes look like:

vanilla - 4
chocolate - 3
mint - 2

If John and Sally vote dishonestly for chocolate, a more popular candidate, they won't get their first choice but they won't get their last choice either:

chocolate - 5
vanilla - 4
mint - 0

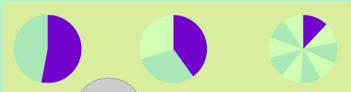
1

Ballot: 1 2 3
B C A
A A C
C B B

Points for **A**: 2x1 + 1x2 = 4
Points for **B**: 2x1 + 1x0 = 2
Points for **C**: 2x1 + 1x1 = 3

A wins!

2



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Ballot: 1 2 3
C B A
A A C
B C B

A vs **B**: **A** **B** vs **C**: **B** **A** vs **C**: **A**

A is the winner!

4

Ballot: 1 2 3 4 5
A A B B C
B C A C B
C B C A A

C is eliminated

Ballot: 1 2 3 4 5
A A B B B
B B A A A

Candidate B wins

6

For example: A family is voting on what to have for dinner:

- Pizza
- Burgers
- Soup

This voter is okay with either pizza or soup for dinner.

Candidates - Votes
- **Pizza** - 4
- Burgers - 1
- Soup - 2

The family will have pizza for dinner!

5

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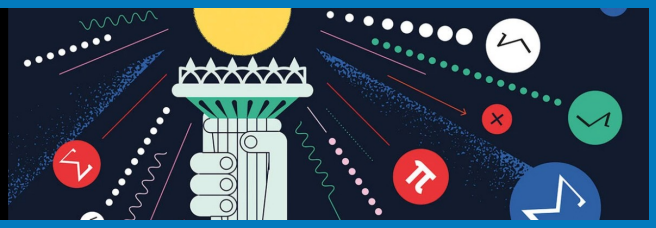

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2. Pay closer attention to "Plurality," "Ranked Choice Voting" (also known as "Instant Runoff") and "Condorcet Method." What could be their negative consequences or drawbacks?

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3. According to you, which would be the best method? Why?

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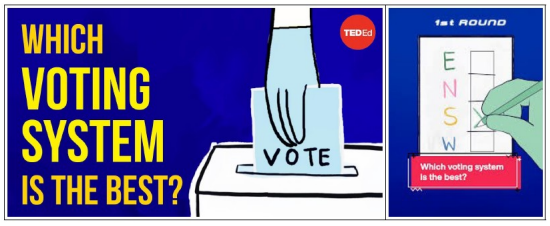
4. Assess the flaws of the different voting methods thanks to Alex Gendler's video "Which voting system is the best?" (https://www.youtube.com/watch?v=PaxVCsnox_4 00:00-05:05).



Plurality:.....
.....
Instant Runoff:.....
.....
Multiple Rounds:.....
.....
Condorcet Method:.....
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5. The very end of the video alludes to "Arrow's Impossibility theorem." What could it consist in?

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6. Would you think about another social issue math could or should help redress?

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7. Solve these two voting problems.

In every round of a certain game show, v votes are cast by the public to decide which contestants out of c contestants continue to the next round. The contestant with the lowest amount of votes in every round is eliminated. The next round proceeds with $c-1$ contestants, and so on. What is the minimum number of votes needed to guarantee that a contestant will proceed to the next round, assuming that he/she does not forfeit?

- c is updated at the start of every round to represent the number of **remaining** contestants.
- v may vary with each round.
- Every round, one contestant must be eliminated by voting, forfeit, or tiebreaker.
- $c \geq 2$.

- $\frac{v}{2}$
- $\frac{v}{c}$
- $\frac{v}{2}+1$
- $\frac{v}{c}+1$

Answer :

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WHAT IS *the best ranked voting method?*

So far, we've examined several ordinal voting methods:

- **Plurality**
- **Instant runoff**
- **Borda count**
- **Condorcet method**

Instant runoff, Borda count, and Condorcet method are all better than plurality, but which is the **fairest**? That depends on how we define "fair" and what aspects of an election we deem important.

Consider the following ranked choice profile:

Ballots:						
# of voters:	36	24	20	18	8	4
	A	B	C	D	E	E
	D	E	B	C	B	C
	E	D	E	E	D	D
	C	C	D	B	C	B
	B	A	A	A	A	A

Calculate the winner using each of the following methods: plurality, instant runoff, Borda count, and Condorcet method.

Answer :