Morgane AUGRIS

Présentation du cours

Présentation du cours

- Déroulé du TD de L3 S6 L.V. Anglais
- Bibliographie
- Modalités de contrôle des connaissances
- Introduction

Présentation du cours

- **Déroulé du TD de L3 L.V. Anglais**
  - Contact : morgane.augris@univ-orleans.fr
  - Descriptif du cours

Absences

- La présence au TD est obligatoire.
- Appel toutes les semaines.
- En cas d'absence justifiée, vous disposez de 5 jours pour transmettre le motif ou le certificat.
- À partir de 20 % d'absence au TD, vous serez considéré comme ABI.

UNIVERSITÉ D'ORLÉANS

Orléans / Bourges / Blois / Chartres / Châteauroux / Issoudun / Tours Fondettes

Bilan des sanctions prononcées par la commission de discipline

Sébastien Ringuedé
VP CFVU

Novembre 2025

ATHENA
EUROPEAN UNIVERSITY

www.univ-orleans.fr

Rapide bilan

9 étudiants ont été convoqués en commission de discipline pour faux et usage de faux certificats médicaux

Sanction maximum : Exclusion de 6 mois de l'université

33 étudiants ont été convoqués pour possession ou usage d'un téléphone portable lors d'un examen.

Sanctions : Exclusions de 6 mois / de 12 mois avec sursis de l'université

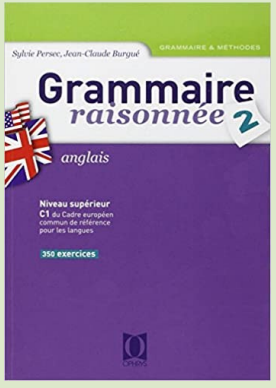
5 étudiants ont été convoqués pour plagiat

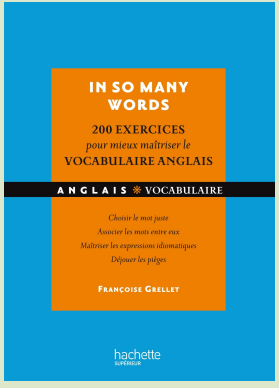
Sanctions : Exclusions de 6 mois avec sursis, exclusions de 6 mois ferme

Université d'ORLÉANS

Présentation du cours

- **Bibliographie**





Célène :
« Anglais maths Semestre 6 »
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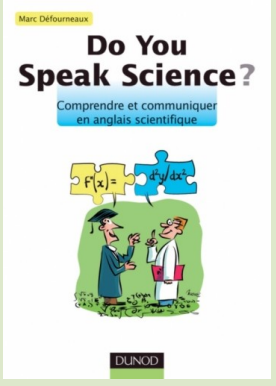
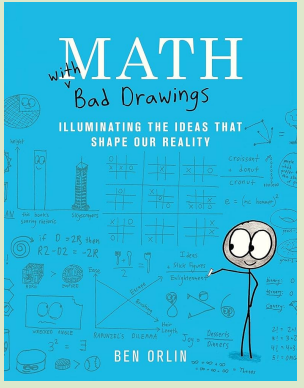
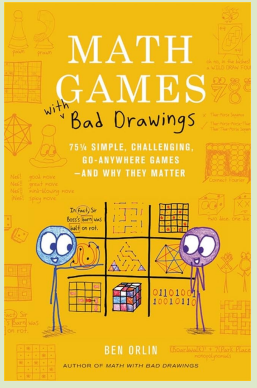
Présentation du cours

- **Bibliographie**







Célène :
« Anglais maths Semestre 6 »
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L3 S6 L.V. Anglais (maths)

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Groupe verbal :

1. Oubli du -s à la troisième personne du singulier du présent (forme simple et formes auxiliées), ainsi que tout autre erreur de conjugaison de base (**you is*)
2. Les verbes irréguliers
3. Les constructions auxiliées, en particulier *have + en*, *be + ing*, *be + en*
4. Forme du verbe après un auxiliaire de modalité

Groupe nominal :

1. Les adjectifs : invariables et placés avant le nom qu'ils qualifient
2. "article zéro" (= pas d'article) devant les noms "abstraits" (*life*, *death*, *nature*...) et les noms propres, y compris accompagnés d'un titre
3. Les noms à pluriel irrégulier (*teeth*, *children*...)
4. Les pronoms : respecter l'accord en genre et en nombre avec l'antécédent

Syntaxe :

1. Construction des phrases négatives
2. Construction des questions, directes et indirectes
3. Ordre Verbe-Objet-Adverbe dans la phrase simple (ex : *He likes coffee very much*)

NB : La présence dans une copie de trois erreurs dans ces rubriques entraîne une baisse de 20% de la note globale.

Présentation du cours

- **Modalités de contrôle des connaissances**

Modalités de contrôle des connaissances :
Contrôle continu
***Etudiants régime général :**
1 devoir de CC1 (séance 6 – CE/CO/Cours) : 50 %
1 devoir de CC2 (séances 7 et 8 – oral – présence obligatoire) : 50 %
***Etudiants régime spécial :**
1 examen terminal (écrit 1h – fin du semestre)

Les notes ne se négocient pas.

Science in Society : A Matter of Sense and Sensibility

- **Science** has an **impact on society**
- But **science** is also **impacted by society** and its **bias**, because scientists belong to society and as individuals, they have **internalized** sometimes flawed **representations**
  > there is no pure objectivity in science, and one must be aware of their influences, distorted views sometimes, to be better as a scientist or a researcher

Examples:

Presentations at the end of the semester could focus on a **case study** about an example of **science** or experimentation which was **deliberately delayed or hindered because of outdated social norms** or issues.

One might also discuss **unconscious bias or discriminations still present in science** that plague some scientists, and even think about ways to overcome those obstacles through **education**, or effective **awareness campaigns** to be imagined.

You might also want to pay attention to particular **wrongs in society, cultural discriminations or injustices that scientists are working to redress**, whether the social issues be contemporary or not.

You may show that **science evolves with society**, or that **society evolves with science**.

- A final question would be the **perception of science itself in society** – which may constitute an obstacle. Some prejudices about science must be fought so scientists can still do their jobs in the future. So what about the image of science in society?

Science in Society : A Matter of Sense and Sensibility

Should we beware of science because its capabilities are so life-threatening? Or should we cherish science because it is so precious? Or, alternatively, should we transform science into a life-affirming pursuit by caring labour? All three, says ***Patsy Hallen.***

Science in Society : A Matter of Sense and Sensibility

WE NEED TO BE careful of science because of its life-destroying potential. Half of all scientists and technologists work on war-related research while a third work for large corporations, mainly in teams on profit-motivated research projects not of their own choosing. Science has become incorporated into the military-industrial complex and often serves the interests of profit-making and social control.

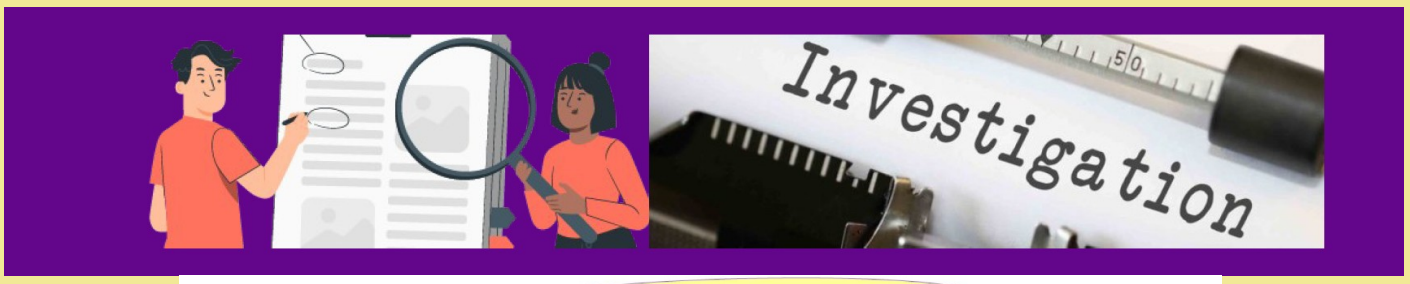
The mind-set of our age is that science is neutral and value-free. But this picture of science functions as a smokescreen. It succeeds in directing our attention away from facts about the social structure of science and its practices. The discourse of value-neutrality performs an ideological service in favour of the status quo and prevents us for examining how science is actually organised (its take-over by the military-industrial complex, its social stratification, its exclusion of female practitioners, its culture, gender and species biases) and what science actually does

Science in Society : A Matter of Sense and Sensibility

Human values and interests shape science in the following ways:

- the selection of goals for science;
- the choice of problems and research projects on which science concentrates;
- the methodologies and knowledge-producing practices of science;
- the choice of experimental design;
- the way we behave towards our research subjects;
- the language we use (for example, the terminology, the 'hard' sciences: are women less well-equipped to penetrate nature's secrets?);
- the very content of our theoretical formulations in science;
- the evaluation and interpretation of scientific results; and
- whom we consider as scientists (depending on one's gender or class, identical work earns the label of lab assistant or scientist).

Science in Society : A Matter of Sense and Sensibility



« Unmannered dog, stand thou when I command ! »

« If ever he have child, abortive be it,
Prodigious, and untimely brought to light,
Whose ugly and unnatural aspect
May fright the hopeful mother at the view »

« Never hung poison on a fouler toad.
Out of my sight ! Thou dost infect mine eyes.
[...]
Would they were basilisks, to strike thee dead. »

« – Blush, blush, thou lump of foul deformity »

« Vouchsafe, diffused infection of a man »

« Thou elvish-marked, abortive, rooting hog,
Thou that wast sealed in thy nativity
The slave of nature and the son of hell ;
Thou slander of thy heavy mother's womb,
Thou loathed issue of thy father's loins,
Thou rag of honour, thou detested – »

« Fouler than heart can think thee, thou canst make
No excuse current but to hang thyself »

« Hie thee to hell for shame, and leave this world,
Thou cacodemon. There thy kingdom is. »

Science in Society : A Matter of Sense and Sensibility

Common points regarding form	Common points regarding content	Your guess

Politics : From Propaganda to Democracy

Common points regarding form	Common points regarding content	Your guess
Lines (in verse), addressed to one person > **probably a theatrical play** Words in early modern English ("thou," "thee," "thy", "dost") > **work of an English playwright from the Renaissance** Pentameter, most of the time iambic, and unrhymed verse (blank verse) > **probably a play by Shakespeare, about noble characters**	**Insults and curses** Accusing the target of being an **animal** ("hog", "dog", "toad"), a **monster or a demon** ("elvish-marked" "son of hell" "cacodemon"), in other words **inhuman** ("infection of a man", "fouler" "detested") Abuse specifically focuses on **physical appearance** ("ugly", "unnatural aspect", "lump of foul deformity" "Thou dost infect mine eyes") Mention of a **kingdom** **> probably a Shakespearean play about a cruel king who is said to be devilish and suffers from physical disabilities**	**William Shakespeare's *Richard III*** 

Politics : From Propaganda to Democracy

Richard III abridged

RICHARD: Mwahahaha! Mwahahahahaha! Mwahaha!

CLARENCE: Hey brother! So, I guess I'm being sent to the Tower of London. Sucks, right?

RICHARD: Don't worry, Clarence, you'll be fine. I'll try and get you out, and certainly won't hire assassins to kill you or anything.

CLARENCE: Awesome! You're the best!

RICHARD: Mwahahaha!

ANNE: You killed my husband and my son in the last play, you asshole! I HATE YOU SO MUCH!

RICHARD: I only killed your husband because you're so fucking hot.

ANNE: OMG TAKE ME RIGHT NOW.

RICHARD: Mwahahahaha.

MARGARET: YOU'RE ALL GONNA DIE BECAUSE OF RICHARD! I CURSE ALL YOU FUCKERS!

EVERYONE: *ignores Margaret*

EDWARD V: The king is dead! Good thing he made you Lord Protector, Richard, so now you can make sure no one steals my throne.

RICHARD: Yeah...so you know what the best hotel in town is? The tower of London. I got you and your brother a suite.

EDWARD V: Awesome! Let's go!

RICHARD: Mwahahahahahaha! Hey everybody, those kids are bastards. I should be king instead.

EVERYONE: WOOOO! WE LOVE RICHARD!

HASTINGS: Okay, I was fine with all the usurping and murdering up until now but...seriously dude, I gotta ask: WHY ARE YOU SO EVIL?

RICHARD: I'm a limeblock. Discussion closed. Oh, and you're under arrest. Say hi to my dead nephews for me.

BUCKINGHAM: Wait, what? Dude, you have officially overdone it. I'm out.

RICHARD: Memo to self: get Buckingham killed. Mwahaha.

ELIZABETH: Okay Richard, you've now killed my two sons, my brothers, and I'm not completely sure you didn't kill my husband too. What else could you possibly do to me?

RICHARD: Well, your daughter's kinda hot. And fortunately, my wife just came down with a deadly illness - the symptoms include head/neck separation, very gross - and I've got to start making some legitimate heirs, *if ya know what I mean*.

ELIZABETH: Fuck you. The Earl of Richmond is gonna kill you so hard.

RICHARD: Oh shit, that's right, we're being invaded. Guess I'd better head over to Bosworth field, then.

AUDIENCE: Oh thank god, it's nearly over.
(For those of you keeping score at home, Richard's current body count is ELEVEN FUCKING PEOPLE.)

GHOSTS OF LITERALLY EVERYONE IN THE PLAY: BOOGEDY BOOGEDY BOOGEDY! You're so gonna die tomorrow, Richard, because we're on the Earl of Richmond's side. In fact, we've already started calling him Henry VII. DESPAIR AND DIE, MOTHERFUCKER!

RICHARD: Mwaha...ha?

AUDIENCE: SERIOUSLY CAN HE PLEASE GET KILLED ALREADY SO I CAN GO TO THE BATHROOM!

RICHARD: All right men, first let me say thanks for sticking with me, despite the fact that all my close friends seem to mysteriously die whenever they disagree with me. Secondly: Yorkists, ready your breakfasts and eat hearty. FOR TONIGHT, WE DINE IN HELL! MWAHAHAHAHAHA-aaaack! That hurt! AND WHERE THE FUCK DID MY HORSE GO?

HENRY VII: I KEEL YOU!!

RICHARD: Oh, fuck. *dies*

AUDIENCE: Thank god. *sprints for the bathrooms*

THE END.

What happens in the play?

1. Read the summary of Shakespeare's play* Richard III *and briefly rephrase in your own words what happens: what crimes could Richard be accused of? Make a list of all the charges that could be held against him.

List of charges held by William Shakespeare against Richard III:

- .....
- .....
- .....
- .....
- .....
- .....
- .....
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- .....
- .....
- .....
- .....

2. Does the summary help you understand all the abuse regarding his physical appearance?
Why does Shakespeare insist upon the way he looks?
To what extent does it account for a specific vision of the cosmos in the Renaissance?

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Historical investigation

3. Let's check the historical and scientific veracity of some of the charges held by Shakespeare against Richard III with the following video, entitled "Facts about Richard III: History's Most Reviled King." (01:04-06:27 https://www.youtube.com/watch?v=PsntI4tn3II)

Which charge(s) have actually been proven true by historians and scientists?

.....
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4. Spot at least one joke in the video (whether you find it funny or not).

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5. Let's try and answer Hastings's question: "WHY [IS RICHARD] SO EVIL?". In other words, why does Shakespeare portray him as a monster more than one century after his death (1593)?

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6. How does the play* Richard III *by Shakespeare demonstrate the performative power of words, here overpowering history and science?

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Politics : From Propaganda to Democracy

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Politics : From Propaganda to Democracy

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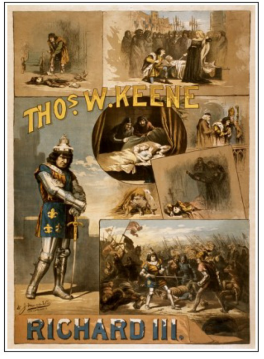

List of charges held by William Shakespeare against Richard III:

- He betrayed his brother Clarence so he was sent to prison (without his knowing it was because of Richard) and hired assassins to kill him while he was there.
- He killed the husband (and father-in-law actually) of Anne Neville, before marrying her.
- As he became Lord Protector after the death of his brother the king (Edward IV), he locked up his nephews, Edward V and his brother, in the Tower of London.
- Then he told everybody they were illegitimate children, so he could access the throne instead.
- He eventually had the "Princes in the tower" killed.
- He sent Hastings to jail (and had him killed).
- He suffered from physical deformity (he was notably pictured as a hunchback).
- He was evil.
- He had Buckingham killed.
- Before killing the sons of the Queen (Elizabeth Woodville, his late brother the King's wife), he had gotten rid of her brothers (and who knows, maybe of the King himself).
- Richard killed his wife, so he could court and try to get married to his niece (to secure the throne).
- Basically, he is accused of killing anybody who disagrees with him and might prevent him from usurping the throne.

2. Does the summary help you understand all the abuse regarding his physical appearance?
Why does Shakespeare insist upon the way he looks?
To what extent does it account for a specific vision of the cosmos in the Renaissance?

His inhumanity is supposed to show physically ("inner crookedness represented externally" "disability caused by inner evil"). The Renaissance insists on the connection and reflection of all elements: inner worth is supposed to show externally, and human (dis)order is echoed by natural (dis)order.

Politics : From Propaganda to Democracy

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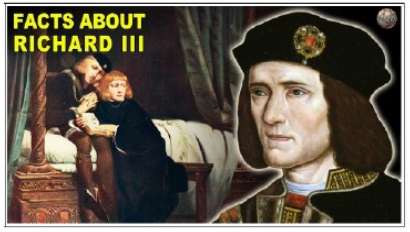

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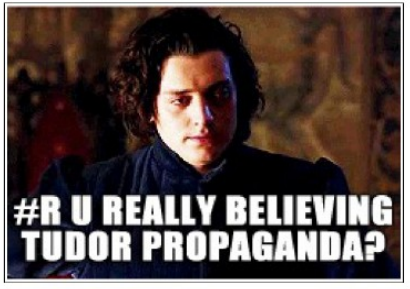

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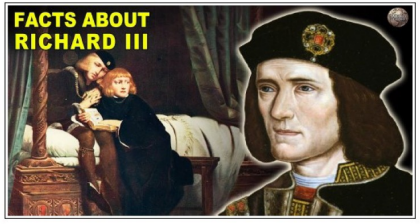


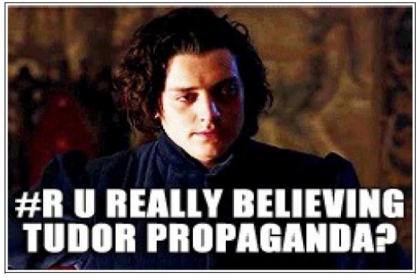
Which charge(s) have actually been proven true by historians and scientists?

Richard had his nephews declared illegitimate and he imprisoned them in the Tower of London (under the guise of protection): that is the only thing historians know for sure.

Actually, he probably wasn't a hunchback. If the blood of his nephews might be on his hands, there is no historical evidence. He was accused of killing his wife, but tuberculosis is the most likely culprit. And despite rumours that Richard killed his wife to marry his niece, Elizabeth of York, he never made a move on her – on the contrary he courted Princess Joana of Portugal.

4. Spot at least one joke in the video (whether you find it funny or not).

Examples: Parking over 24hrs, that's going to be a big fee; Locking up children in the tower is not good PR (public relations) ; Children have been discovered in a chamber that had been walled-up > not a good place to grow up in ; Repetition of "illegitimate" ; Richard's courting of Joana hit a snag when he died on the battlefield.



5. Let's try and answer Hastings's question: "WHY [IS RICHARD] SO EVIL?". In other words, why does Shakespeare portray him as a monster more than one century after his death (1593)?

Shakespeare wrote his play under the reign of Queen Elizabeth I, the granddaughter of Henry Tudor, who became King Henry VII after defeating Richard III at the battle of Bosworth Field. The idea behind Tudor propaganda is to legitimize the Tudor dynasty by portraying the previous king as devilish ("Eager to legitimize the Tudor dynasty, propagandists denounced Richard as a devilish usurper").

6. How does the play* Richard III *by Shakespeare demonstrate the performative power of words, here overpowering history and science?

Through his play, Shakespeare created the monstrous image we still have of Richard III, without concerns for historical truth.

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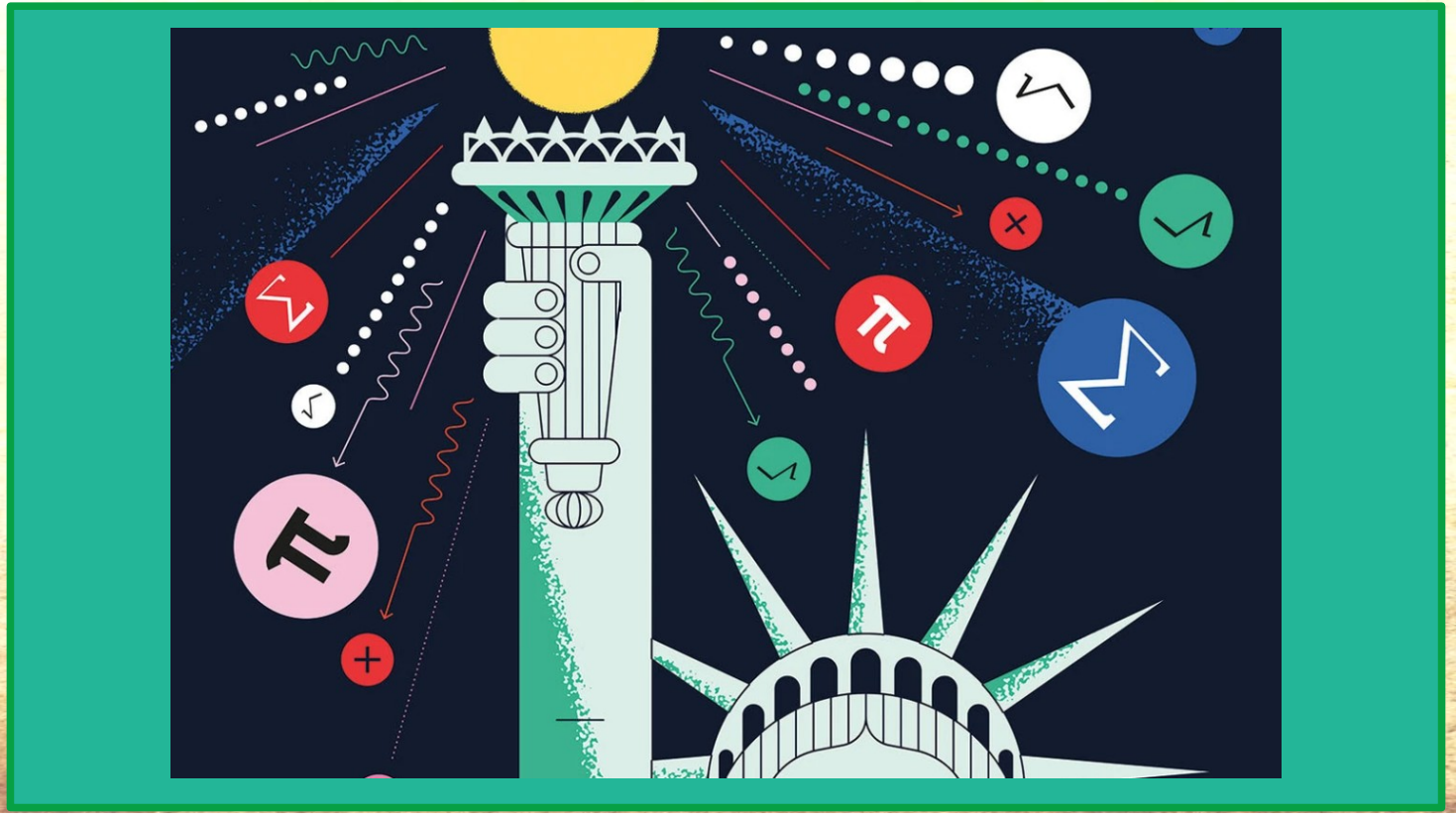

Politics : From Propaganda to Democracy

Relationship Between Electoral System and Mathematics in Contemporary Areas :

- **Designing different voting methods**
  - ✔ Majority, first-past-the-post, proportional representation, ranked-choice voting (arithmetic)
  - ✔ Mathematical formulas for seat allocation
- **Analysis and evaluation to make sure elections are transparent, fair and logical**
  - ✔ Counting methods, error checking
  - ✔ Detecting irregularities and minimizing bias
  - ✔ Determining the good and bad sides of the system: decision-making and policy planning
  - ✔ Drawing the boundaries of voter areas (geometry, statistics, graph theory)
- **Predicting voter behaviour and outcomes through modeling**
  - ✔ Opinion polls, surveys and historical voting data
  - ✔ Statistical analysis
  - ✔ Simulations and probabilistic models (predicting results, testing voting methods)
  - ✔ Predicting election outcomes and "what-if" scenarios (predictive models, regression analysis, machine learning, Bayesian statistics)
  - ✔ Campaign strategy (game theory)

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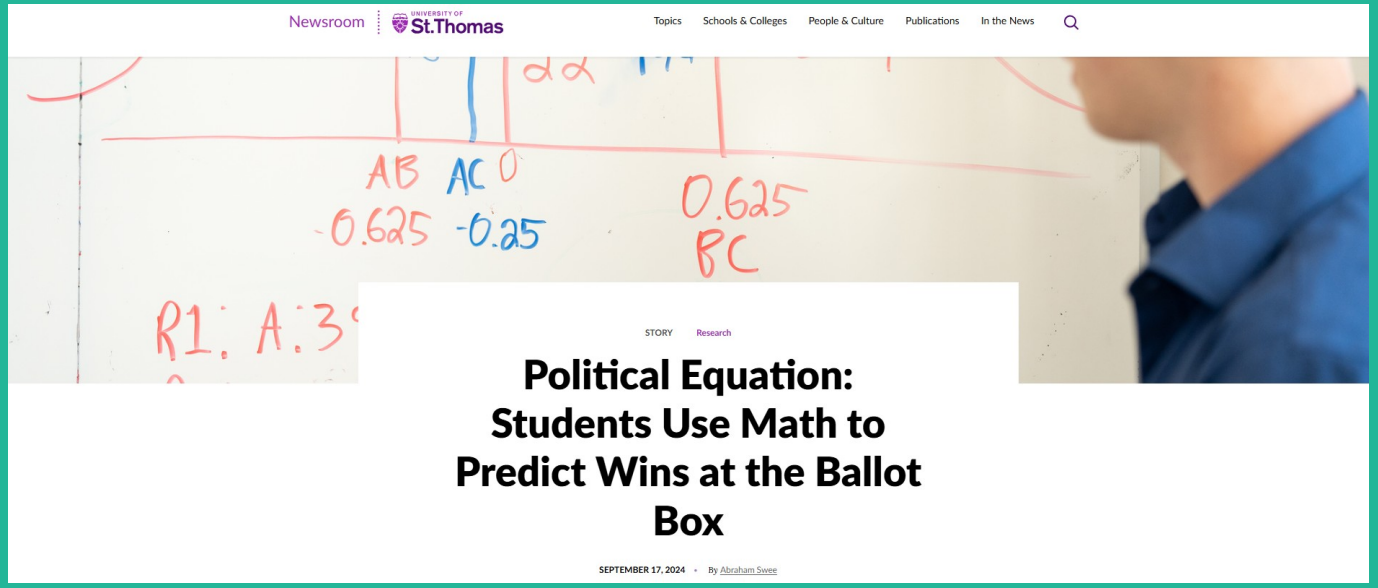

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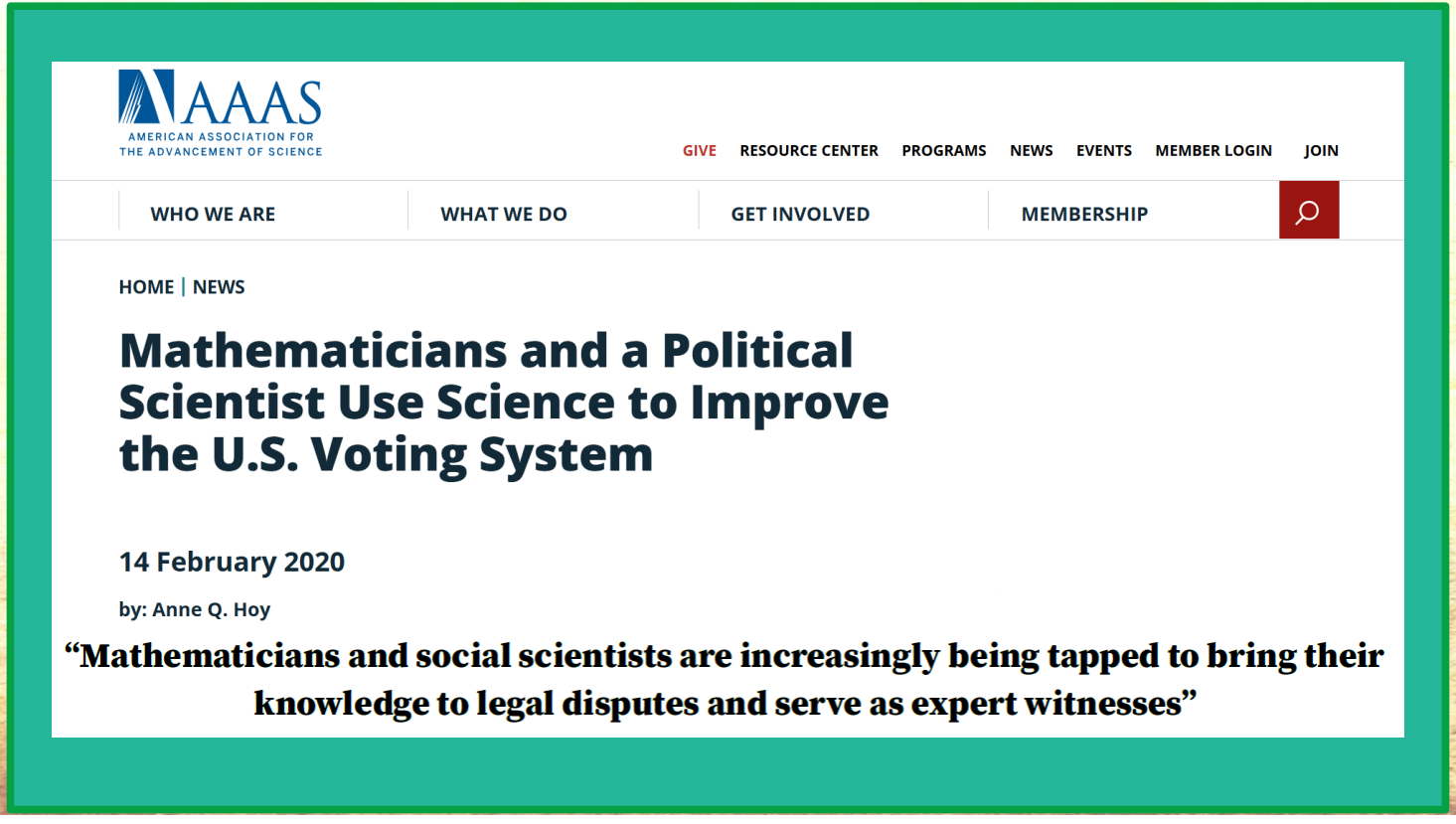

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Mathematicians and a Political Scientist Use Science to Improve the U.S. Voting System

14 February 2020

by: Anne Q. Hoy

"Mathematicians and social scientists are increasingly being tapped to bring their knowledge to legal disputes and serve as expert witnesses"

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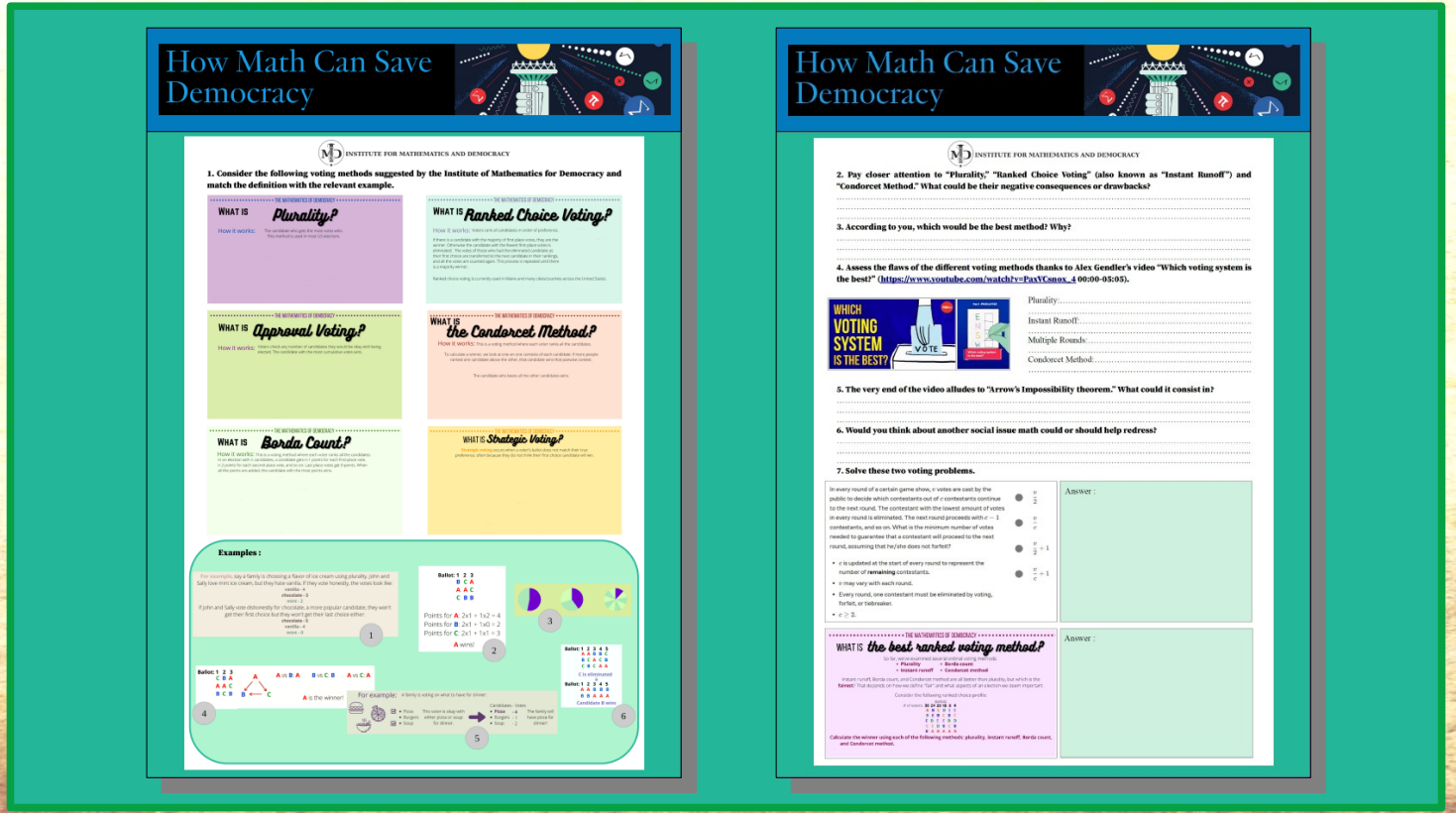

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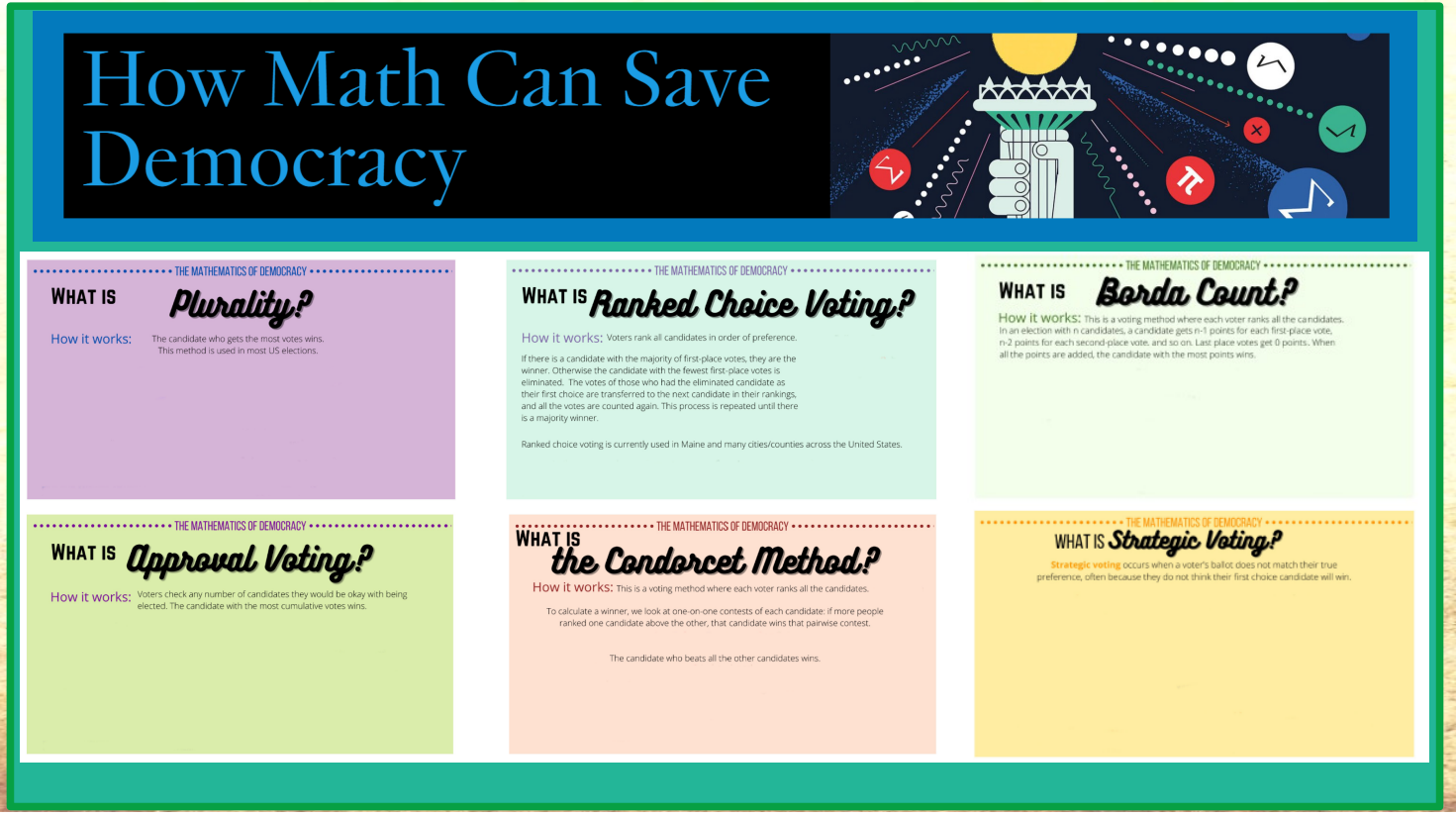

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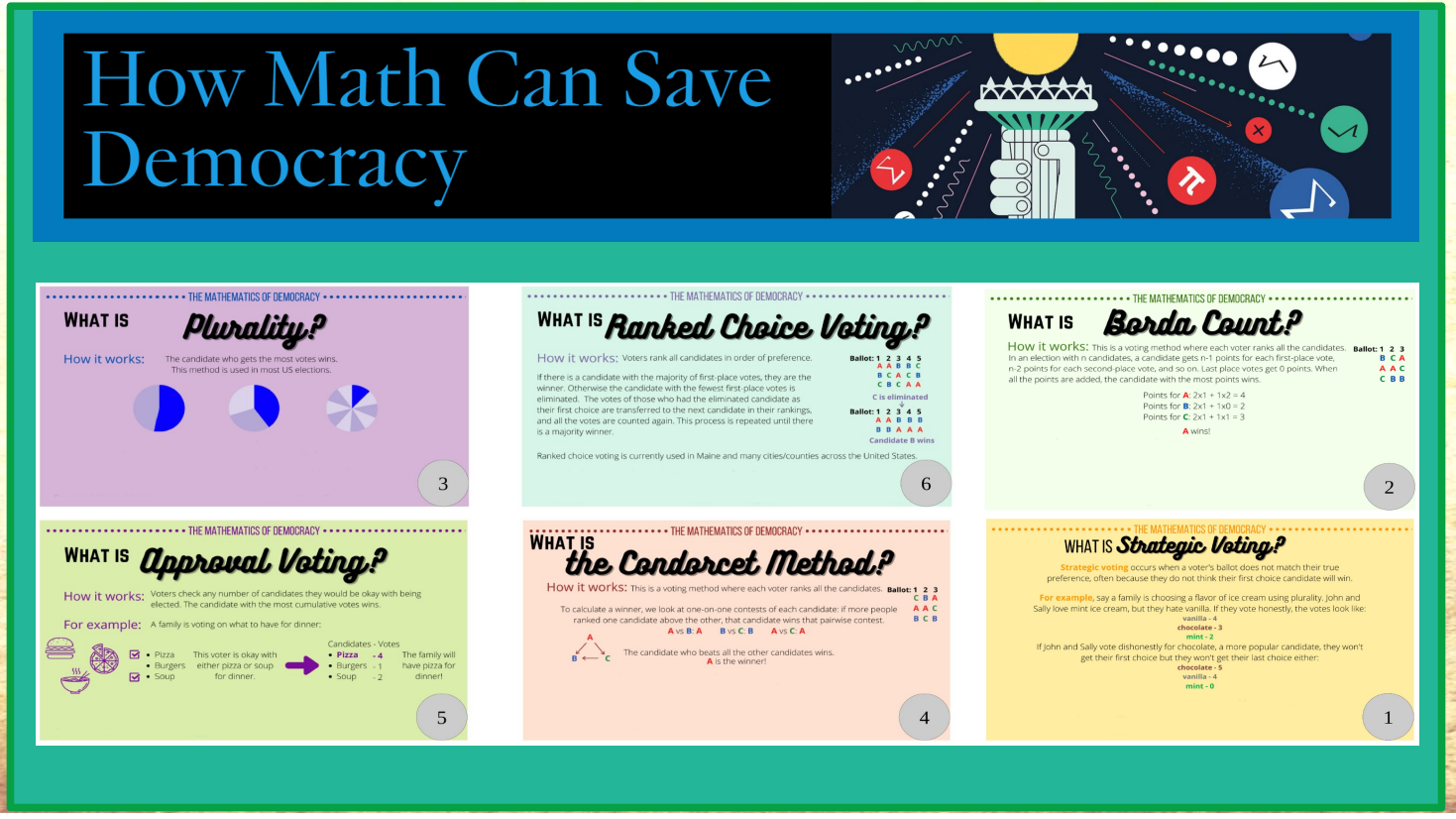


Politics : From Propaganda to Democracy

INSTITUTE FOR MATHEMATICS AND DEMOCRACY

2. Pay closer attention to "Plurality," "Ranked Choice Voting" (also known as "Instant Runoff") and "Condorcet Method." What could be their negative consequences or drawbacks?

……

……

……

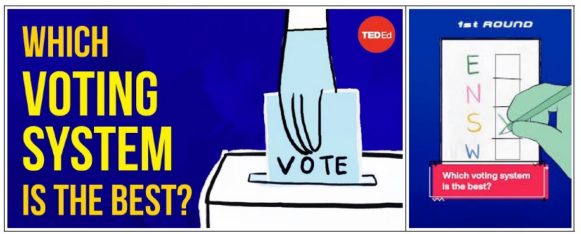
3. According to you, which would be the best method? Why?

……

……

……

4. Assess the flaws of the different voting methods thanks to Alex Gendler's video "Which voting system is the best?" (https://www.youtube.com/watch?v=PaxVCsnox_4 00:00-05:05).



Plurality:……

……

Instant Runoff:……

……

Multiple Rounds:……

……

Condorcet Method:……

……

Politics : From Propaganda to Democracy

How Math Can Save Democracy

4. Assess the flaws of the different voting methods thanks to Alex Gendler's video "Which voting system is the best?" (https://www.youtube.com/watch?v=PaxVCsnox_4 00:00-05:05).

Plurality: a candidate may be chosen even though most think it is the worst.
Instant Runoff: in the example, the chosen candidate started out in second-to-last place and a majority ranked it among their two least preferred options.
Multiple Rounds: (the top two winners proceed to a separate runoff) encourages tactical voting and lying (skewing the votes in their favor)
Condorcet Method: may fail to select a winner

5. The very end of the video alludes to "Arrow's Impossibility theorem." What could it consist in?
(Video) It's possible that the voting methods deliver unfair results. Our intuitive concept of fairness contains a number of assumptions that may contradict each other. For any election with more than two options, it's impossible to design a voting system that doesn't violate at least some theoretically desirable criteria.
Arrow's Theorem says that no ranked choice voice is perfect. It's impossible to fulfill the three desirable features for a voting system (unanimity, no dictators, independence of irrelevant alternatives) at the same time in any ranked voting system.

6. Would you think about another social issue math could or should help redress?

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In every round of a certain game show, v votes are cast by the public to decide which contestants out of c contestants continue to the next round. The contestant with the lowest amount of votes in every round is eliminated. The next round proceeds with $c-1$ contestants, and so on. What is the minimum number of votes needed to guarantee that a contestant will proceed to the next round, assuming that he/she does not forfeit?

- c is updated at the start of every round to represent the number of **remaining** contestants.
- v may vary with each round.
- Every round, one contestant must be eliminated by voting, forfeit, or tiebreaker.
- $c \geq 2$.

- $\frac{v}{2}$
- $\frac{v}{c}$
- $\frac{v}{2}+1$
- $\frac{v}{c}+1$

Answer :

THE MATHEMATICS OF DEMOCRACY

WHAT IS *the best ranked voting method?*

So far, we've examined several ordinal voting methods:

- **Plurality**
- **Instant runoff**
- **Borda count**
- **Condorcet method**

Instant runoff, Borda count, and Condorcet method are all better than plurality, but which is the **fairest**? That depends on how we define "fair" and what aspects of an election we deem important.

Consider the following ranked choice profile:

Ballots:					
# of voters:	36	24	20	18	8	4
	A	B	C	D	E	E
	D	E	B	C	B	C
	E	D	E	E	D	D
	C	C	D	B	C	B
	B	A	A	A	A	A

Calculate the winner using each of the following methods: plurality, instant runoff, Borda count, and Condorcet method.

Answer :

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The correct answer is:

$$\frac{v}{c}+1$$

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	D	E	B	C	B	C
	E	D	E	E	D	D
	C	C	D	B	C	B
	B	A	A	A	A	A

Calculate the winner using each of the following methods: plurality, instant runoff, Borda count, and Condorcet method.

No majority winner.
A is the plurality winner.
B is the runoff winner.
C is the instant runoff winner.
D is the Borda Count winner.
E is the Condorcet winner.

Morgane AUGRIS

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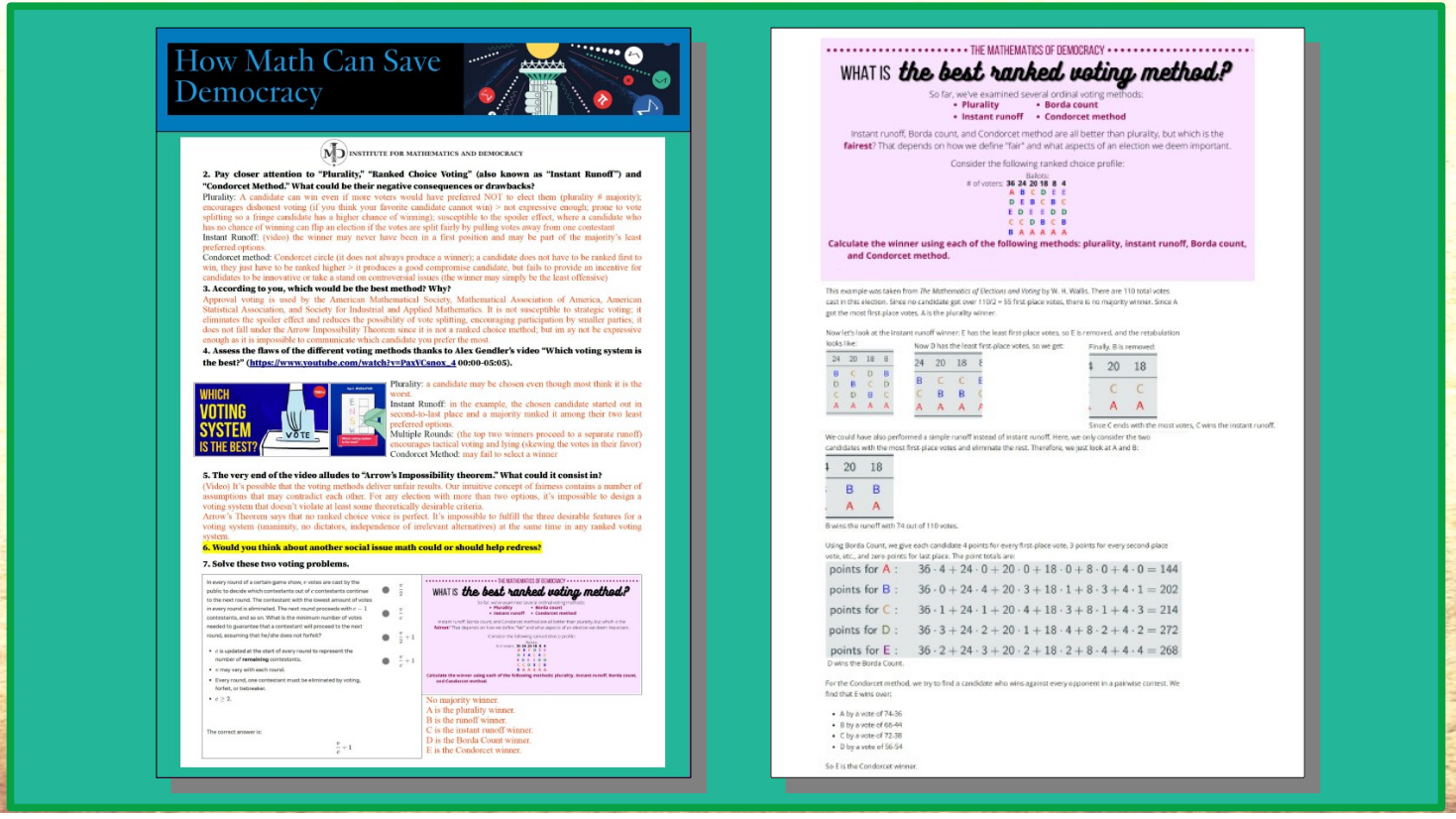

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Arrow's Impossibility Theorem

Three desirable features for a voting system are as follows:

- **Unanimity**: If everyone prefers A to B, A should win.
- **No Dictators**: There should not be anyone whose individual preferences always determine who wins.
- **Independence of Irrelevant Alternatives (IIA)**: Adding extra options should not make existing relations change. That is, if $A \geq B$, adding option C should not make $B \geq A$.

Independence of irrelevant alternatives is an important criterion since ideally a voting system will not be susceptible to **strategic voting**, in which voters will rank options in ways that do not reveal their true preferences so as to attempt to make sure their top choice is elected.

THEOREM

Arrow's Impossibility Theorem

It is impossible to fulfill all of the three above features (Unanimity, No Dictators, IIA) at the same time in any ranked voting system.

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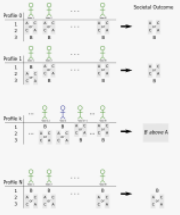
PROOF

Consider a vote with N voters and three options A, B, and C. In the diagram to the right, it is shown that given unanimity and independence of irrelevant alternatives, one of the voters must be a dictator [3].



Voter profiles and the resulting aggregate preferences [2].

If everyone individually votes B as the worst option, then by unanimity, B is the aggregate worst option. This is profile 0 in the diagram. Similarly, if everyone thinks B is the best option, then B is the aggregate best option. This is profile N. Between 0 and N are profiles i where the first i people vote B as best, and $N - i$ people vote B as worst. At one of these values, the aggregate ranking must switch so that B is the aggregate best. Let this switching value be k, the *pivotal voter* for B. If k ranks B first, then B wins, and if k ranks B last, then B does not win.

The rest of the proof demonstrates that, if unanimity and independence of irrelevant alternatives hold, then voter k always determines the outcome of the vote. That is, voter k is a dictator.

First, consider the case where k ranks A over C, and all other voters have some arbitrary ranking. Now, for k, move A to their first preference and B to their second. By IIA, this can't change the aggregate ranking of A versus C. Similarly, for voters 0 through $k - 1$, move B to their first preference, and for voters $k + 1$ through N, move B to their bottom preference. Again, by IIA, this can't change the ranking of A and C.

Now all voters 0 through k prefer A to B and all voters $k + 1$ through N prefer B to A. By the definition of k, that means the aggregate preference must be $A > B$. Similarly, all voters 0 through k prefer B to C and all voters $k + 1$ through N prefer C to B. By the definition of k, that means the aggregate preference must be $B > C$. By transitivity, the overall preference must be $A > C$. Therefore, k is a dictator. □