THE MATHEMATICS OF DEMOCRACY

WHAT IS *the best ranked voting method?*

So far, we've examined several ordinal voting methods:

- **Plurality**
- **Borda count**
- **Instant runoff**
- **Condorcet method**

Instant runoff, Borda count, and Condorcet method are all better than plurality, but which is the **fairest**? That depends on how we define "fair" and what aspects of an election we deem important.

Consider the following ranked choice profile:

Ballots:						
# of voters:	**36**	**24**	**20**	**18**	**8**	**4**
	A	B	C	D	E	E
	D	E	B	C	B	C
	E	D	E	E	D	D
	C	C	D	B	C	B
	B	A	A	A	A	A

Calculate the winner using each of the following methods: plurality, instant runoff, Borda count, and Condorcet method.

This example was taken from *The Mathematics of Elections and Voting* by W. H. Wallis. There are 110 total votes cast in this election. Since no candidate got over 110/2 = 55 first-place votes, there is no majority winner. Since A got the most first-place votes, A is the plurality winner.

Now let's look at the instant runoff winner: E has the least first-place votes, so E is removed, and the retabulation looks like:

24	20	18	8
B	C	D	B
D	B	C	D
C	D	B	C
A	A	A	A

Now D has the least first-place votes, so we get:

24	20	18	8
B	C	C	E
C	B	B	C
A	A	A	A

Finally, B is removed:

4	20	18
	C	C
	A	A

Since C ends with the most votes, C wins the instant runoff.

We could have also performed a simple runoff instead of instant runoff. Here, we only consider the two candidates with the most first-place votes and eliminate the rest. Therefore, we just look at A and B:

4	20	18
	B	B
	A	A

B wins the runoff with 74 out of 110 votes.

Using Borda Count, we give each candidate 4 points for every first-place vote, 3 points for every second-place vote, etc., and zero points for last place. The point totals are:

$$\text{points for A}: \quad 36\cdot 4 + 24\cdot 0 + 20\cdot 0 + 18\cdot 0 + 8\cdot 0 + 4\cdot 0 = 144$$

$$\text{points for B}: \quad 36\cdot 0 + 24\cdot 4 + 20\cdot 3 + 18\cdot 1 + 8\cdot 3 + 4\cdot 1 = 202$$

$$\text{points for C}: \quad 36\cdot 1 + 24\cdot 1 + 20\cdot 4 + 18\cdot 3 + 8\cdot 1 + 4\cdot 3 = 214$$

$$\text{points for D}: \quad 36\cdot 3 + 24\cdot 2 + 20\cdot 1 + 18\cdot 4 + 8\cdot 2 + 4\cdot 2 = 272$$

$$\text{points for E}: \quad 36\cdot 2 + 24\cdot 3 + 20\cdot 2 + 18\cdot 2 + 8\cdot 4 + 4\cdot 4 = 268$$

D wins the Borda Count.

For the Condorcet method, we try to find a candidate who wins against every opponent in a pairwise contest. We find that E wins over:

- A by a vote of 74-36
- B by a vote of 66-44
- C by a vote of 72-38
- D by a vote of 56-54

So E is the Condorcet winner.