How Math Can Save Democracy

INSTITUTE FOR MATHEMATICS AND DEMOCRACY

1. Consider the following voting methods suggested by the Institute of Mathematics for Democracy and match the definition with the relevant example.

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WHAT IS *Plurality?*

How it works: The candidate who gets the most votes wins. This method is used in most US elections.

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WHAT IS *Ranked Choice Voting?*

How it works: Voters rank all candidates in order of preference.

If there is a candidate with the majority of first-place votes, they are the winner. Otherwise the candidate with the fewest first-place votes is eliminated. The votes of those who had the eliminated candidate as their first choice are transferred to the next candidate in their rankings, and all the votes are counted again. This process is repeated until there is a majority winner.

Ballot: 1 2 3 4 5
A A B B C
B C A C B
C B C A A

C is eliminated

Ballot: 1 2 3 4 5
A A B B B
B B A A A

Candidate B wins

Ranked choice voting is currently used in Maine and many cities/counties across the United States.

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WHAT IS *Approval Voting?*

How it works: Voters check any number of candidates they would be okay with being elected. The candidate with the most cumulative votes wins.

For example: A family is voting on what to have for dinner:

- Pizza
- Burgers
- Soup

This voter is okay with either pizza or soup for dinner.

Candidates - Votes
- **Pizza - 4**
- Burgers - 1
- Soup - 2

The family will have pizza for dinner!

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WHAT IS *the Condorcet Method?*

How it works: This is a voting method where each voter ranks all the candidates.

Ballot: 1 2 3
C B A
A A C
B C B

To calculate a winner, we look at one-on-one contests of each candidate: if more people ranked one candidate above the other, that candidate wins that pairwise contest.

A vs B: A B vs C: B A vs C: A

The candidate who beats all the other candidates wins.
A is the winner!

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WHAT IS *Borda Count?*

How it works: This is a voting method where each voter ranks all the candidates. In an election with n candidates, a candidate gets n-1 points for each first-place vote, n-2 points for each second-place vote, and so on. Last place votes get 0 points. When all the points are added, the candidate with the most points wins.

Ballot: 1 2 3
B C A
A A C
C B B

Points for A: 2x1 + 1x2 = 4
Points for B: 2x1 + 1x0 = 2
Points for C: 2x1 + 1x1 = 3

A wins!

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WHAT IS *Strategic Voting?*

Strategic voting occurs when a voter's ballot does not match their true preference, often because they do not think their first choice candidate will win.

For example, say a family is choosing a flavor of ice cream using plurality. John and Sally love mint ice cream, but they hate vanilla. If they vote honestly, the votes look like:

vanilla - 4
chocolate - 3
mint - 2

If John and Sally vote dishonestly for chocolate, a more popular candidate, they won't get their first choice but they won't get their last choice either:

chocolate - 5
vanilla - 4
mint - 0

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Examples :

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If John and Sally vote dishonestly for chocolate, a more popular candidate, they won't get their first choice but they won't get their last choice either:
chocolate - 5
vanilla - 4
mint - 0

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Ballot: 1 2 3
B C A
A A C
C B B

Points for A: 2x1 + 1x2 = 4
Points for B: 2x1 + 1x0 = 2
Points for C: 2x1 + 1x1 = 3

A wins!

2

3

Ballot: 1 2 3
C B A
A A C
B C B

A vs B: A B vs C: B A vs C: A

A is the winner!

4

Ballot: 1 2 3 4 5
A A B B C
B C A C B
C B C A A

C is eliminated

Ballot: 1 2 3 4 5
A A B B B
B B A A A

Candidate B wins

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For example: A family is voting on what to have for dinner:

- Pizza
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This voter is okay with either pizza or soup for dinner.

Candidates - Votes
- Pizza - 4
- Burgers - 1
- Soup - 2

The family will have pizza for dinner!

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2. Pay closer attention to "Plurality," "Ranked Choice Voting" (also known as "Instant Runoff") and "Condorcet Method." What could be their negative consequences or drawbacks?

Plurality: A candidate can win even if more voters would have preferred NOT to elect them (plurality # majority); encourages dishonest voting (if you think your favorite candidate cannot win) > not expressive enough; prone to vote splitting so a fringe candidate has a higher chance of winning); susceptible to the spoiler effect, where a candidate who has no chance of winning can flip an election if the votes are split fairly by pulling votes away from one contestant

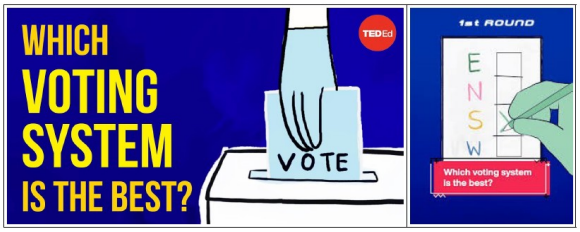
Instant Runoff: (video) the winner may never have been in a first position and may be part of the majority's least preferred options.

Condorcet method: Condorcet circle (it does not always produce a winner); a candidate does not have to be ranked first to win, they just have to be ranked higher > it produces a good compromise candidate, but fails to provide an incentive for candidates to be innovative or take a stand on controversial issues (the winner may simply be the least offensive)

3. According to you, which would be the best method? Why?

Approval voting is used by the American Mathematical Society, Mathematical Association of America, American Statistical Association, and Society for Industrial and Applied Mathematics. It is not susceptible to strategic voting; it eliminates the spoiler effect and reduces the possibility of vote splitting, encouraging participation by smaller parties; it does not fall under the Arrow Impossibility Theorem since it is not a ranked choice method; but im ay not be expressive enough as it is impossible to communicate which candidate you prefer the most.

4. Assess the flaws of the different voting methods thanks to Alex Gendler's video "Which voting system is the best?" (https://www.youtube.com/watch?v=PaxVCsnox_4 00:00-05:05).



Plurality: a candidate may be chosen even though most think it is the worst.

Instant Runoff: in the example, the chosen candidate started out in second-to-last place and a majority ranked it among their two least preferred options.

Multiple Rounds: (the top two winners proceed to a separate runoff) encourages tactical voting and lying (skewing the votes in their favor)

Condorcet Method: may fail to select a winner

5. The very end of the video alludes to "Arrow's Impossibility theorem." What could it consist in?

(Video) It's possible that the voting methods deliver unfair results. Our intuitive concept of fairness contains a number of assumptions that may contradict each other. For any election with more than two options, it's impossible to design a voting system that doesn't violate at least some theoretically desirable criteria.

Arrow's Theorem says that no ranked choice voice is perfect. It's impossible to fulfill the three desirable features for a voting system (unanimity, no dictators, independence of irrelevant alternatives) at the same time in any ranked voting system.

6. Would you think about another social issue math could or should help redress?

7. Solve these two voting problems.

In every round of a certain game show, v votes are cast by the public to decide which contestants out of c contestants continue to the next round. The contestant with the lowest amount of votes in every round is eliminated. The next round proceeds with $c-1$ contestants, and so on. What is the minimum number of votes needed to guarantee that a contestant will proceed to the next round, assuming that he/she does not forfeit?

- c is updated at the start of every round to represent the number of **remaining** contestants.
- v may vary with each round.
- Every round, one contestant must be eliminated by voting, forfeit, or tiebreaker.
- $c \geq 2$.

- $\frac{v}{2}$
- $\frac{v}{c}$
- $\frac{v}{2}+1$
- $\frac{v}{c}+1$

The correct answer is:

$$\frac{v}{c}+1$$

THE MATHEMATICS OF DEMOCRACY

WHAT IS *the best ranked voting method?*

So far, we've examined several ordinal voting methods:

- **Plurality**
- **Borda count**
- **Instant runoff**
- **Condorcet method**

Instant runoff, Borda count, and Condorcet method are all better than plurality, but which is the **fairest**? That depends on how we define "fair" and what aspects of an election we deem important.

Consider the following ranked choice profile:

Ballots:						
# of voters:	36	24	20	18	8	4
	A	B	C	D	E	E
	D	E	B	C	B	C
	E	D	E	E	D	D
	C	C	D	B	C	B
	B	A	A	A	A	A

Calculate the winner using each of the following methods: plurality, instant runoff, Borda count, and Condorcet method.

No majority winner.
A is the plurality winner.
B is the runoff winner.
C is the instant runoff winner.
D is the Borda Count winner.
E is the Condorcet winner.