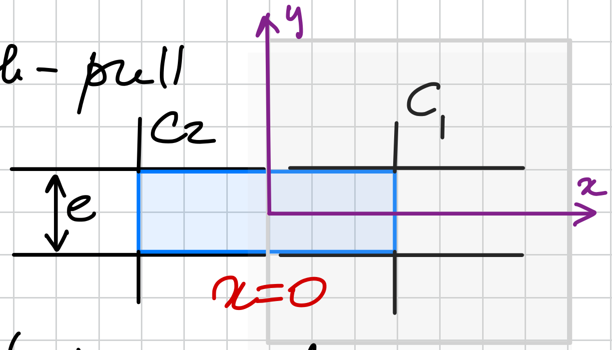
TD5 Capteurs capacitifs push-pull



Sans noyau, chaque capacité est :

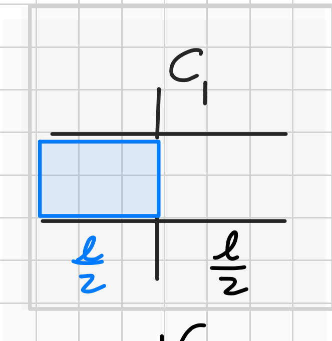
$$C = \frac{\varepsilon_0}{e} \times A$$

Avec le noyau occupant tout l'espace entre ses armatures :

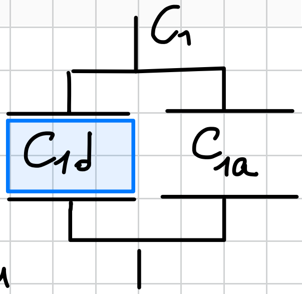
$$C = \frac{\varepsilon_0 \varepsilon_r}{e} \times A$$

ε_0 : permittivité de l'air
↳ Cste (tjs la même)
ε_r : permittivité du noyau
↳ dépend uniquement du matériau !

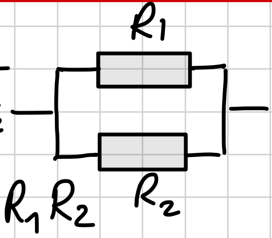
$\boxed{x : \text{position du centre du noyau}}$



Chaque condensateur C_1 et C_2 peut être vu comme deux condensateurs en parallèle : l'un avec un noyau et l'autre sans.

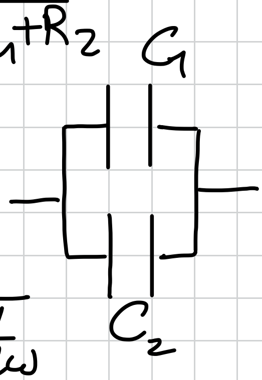


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$\Leftrightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$C_{eq} = C_1 + C_2$$



$$\frac{1}{\frac{1}{jC_{eq}\omega}} = \frac{1}{\frac{1}{jC_1\omega}} + \frac{1}{\frac{1}{jC_2\omega}}$$

Pour $x = 0$

$$C_{1d} = \frac{\varepsilon_0 \varepsilon_r}{e} \times \frac{A}{2} \ ;$$

$$C_{1a} = \frac{\varepsilon_0}{e} \times \frac{A}{2}$$

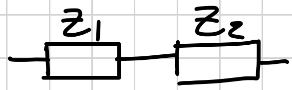
$$C_1 = C_{1d} + C_{1a}$$

$$= \frac{\varepsilon_0 \varepsilon_r}{e} \times \frac{A}{2} + \frac{\varepsilon_0}{e} \times \frac{A}{2} \times 1$$

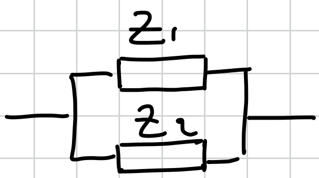
$$= \frac{\varepsilon_0}{e} \times \frac{A}{2} (\varepsilon_r + 1)$$

D'une façon générale

$$Z_{eq} = Z_1 + Z_2$$



$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$



$$Z_R = R$$

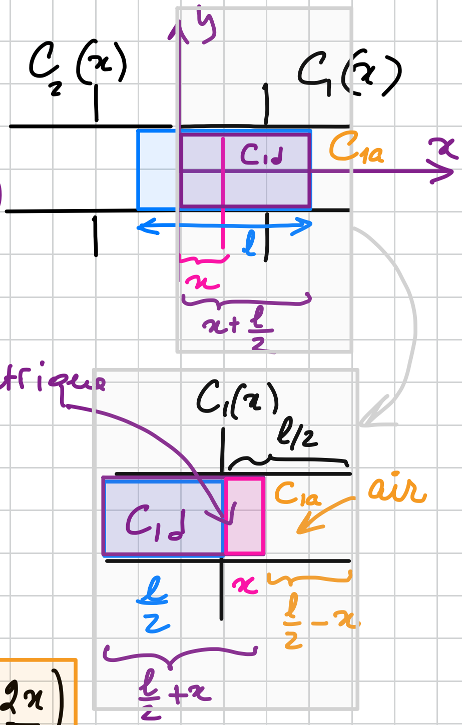
$$Z_C = \frac{1}{jC\omega}$$

$$Z_L = jL\omega$$

pour $x = 0$,

$$\boxed{C_1(0) = C_2(0) = C_0 = \frac{\varepsilon_0 \times A}{2e} (\varepsilon_r + 1)}$$

* $C_1(x) = \boxed{C_{1d}(x)} + \boxed{C_{1a}(x)}$



$$\boxed{\begin{aligned} C_{1d}(x) &= \frac{\varepsilon_0\varepsilon_r}{e}\times\left(\frac{A}{2}+\frac{x}{\ell}\times A\right) \\ &= \frac{\varepsilon_0\varepsilon_r}{e}\,\frac{A}{\ell}\left(\frac{\ell}{2}+x\right) \\ &= \frac{\varepsilon_0\varepsilon_r}{2e}\cdot A\left(1+\frac{2x}{\ell}\right) \end{aligned}}$$ (zone diélectrique)

$$\boxed{C_{1a}(x) = \frac{\varepsilon_0}{2e}\cdot A\left(1-\frac{2x}{\ell}\right)}$$ (zone air)

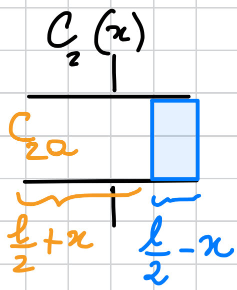
$$C_1(x) = C_{1d}(x) + C_{1a}(x) = \boxed{\frac{\varepsilon_0\varepsilon_r}{2e}A\left(1+\frac{2x}{\ell}\right)} + \boxed{\frac{\varepsilon_0}{2e}A\left(1-\frac{2x}{\ell}\right)}$$

$$= \frac{\varepsilon_0}{2e}A\left[\varepsilon_r\left(1+\frac{2x}{\ell}\right)+\left(1-\frac{2x}{\ell}\right)\right] = \frac{\varepsilon_0}{2e}A\left[\boxed{\varepsilon_r}+\boxed{\frac{2x\varepsilon_r}{\ell}}+\boxed{1}-\boxed{\frac{2x}{\ell}}\right]$$

$$= \frac{\varepsilon_0}{2e}A\left[\boxed{(\varepsilon_r+1)}+\boxed{\frac{2x}{\ell}(\varepsilon_r-1)}\right] = \frac{\varepsilon_0}{2e}A\boxed{(\varepsilon_r+1)}\left[\boxed{1}+\frac{2x}{\ell}\frac{(\varepsilon_r-1)}{\boxed{(\varepsilon_r+1)}}\right]$$

$$\boxed{C_1(x) = \underbrace{\boxed{\frac{\varepsilon_0 A}{2e}(\varepsilon_r+1)}}_{C_0}\left(1+\frac{2x(\varepsilon_r-1)}{\ell(\varepsilon_r+1)}\right) = \boxed{C_0}\left(1+\frac{2x(\varepsilon_r-1)}{\ell(\varepsilon_r+1)}\right)}$$

* $C_2(x) = \boxed{C_{2d}(x)} + \boxed{C_{2a}(x)}$



$$\boxed{C_{2d}(x) = \frac{\varepsilon_0\varepsilon_r}{e}\cdot A\left(\frac{\ell}{2}-x\right)} \; ; \; \boxed{C_{2a}(x) = \frac{\varepsilon_0}{e}A\left(\frac{\ell}{2}+x\right)}$$

En s'aidant de l'expression déterminée pour $C_1(x)$, on trouve :

$$C_2(x) = \boxed{\frac{\varepsilon_0\varepsilon_r}{2e}\cdot A\left(1-\frac{2x}{\ell}\right)} + \boxed{\frac{\varepsilon_0}{2e}A\left(1+\frac{2x}{\ell}\right)}$$

$$= \frac{\varepsilon_0 A}{2e}\left(\varepsilon_r-\frac{2x}{\ell}\varepsilon_r+1+\frac{2x}{\ell}\right) = \frac{\varepsilon_0 A}{2e}\left(\varepsilon_r+1-\frac{2x}{\ell}(\varepsilon_r-1)\right)$$

$$= \underbrace{\boxed{\frac{\varepsilon_0 A}{2e}(\varepsilon_r+1)}}_{C_0}\left(1-\frac{2x}{\ell}\frac{(\varepsilon_r-1)}{(\varepsilon_r+1)}\right) = C_0\times\left(1-\frac{2x}{\ell}\frac{(\varepsilon_r-1)}{(\varepsilon_r+1)}\right)$$

Conclusion :

$$C_1(x) = C_0 + \underbrace{\boxed{C_0 \times \frac{2x}{\ell} \times \frac{(\varepsilon_r - 1)}{\varepsilon_r + 1}}}_{}^{\Delta C(x)} = C_0 + \Delta C(x)$$

$$C_2(x) = C_0 - \underbrace{\boxed{\left(C_0 \times \frac{2x}{\ell} \times \frac{(\varepsilon_r - 1)}{\varepsilon_r + 1}\right)}}_{\Delta C(x)} = C_0 - \Delta C(x)$$

avec $\boxed{\Delta C(x) = \frac{2C_0 x}{d} \quad \frac{\varepsilon_r - 1}{\varepsilon_r + 1}}$ et $\boxed{C_0 = \frac{\varepsilon_0 A}{e}(\varepsilon_r + 1)}$