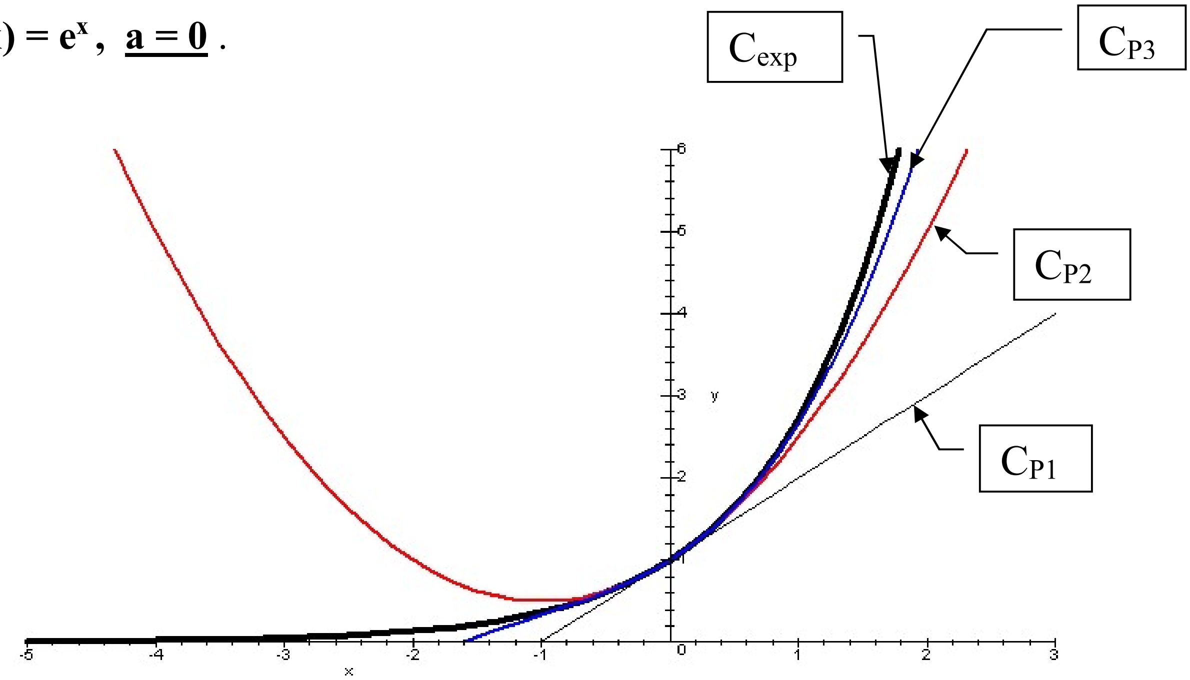
Développements limités à connaître

a) $f(x) = e^x$, $\underline{a = 0}$.



$P_1(x) = 1 + x$

$P_2(x) = 1 + x + \dfrac{x^2}{2!}$

$P_3(x) = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!}$

$$e^x = 1 + x + o(x)$$
$$e^x = 1 + x + \frac{x^2}{2} + o(x^2)$$
$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)$$

<u>En effet</u> :

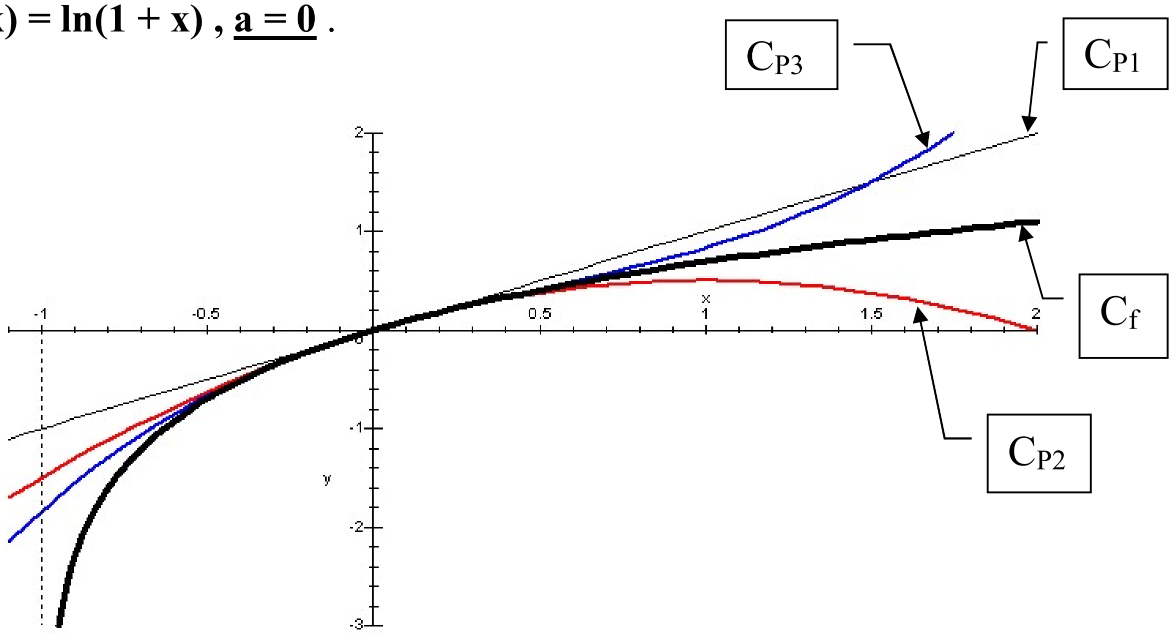
$f(x) = f'(x) = f''(x) = f'''(x) = e^x$.

$f(0) = f'(0) = f''(0) = f'''(0) = e^0 = 1$

$$f(x) = 1 + 1\,x + \frac{1}{2}\,x^2 + \frac{1}{6}\,x^3 + o(x^3).$$

(fonction de la forme $x^3\,\varepsilon(x)$ avec $\lim\limits_{\substack{x\to 0\\ x\neq 0}} \varepsilon(x) = 0$)

b) $f(x) = \ln(1 + x)$, $\underline{a = 0}$.



$$\ln(1+x) = x + o(x)$$
$$\ln(1+x) = x - \frac{x^2}{2} + o(x^2)$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

<u>En effet</u> :

$f(x) = \ln(1 + x)$

$f'(x) = \dfrac{1}{1+x}$ (u'/u) $= (1 + x)^{-1}$

$f''(x) = (-1)(1)(1 + x)^{-1-1} = -(1 + x)^{-2}$ ($nu'u^{n-1}$)

$f'''(x) = -(-2)(1)(1 + x)^{-2-1} = 2(1 + x)^{-3}$

$f(0) = \ln(1) = 0$

$f'(0) = (1)^{-1} = 1$

$f''(0) = -(1)^{-2} = -1$

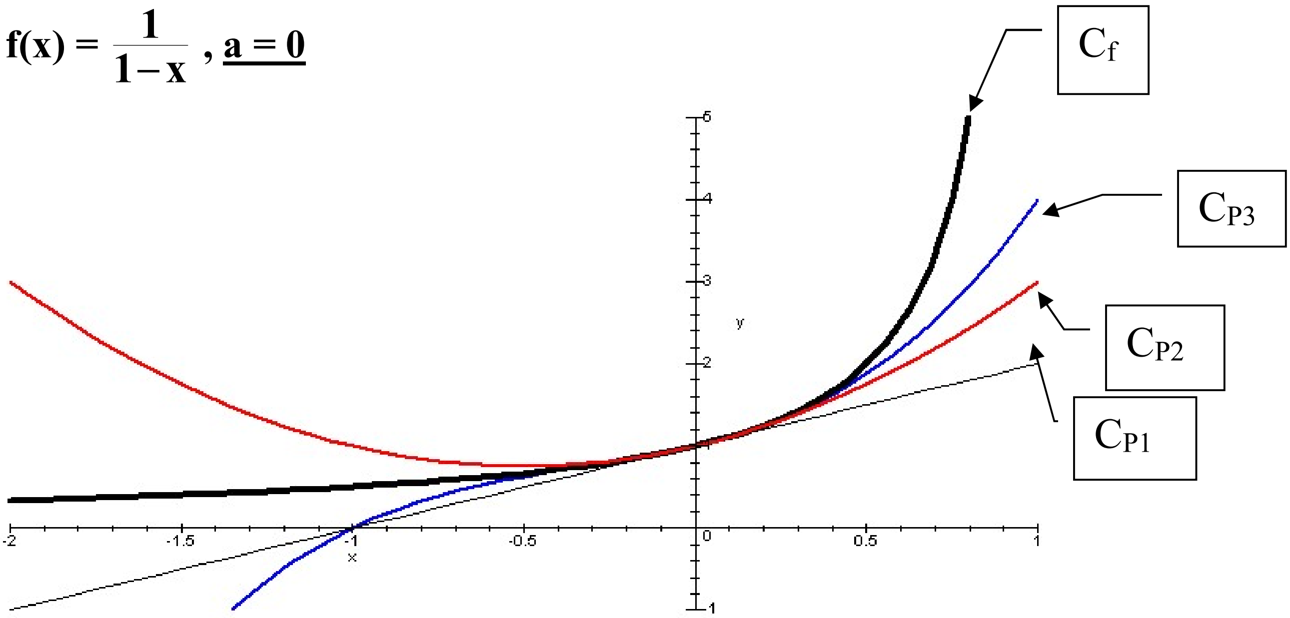
$f'''(0) = 2(1)^{-3} = 2(1) = 2$

$$f(x) = 0 + 1\,x + \frac{-1}{2}\,x^2 + \frac{2}{6}\,x^3 + o(x^3).$$

(fonction de la forme $x^3\,\varepsilon(x)$ avec $\lim\limits_{\substack{x\to 0\\ x\neq 0}} \varepsilon(x) = 0$)

$$= x - \frac{1}{2}\,x^2 + \frac{1}{3}\,x^3 + o(x^3).$$

c) $\mathbf{f(x) = \dfrac{1}{1-x}}$ **,** **<u>a = 0</u>**



$$\frac{1}{1-x} = 1 + x + o(x)$$
$$\frac{1}{1-x} = 1 + x + x^2 + o(x^2)$$
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + o(x^3)$$

En effet :

$f(x) = \dfrac{1}{1-x} = (1 - x)^{-1}$

$f\,'(x) = (-1)\,(-1)\,(1 - x)^{-1-1} = (1 - x)^{-2}$ ($nu'u^{n-1}$)

$f\,''(x) = (-2)\,(-1)\,(1 + x)^{-2-1} = 2\,(1 - x)^{-3}$

$f\,'''(x) = 2(-3)\,(-1)\,(1 - x)^{-3-1} = 6\,(1 + x)^{-4}$

$f(0) = \dfrac{1}{1} = 1$

$f\,'(0) = (1)^{-2} = 1$

$f\,''(0) = 2\,(1)^{-2} = 2(1) = 2$

$f\,'''(0) = 6\,(1)^{-4} = 6(1) = 6$

$$f(x) = 1 + 1\,x + \frac{2}{2}\,x^2 + \frac{6}{6}\,x^3 + o(x^3).$$

(fonction de la forme $x^3\,\varepsilon(x)$ avec $\lim\limits_{\substack{x\to 0\\ x\neq 0}} \varepsilon(x) = 0$)

$$= 1 + x + x^2 + x^3 + o(x^3).$$