*Polytech'Orléans*

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***Institut***

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# Master AESM

Aesm 8

**VECTORIAL MODELING OF A SYNCHRONOUS MACHINE**

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**1er semester**

*Bruno Bonheur*

**VECTORIAL MODELING OF A SYNCHRONOUS MACHINE**

**1 OBJECTIVES**

In the traditional use of a synchronous machine, speed is given by the grid pulse 

In the case of passive electronic switching,  is variable but with non-autonomous inverter-type converters a new command must be given every or in phase.

The torque obtained is very similar to a stepper motor and can be changed only every 1/6th of the rotation period.

Ex : ENGINE 3000 rpm ⇒ 100 Π rd.s



This is the basic limitation. This is acceptable in electric rail traction because the time to reach speed is on the order of a minute, but this is too long in Robotics where the mechanical time constants are on the order of a few 10 ms. One must be able to control the pulse current ω at least 100 times more quickly, i.e. every 100 μs.

Ex : current wave 50 Hz ⇒ T = 20 ms. Number of points = points per period

I(t) = Î cos(ωt+ϕ) One must be able to modify the current by its amplitude and its phase ϕ(t). Thus the complex notation is more appropriate because the electrical system is more typically sinusoidal.

The objective of this course is therefore to create a new modeling of the machine in order to achieve instant knowledge of the machine via linear algebra.

**2 PRESENTATION OF THE TRANSFORMATION**

**2.1 Modeling assumptions**

- no magnetic saturation ⇒ the relations between flows and currents are linear

- skin effect neglected ⇒ the resistances do not depend on frequency

value of the so-called characteristic skin thickness

example 0.67 mm at 10Khz for copper

- capacitive coupling between windings neglected.

- no influence of temperature ⇒ 

- Iron losses not modeled

.

- non-salient pole rotor

- absence of Leblanc-type dampers.

**2.2 Transformation de Concordia**

# 2.2.1 Ferraris Theorem

This theorem states that for 3 coils placed at  powered by 3 currents constituting a three-phase system (a, b, c), a rotating field  is created in a cylindrical geometry such as that of the synchronous motor.

This property also holds for two-phase systems if 2 currents are defined such that:



Figure 1

Thus, for each rotating field there are these 2 visions:

The three-phase system is the real vision

The two-phase system is the result of a transformation where a real engine exists

**2.2.2 Definition of the Concordia transformation**

For vectors [X] can be voltages, currents, or flows

[X]s stator magnitudes

representation

in the actual base 

in the two-phase base  phase ξ = ωt + ϕ







Hence the natural **Concordia** construction



thus









Improvement of the transformation so that there is conservation of the norm  

Main remark: the non-square matrix can be surprising and can complicate a general vision of machines.

It is possible to define a more complete transformation in which a third component is introduced in α, β. This is the homopolar component indexed in 0, hence the complete transformation α, β, 0.



thus





Most of the time the homopolar component will be zero for star-coupled currents ⇒ iN = 0 = i0

For voltages, we are interested only in the fundamental.

For simplicity’s sake, we keep the passage of the former base matrix abc to the new base αβ.



Properties



**2.3 Rotation matrix**

This transformation makes it possible to translate from the rotating coordinate system to that of the rotor (d,q)



Figure 51

Coordinate system space time (constant speed)

θmech pθelectrical ωt = pΩt

pθ

Search for R such that 



The inverse matrix of a rotation is the inverse angular rotation.



constant speed vector [X] dq is a constant.



At constant speed, pθ = pΩt = ωt = ξ hence  is a vector whose components are the constants therefore a continuous current

 **2.4 Park Transformation**  

It is the association of the two transformations

a) If one wishes to impose the current coordinates in the coordinate system of the rotor, then these components are known. This can only be done by real magnitudes, thus 





This is the direct definition of Park as defined by Chatelain

It corresponds to the inverse definition as defined by Sequier.

b) If one wishes to calculate the components real dq by measuring the actual components of abc**,** it is necessary to find the inverse transform.

Measurement case (feedback). h

Multiply on the left by the inverse matrix 

The advantage of this linear modeling of a synchronous machine is that it is computerizable if θ is known.

3 blocks have to be developed:.

- **equations of the stator voltages in the coordinate system dq**

- **the torque from the current vector equations dq.**

- **mechanical equations depending on the load C.**

**3. Torque calculation**

**3.1 Voltage equations**

Equation in the coordinate system of **abc**.



Figure 52

Let us keep the expressions with the flow

Thus in the abc coordinate system:

 We wish to establish the same relationship to the coordinate system d, q with  because the three resistors of the rotor are

identical in the coordinate system αβ



 by commutativity on the real numbers

 all  are constant

hence  because the coefficients of T32 are constant,

Passage to the rotating coordinate system thanks to the rotation matrix [R(pθ)]

thus



d'où



in the 1st term, the matrices [Id] and t [R(pθ)] can commute, thus:



The second term is more difficult because the rotating matrix contains θ which depends on t the time



thus



Two calculations have to be performed

a)  



Hence by introducing these relations and multiplying on the left by [R(pθ)]-1, we get [Vs]dq.

 

Supplementary term

**3.2 Matrix of the stator flux in the dq coordinate system**

By definition



[L]abc has two forms:



a b

with main and mutual inductances a)

with cyclic inductances b (Behn Eshenburg model)

The inductances are constant with a non-salient pole rotor.

Don’t forget that there are other models for salient pole rotors

 go to [1] if necessary

adopt the 2nd form, because the matrix is diagonal

let us describe 

This is the flow from the rotor received by the stator windings. It is unnecessary to calculate these coefficients in abc because they are known in dq.

generally



Figure 4

Remark thus 

calculation



 is commutated with any matrix thus



 

**3.3 Power conservation**

The instantaneous power in the abc coordinate system is: 



= 

Identically  then

**3.4 Torque**

Hence the easy calculation in the d q coordinate system







Joule effect electromagnetic term magnetic energy



Remark 1 :

ΨsdΨsq are the virtual components of the stator flux

isd et isq are the virtual components of stator currents

at constant speed, θ = Ωt, then these quantities are constant, continuous in the electrical sense.

One can therefore express the torque based on continuous quantities

TE = kφIa DCmotor Form







Remark 2:

There remains a degree of freedom on the choice of the rotor coordinate system. If the coordinate system is such that ψrq = 0, i.e. if the coordinate system can be fixed on the poles of the rotor axis, we must have ⇒ control the field of the rotor to maintain ψrq = 0.

Make a drawing

 compare with kφIa

Ψrd is like the flow Φ

isq = plays the role of the induced current in the continuous motor:

Ψrd and isq can be time-dependent in transitional regimes

When autopiloting is implemented, Ψrd is always constant.

**3.5 Implementing autopiloting**:

It is necessary for the θ that identifies the axis of the rotor field to be calibrated by the phase currents.

Therefore, the system must therefore be adjusted before being switched on; this procedure is generally called autophasing. In modern systems it is an automatic procedure.



**Modèle vectoriel de la machine synchrone**



This is an example of command dq or it regulates the real currents in abc from the recorded instructions iq\* and id\* in the dq system of coordinates

From B Multon

Previous figure



θ

Cos θ

Sin θ

R(pθ)

T32