Licence EG - Semester 3 - 2022-2023

QUANTITATIVE TECHNIQUES

Taylor expansions (tutorial sheet)

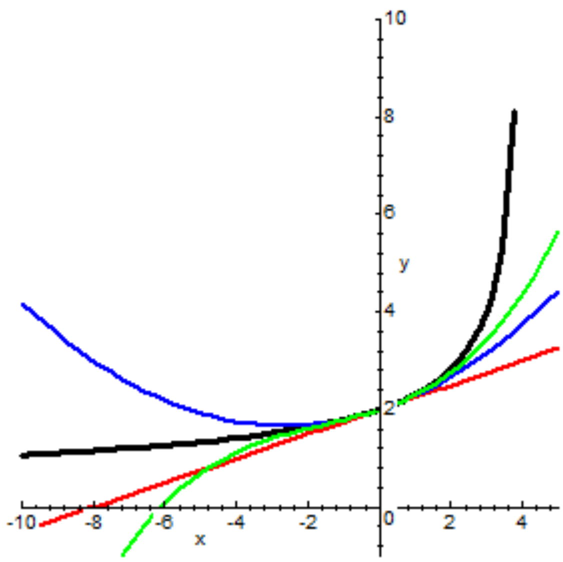
Exercise 1:

Using the Taylor-Young formula, find the third order Taylor expansion of the following functions about 0.

a) $f(x) = 3\ln(x+2)$ b) $g(x) = e^{1+3x} + \ln(2x+2)$ c) $h(x) = \sqrt{1-3x}$

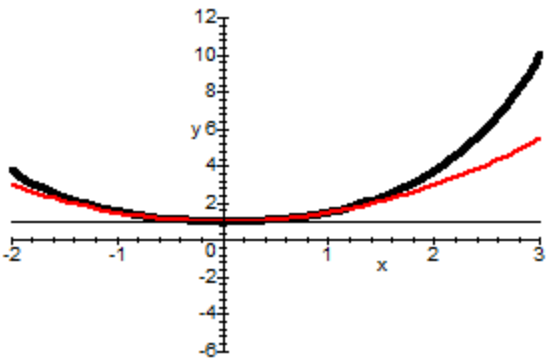
Exercise 2: Let f be the function defined for any x<4 by: $f(x) = \dfrac{4}{\sqrt{4-x}}$.

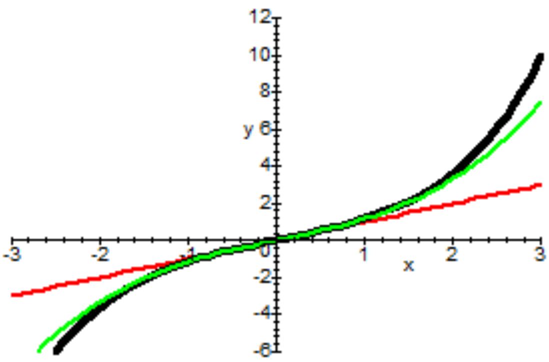
1) Using the Taylor-Young formula, check that the third order Taylor expansion of f about 0 is:
$$f(x) = 2 + \frac{1}{4}x + \frac{3}{64}x^2 + \frac{5}{512}x^3 + o_{x\to 0}(x^3).$$
2) Use 1) to deduce an approximation of the value $\dfrac{4}{\sqrt{4.08}}$.
3) Use 1) to deduce the third order Taylor expansions of $g(x) = \dfrac{4}{\sqrt{4+x}}$ and of $h(x) = \dfrac{1}{4+x}$ at x=0.



Exercise 3: *Hyperbolic Functions*.

Let cosh be the « hyperbolic cosine» and sinh be the « hyperbolic sine », i.e the functions from ℝ to ℝ





respectively defined by: $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$ and $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$.

Find the third order Taylor expansion of cosh and sinh about 0.

Exercise 5:

1) Using Taylor expansions of common functions, find the third order T.E of the following functions at x=0:

$f_1(x) = e^{-3x+5}$, $\quad f_2(x) = \dfrac{1}{1 - x^2}$, $\quad f_3(x) = \dfrac{1}{1 - x^3}$, $\quad f_4(x) = e^{3x}\ln(3 - 4x)$,

$g(x) = 4 - 2x + x^2 + 2x^3 + 8x^4$,

$t(x) = \dfrac{\ln(1 + x^2)}{2x + 5}$, $\quad v(x) = \dfrac{4 - 2x + x^2 + 2x^3 + 8x^4}{2x + 5}$.

2) For the function f_2 above, give the equation of the tangent line (T_2) of C_{f_2} at the point $(0;f_2(0))$ and a local graphical representation of C_{f_2} near this point. Do the same with f_3.

Exercise 6: Using appropriate common Taylor expansions, determine the following limits:

1) $\displaystyle\lim_{x\to 0} \frac{\ln(1 + x)}{x}$ $\qquad$ 2) $\displaystyle\lim_{x\to 0} \frac{e^x - 1}{x}$ $\qquad$ 3) $\displaystyle\lim_{x\to 0} \frac{\cosh(x) - 1}{x^2}$